Linear Physical-layer Network Coding for the Fading Y-channel without Transmitter Channel State Information

Jiajia Guo[1], Tao Yang[2], Jinhong Yuan[1], and Jian A. Zhang[3]
[1]University of New South Wales, Sydney, Australia, [2]University of Technology Sydney, Sydney, Australia,
[3]Digital productivity flagship, CSIRO, Sydney, Australia
Email: jiajia.guo@students.unsw.edu.au; Andrew.Zhang@csiro.au;

Abstract—In this paper, we propose a new linear physical-layer network coding (NC) scheme for the fading Y-channel, assuming that the channel state information (CSI) is not available at transmitters. In this scheme, each user transmits one message to a relay and intends to obtain both other two users' messages. Based on the receiver-side CSI, the relay determines two NC generator vectors for linear network coding, and reconstructs the associated two linear NC codewords. For the case when there is one time-slot in the uplink phase, we present an explicit solution for the generator vectors that minimizes the error probability at a high SNR, and a lower bound of the error performance of the proposed scheme using our optimized generator vectors. Extending to multiple time-slots in the uplink, two typical scenarios are discussed. Numerical results show that the proposed scheme significantly outperforms existing schemes, and match well with our analytical results.

I. INTRODUCTION

Multi-way relay channels have drawn many attentions in recent studies. Multi-way communications were first studied by Shannon in [1] where a two-way channel was considered. By combining multi-way communications and relaying, the multi-way relay channel was studied in [2] in a multi-cast scenario. A similar setup, where all users belong to the same cluster and all channel gains are equal was studied in [3], and the sum-capacity of this Gaussian setup with more than two users was obtained. A broadcast variant of this multi-way relaying setup, the so called Y-channel, was considered in [4] [5] [6] [7]. In the Y-channel, three users attempt to communicate with each other via a relay. Each user sends two independent messages, one to each other user.

Current studies on the Y-channel intensely focused on achievable degrees of freedom using signal alignment [5] [6] [7]. For example, in [5], a transmission scheme exploiting interference alignment was proposed, and its corresponding achievable degrees of freedom were calculated. Note that the capacity of the Y-channel is not known in general. Most existing works, especially those on the degrees of freedom for Y-channel, require global channel state information (CSI) at each node. Unfortunately, global CSI requires considerable overhead cost in real networks. which makes this assumption quite strong in practice.

Physical-layer network coding (PNC) has been recognized as a powerful tool to improve the throughput and reliability of multi-user wireless communication networks [8] [9]. Designing a practical Y-channel PNC scheme without transmitter-side CSI (CSIT) is an important but challenging task. This is mainly due to the existence of carrier-phase offset between all the users. In such a scenario, the error probability performance of the conventional PNC can be significantly degraded. There have been several PNC works not requiring CSIT for the two-way reley channel (TWRC) [10] [11]. For example, a linear PNC scheme was proposed for TWRCs without CSIT [12], [13]. It focused on the optimization of the PNC constellation by exploiting the structure of linear PNC mapping. Yet, results on the practical coding and modulation design for the Y-channel scheme without CSIT remain limited.

In this paper, we propose a practical PNC scheme for a Y-channel system with three users and one relay where the transmitters have no CSI and each node has a single antenna. Throughout the paper, we focus on the case where each user intends to realize full data exchange with all other users, i.e., each user wants all messages from all other users. Each user in our model is thus a source of one messages and a destination of two messages.

We also present an explicit solution for the generator vectors that minimize the error probability at a high SNR, and analyze the lower bound of the error performance at the relay. Numerical results show that our proposed method obtains about 3 dB over conventional schemes at a high SNR, and match well with our analytical results. We also extend the results from one to multiple time-slots in the uplink by considering two typical scenarios and making comparisons of their error performances and computation complexities.

II. SYSTEM MODEL

In this paper, we consider a typical Y-channel system consisting of three users A, B, C and one relay where each node has a single antenna as shown in in Fig. 1. Each round of information exchange runs in two stages, referred to as the uplink phase and the downlink phase in the standard protocol of PNC. A flat block fading channel for both uplink and downlink is considered throughout the paper.

