Gaussian Processes for Information-theoretic Robotic Mapping and Exploration

by

Maani Ghaffari Jadidi

Submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

at the
Centre for Autonomous Systems
Faculty of Engineering and Information Technology
University of Technology, Sydney

March 2017

Declaration of Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signed:

Date:

Gaussian Processes for Information-theoretic Robotic Mapping and Exploration

by

Maani Ghaffari Jadidi

Submitted to the Faculty of Engineering and Information Technology
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Abstract

This thesis proposes a framework for autonomous robotic mapping, exploration, and planning that uses Gaussian Processes (GPs) to model high-dimensional dense maps and solve the problem of infinite-horizon planning with imperfect state information.

Robotic exploration is traditionally implemented using occupancy grid representations and geometric targets known as frontiers. The occupancy grid representation relies on the assumption of independence between grid cells and ignores structural correlations present in the environment. We develop an incremental GP occupancy mapping technique that is computationally tractable for online map building and represents a continuous model of uncertainty over the map spatial coordinates. The standard way to represent geometric frontiers extracted from occupancy maps is to assign binary values to each grid cell. We extend this notion to novel probabilistic frontier maps computed efficiently using the gradient of the GP occupancy map and propose a mutual information-based greedy exploration technique built on that representation. A primary motivation is the fact that high-dimensional map inference requires fewer observations, leading to a faster map entropy reduction during exploration for map building scenarios.

The uncertainty from pose estimation is often ignored during current mapping strategies as the dense belief representation of occupancy maps makes the uncertainty propagation impractical.

Additionally, when kernel methods are applied, such maps tend to model structural shapes of the environment with excessive smoothness. We show how the incremental GP occupancy mapping technique can be extended to accept uncertain robot poses and mitigate the excessive smoothness problem using Warped Gaussian Processes. This approach can model non-Gaussian noise in the observation space and capture the possible non-linearity in that space better than standard GPs.

Finally, we develop a sampling-based information gathering planner, with an information-theoretic convergence, which allows dense belief representations. The planner takes the present uncertainty in state estimation into account and provides a general framework for robotic exploration in *a priori* unknown environments with an information-theoretic stopping criterion. The developed framework relaxes the need for any state or action space discretization and is a fully information-driven integrated navigation technique.

The developed framework can be applied to a large number of scenarios where the robot is tasked to perform exploration and information gathering simultaneously. The developed algorithms in this thesis are implemented and evaluated using simulated and experimental datasets and are publicly available as open source libraries.

Thesis Supervisors: Jaime Valls Miro and Gamini Dissanayake

Acknowledgements

First and foremost I would like to express my sincere gratitude to my supervisors, Prof. Jaime Valls Miro and Prof. Gamini Dissanayake, for being enthusiastic in guiding my research, for the freedom to learn and explore exciting topics, and for their patience. I hope I continue to collaborate with you in the future. I acknowledge the scholarships to support my research and make my study possible.

Besides my advisors, I would like to thank Dr. Teresa Vidal Calleja, Prof. Shoudong Huang, Prof. Juan Andrade Cetto, and Dr. Rafael Valencia for their guidance and friendship. I learned a lot from our weekly SLAM meetings organized by Shoudong and Teresa. Collaborating with Juan and Rafael during my first year was prolific.

I am deeply grateful to my external examiners, Professors Geoffrey Hollinger and Jon Kim, for reviewing this thesis, their comments, suggestions, and explanations.

My sincere thanks also goes to Prof. Kazuhiro Nakadai and Dr. Keisuke Nakamura, who provided me an opportunity to join their team as an intern at HRI-JP and experience living in Japan. During my six month visit, I made great friends and memories. I also thank Dr. Mitesh Patel for hosting me at FXPAL as an intern; I enjoyed working in a friendly environment and living in California.

I would like to thank my friends and colleagues at the Centre for Autonomous Systems for the numerous stimulating and helpful discussions and their friendship. It has been great living in beautiful Sydney and spending time with all of them. Special thanks to my dear friend Kasra Khosoussi who has been like a brother to me during the past five years. I guess we've also had the same number of fights I've had with my brothers back home! Thanks to my labmate Raphael Falque for always finding a way to be next to me, despite changing my desk two times plus moving to a new building. Over time, we have developed a non-verbal language that surprises others. Needless to say, the 2015 IROS together and three weeks traveling in Europe are among our memorable times.