A. Uplink

Suppose that T time-slots are used for the uplink phase. Note that we will not consider the case where $T > 2$. This is because for $T > 2$, the relay will be able to well distinguish the three users' messages based on its observations in three or more time-slots and this is not the appealing zone of network coding. Also, it is of more interest when we use as less time-slots as possible.

Denote the signals from user l, $l \in \{A, B, C\}$, by a length-T signal vector $\mathbf{x}_l$ with its average power normalized to one. The
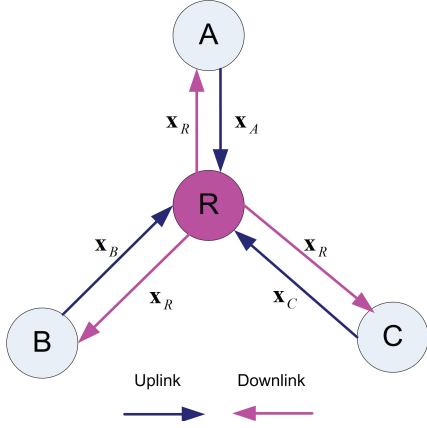
Fig. 1. System model for the Y-channel.

fading channel coefficient between user l and the relay is denoted by h_l. The received signal vector at the relay, denoted by $\mathbf{y}$, is

$$\mathbf{y} = \sqrt{E_s}h_A\mathbf{x}_A + \sqrt{E_s}h_B\mathbf{x}_B + \sqrt{E_s}h_C\mathbf{x}_C + \mathbf{n}. \quad (1)$$

where E_s is the average transmitted power per symbol vector, and $\mathbf{n}$ is the additive white Gaussian noise (AWGN) vector at the relay whose entries are i.i.d. with zero mean and variance σ_z^2. The signal-to-noise ratio (SNR) is defined as $\rho = E_s/\sigma_z^2$.

Remark 1 (Degree of CSI): In this paper, we assume that CSI is not available at the transmitters, and is perfectly known at the intended receivers. This applies to practical scenarios where there is no feedback from the receiver to the transmitter or the channel reciprocity does not hold.

B. Downlink

Upon receiving $\mathbf{y}$, the relay generates a signal vector $\mathbf{x}_R$ which contains some function of the three users' signals, denoted by

$$\mathbf{x}_R = f\left(\mathbf{x}_A, \mathbf{x}_B, \mathbf{x}_C\right). \quad (2)$$

This signal vector $\mathbf{x}_R$ is then broadcasted to the three users. Here, note that $f(\cdot)$ could be some linear functions [11] or non-linear functions [8] [10] of the three users' messages. In this paper, we focus on linear functions due to its low computational complexity and scalability as we will see later.

In the downlink phase, upon receiving $\mathbf{x}_R$, each user extracts the other two users' messages with the help of the perfect knowledge of its own message. This finishes one round of message exchange.

III. PROPOSED LINEAR NETWORK CODING SCHEME

A. Users

Each user l has a message w_l, $l \in \{A, B, C\}$, which are drawn from a finite field $GF(q)$.

Recall that we will not consider the case where $T > 2$. Thus, we propose our linear network coding schemes for $T = 1$ and $T = 2$ respectively as follows.
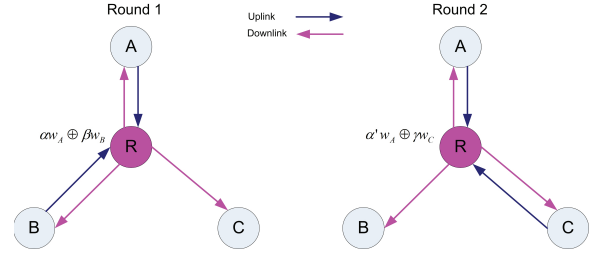

Fig. 2. TDD-TWRC Scheme for the Y-channel

1) $T = 1$: Using quadrature amplitude modulation (QAM) , user l's transmitted message w_l is one-to-one mapped to a modulated symbol x_l by

$$x_l = \frac{1}{\gamma}\left(w_l - \left(\frac{q-1}{2} + j\frac{q-1}{2}\right)\right) \quad (3)$$

where γ is a normalization factor which ensures that $E\left(|x_l^2|\right) = 1$. Note that this mapping is on real number field.