Thanks to Lakshitha Dantanarayana AKA Lakipedia, for his friendship and vast knowledge on nearly anything available on the Internet. I thank my friends and labmates Gibson Hu, Andrew To, Buddhi Wijerathna, Mohammad Norouzi, Daobilige Su, Mahdi Hassan, Phillip Quin, Kanzhi

Wu, Liye Sun, Liang Zhao, Alen Alempijevic, Bradley Skinner, Teng Zhang, Peter Ward, Nalika Ulapane, Linh Van Nguyen, Asok Aravinda, Julien Collart, Alexander Virgona, Dinuka Abeywardena, Lasitha Piyathilaka, Leo Shi, James Poon, Brendan Emery, Katherine Waldron, Cédric Le Gentil, Karthick Thiyagarajan, David Valiente Garcia, and many others.

I would also like to thank my former supervisors, Prof. Morteza Mousakhani, Dr. Ehsan Hashemi, Prof. Alireza Mohammad Shahri, and Prof. Houman Sadjadian for their guidance and encouraging me to pursue my studies.

I thank my parents and my brothers, Navid and Houtan, for their unconditional love and care. I would not have made it this far without them.

Contents

List of Figures

List of Tables

List of Algorithms

Acronyms

2D	Two-Dimensional. 2, 36
APS	Active Pose SLAM. 117, 118, 128, 129
ARD	Automatic Relevance Determination. 80
ASV	Autonomous Surface Vehicle. 110
AUC	the Area Under the receiver operating characteristic Curve. 43, 44, 79
BCM	Bayesian Committee Machine. 15, 42–45, 103
COM	Continuous Occupancy Map. 2, 35, 39, 41, 128, 130
EK	Expected Kernel. 79–81, 87
EKF	Extended Kalman Filter. 9, 11
ESM	Expected Sub-Map. 79–81, 87
GP	Gaussian Process. 2, 3, 5, 15, 16, 24, 26, 27, 32, 33, 35, 37–43, 49, 51, 70, 71, 73, 74, 77, 79, 87, 97, 103, 119–121, 125, 126, 129, 130
GPOM	Gaussian Processes Occupancy Map. 15, 16, 41, 49–52, 56, 60, 68, 72, 73, 79–81, 84, 86, 87, 125

RIC	Relative Information Contribution. 96
RIG	Rapidly-exploring Information Gathering. 89–92, 94–97, 110, 124
ROS	Robot Operating System. 27
SE	Squared Exponential. 26, 27, 37, 74, 80, 120
SLAM	Simultaneous Localization And Mapping. 2–4, 8, 9, 12, 13, 29, 30, 69
UGPVR	Uncertain Gaussian Processes Variance Reduction. 98, 104, 110–114, 124
WGP	Warped Gaussian Process. 70, 87
WGPOM	Warped Gaussian Processes Occupancy Map. 72, 73, 79–81, 84, 86, 87, 125

Nomenclature

t	Time step
$\approx$	Approximately equal
$\sim$	Distributed according to
$\triangleq$	Definition
$\varnothing$	Empty set
$\backslash$	Matrix left division
SE(2)	2D special Euclidean group
SO(2)	2D special orthogonal group
$R(\cdot)$	Rotation matrix
$\lvert\cdot\rvert$	Absolute value
a_t	Action at time step t
b_{occ}	Belief for the occupied map point
b_{free}	Belief for the unoccupied map point
$\mathcal{O}(\cdot)$	Big O notation
$\lambda^{[i]}$	Bounded information associated with location i
b_t	Budget at time step t
b	Budget

l	Characteristic length-scale
y_+	Class label for occupied points
y_-	Class label for unoccupied points
$f_c(\cdot)$	Cost function
$k(\cdot,\cdot)$	Covariance function of Gaussian processes
$\boldsymbol{K}$	Covariance matrix of Gaussian processes
$\mathbb{Cov}[\cdot]$	Covariance of random variables/vectors
$\lvert\cdot\rvert$	Determinant of a matrix
$h(\cdot)$	Differential entropy of a random variable
$H(\cdot)$	Entropy of a random variable
$\lVert\cdot\rVert$	Euclidean norm
$\mathbb{E}[\cdot]$	Expected value of a random variable
$\tilde{k}(\cdot,\cdot)$	Expected/Modified covariance function
$\mathcal{X}_f$	Free workspace
$\Gamma(\cdot)$	Gamma function
H_n	Hermite polynomials
$\mathcal{H}$	Hypothesis
$\boldsymbol{I}_n$	Identity matrix of size n
$f_I(\cdot)$	Information function
α	Information gain factor
$\boldsymbol{f}_*$	Latent variable of Gaussian processes
$\int(\cdot)$	Lebesgue integral

γ	Logistic regression classifier weight
$m^{[i]}$	Map occupancy status at location i
h_{sat}	Map saturation entropy
p_{sat}	Map saturation probability
δ_{map}	Map spatial resolution
$\boldsymbol{X}_f^{[k]}$	Matrix of sampled unoccupied points along the k-th sensor beam
$\max(\cdot)$	Maximal element of a set
$f_m(\cdot)$	Mean function of Gaussian processes
$\boldsymbol{z}_t^{[k]}$	Measurement at time step t by the k-th sensor beam
$\boldsymbol{z}_t$	Measurement at time step t
$\alpha_t^{[k]}$	Measurement bearing at time step t by the k-th sensor beam
$r_t^{[k]}$	Measurement range at time step t by the k-th sensor beam
$\boldsymbol{z}_{1:t}$	Measurements up to time step t
$\min(\cdot)$	Minimal element of a set
$K_\nu(\cdot)$	Modified Bessel function of the second kind of order ν
$\hat{I}(\cdot;\cdot)$	Mutual information approximation between two random variables
$I(\cdot;\cdot)$	Mutual information between two random variables
$\log(\cdot)$	Natural logarithm
n_i	Number of inducing points
n_p	Number of map points in the current perception field of the robot
n_m	Number of map points
n_q	Number of query points

n_z	Number of range-finder sensor beams per observation
n_s	Number of samples
n_f	Number of unoccupied sampled points
$h(\cdot,\cdot)$	Observation model
$\boldsymbol{x}_o^{[k]}$	Observed occupied point by the k-th sensor beam
$\boldsymbol{x}_o^{[k,i]}$	Observed occupied point's i-th dimension by the k-th sensor beam
β	Occupied boundaries factor
p_o	Occupied probability threshold
$a_t^\star$	Optimal action at time step t
$\mathcal{P}_t^\star$	Optimal trajectory at time step t
I_{RIC}	Penalized relative information contribution
$\mathcal{I}_t^{[k]}$	Perception field of the k-th sensor beam at any robot location at time step t
$\mathcal{G}$	Planning graph
T	Planning horizon
$\mathcal{T}$	Planning tree
$\sigma_{X_i\|Z}$	Posterior marginal variance of random variable X_i after taking observation Z
σ_{X_i}	Prior marginal variance of random variable X_i
$p(\cdot)$	Probability measure
$\boldsymbol{x}_*$	Query point
Z_t	Random variable for measurement at time step t
M	Random variable for the occupancy map
S	Random variable for the state

δ_{RIC}	Relative information contribution threshold
s_z	Resolution of the numerical integration
$\boldsymbol{Q}$	Robot motion noise covariance
$\boldsymbol{x}_t$	Robot pose at time step t
$\boldsymbol{x}_{1:t}$	Robot trajectory up to time step t
$\mathbb{Z}$	Set of integers
$\mathbb{N}$	Set of natural numbers
$\mathbb{R}_{\geq 0}$	Set of non-negative real numbers
$\mathcal{A}_t$	Set of possible actions at time step t
$\mathcal{A}$	Set of possible actions
$\mathcal{M}$	Set of possible occupancy maps
$\mathcal{Z}$	Set of possible range measurements
$\mathcal{S}$	Set of possible states
$\mathbb{R}$	Set of real numbers
$\mathcal{D}$	Set of training data
$\prod(\cdot)$	Set product
$\sum(\cdot)$	Set summation
$\operatorname{sgn}(\cdot)$	Sign function
σ_f^2	Signal variance
n_t	Size of training data
$u(\cdot)$	Total utility function
$\operatorname{tr}(\cdot)$	Trace of a matrix

$\mathcal{P}_t$	Trajectory at time step t
$\mathcal{P}$	Trajectory
$(\cdot)^T$	Transpose of a matrix
p_f	Unoccupied probability threshold
$\mathbb{V}[\cdot]$	Variance of a random variable
σ_n^2	Variance of the observation noise
$\boldsymbol{\psi}$	Vector of hyperparameters of a warping function
$\boldsymbol{\theta}$	Vector of hyperparameters
$\boldsymbol{t}$	Vector of latent targets
$\boldsymbol{y}$	Vector of targets
$g_w(\cdot)$	Warping function
$\mathcal{X}$	Workspace