In the uplink phase, users transmit their signals simultaneously to the relay. The received signal at the relay is

$$y = \sqrt{E_s}h_Ax_A + \sqrt{E_s}h_Bx_B + \sqrt{E_s}h_Cx_C + n \quad (4)$$

where $n \sim \mathcal{N}\left(0, \sigma_z^2\right)$.

2) $T = 2$: Here, we propose two options for the case where two time-slots are available in the uplink phase.

a) Option 1: Separate Transmission: Since we consider a flat block fading channel with a single antenna at each node, there is no diversity we can explore by transmitting in different time-slots. By introducing more time-slots, a basic thought is to minimize the interference between all users' transmitted signals.

Then, we have an intuitive uplink strategy as follows. Only one user transmits in the first time-slot and the rest two users transmit simultaneously in the second time-slot. For example, the received signals at the relay in each time-slot are

$$y_1 = \sqrt{E_s}h_Ax_A + n_1, \quad (5)$$

and

$$y_2 = \sqrt{E_s}h_Bx_B + \sqrt{E_s}h_Cx_C + n_2 \quad (6)$$

respectively.

b) Option 2: TDD-TWRC: We next consider a time-division duplex (TDD) TWRC scheme, where only two users transmit signals at each time-slot in the uplink phase as shown in Fig. 2.

Considering the power constrain for each user ($E\left(\|\mathbf{x}_l^2\|\right) = 1$), the received signals at the relay are

$$y_1 = \frac{\sqrt{2}}{2}\sqrt{E_s}h_Ax_A + \sqrt{E_s}h_Bx_B + n_1, \quad (7)$$

$$y_2 = \frac{\sqrt{2}}{2}\sqrt{E_s}h_Ax_A + \sqrt{E_s}h_Cx_C + n_2. \quad (8)$$

B. Relay

Based on the received signals and its receiver-side CSI , the relay selects two network-coding coefficients v_{l_1} and v_{l_2} for each user, $v_{l_1}, v_{l_2} \in GF(q)$. It then generates the corresponding NC codewords u_1 and u_2 where

$$u_1 = (v_{A_1} \otimes w_A) \oplus (v_{B_1} \otimes w_B) \oplus (v_{C_1} \otimes w_C), \quad (9)$$

and

$$u_2 = (v_{A_2} \otimes w_A) \oplus (v_{B_2} \otimes w_B) \oplus (v_{C_2} \otimes w_C). \quad (10)$$

We rewrite the above expressions using vectors as

$$\begin{aligned}
\mathbf{u} &= \begin{bmatrix} v_{A_1} & v_{B_1} & v_{C_1} \\ v_{A_2} & v_{B_2} & v_{C_2} \end{bmatrix} \begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B & \mathbf{v}_C \end{bmatrix} \begin{bmatrix} w_A & w_B & w_C \end{bmatrix}^T \\
&= (\mathbf{v}_A \otimes w_A) \oplus (\mathbf{v}_B \otimes w_B) \oplus (\mathbf{v}_C \otimes w_C), \quad (11)
\end{aligned}$$

where $\mathbf{u}$ is referred to as the *linear NC codeword* and vectors $\mathbf{v}_A$, $\mathbf{v}_B$, $\mathbf{v}_C$ are referred to as the *NC generator vectors*. Note that the additions and multiplications above are operated on the finite field $GF(q)$. There have been works using non-linear PNC methods [8] [10]. Here, we choose a linear PNC scheme because of its low computational complexity and scalability.

Note that in order for each user to recover all the other users' messages, the NC coefficient vectors must satisfy the following condition.

Condition 1 (Full-Rank Condition): Matrices $\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B \end{bmatrix}$, $\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_C \end{bmatrix}$ and $\begin{bmatrix} \mathbf{v}_B & \mathbf{v}_C \end{bmatrix}$ must be full rank.

In the uplink phase, the relay attempts to re-construct the linear NC codeword $\mathbf{u}$ in (11) w.r.t. to the selected NC coefficient vectors. Denote the decision by $\hat{\mathbf{u}}$.

Remark 2: The error probability in the uplink phase, also referred to as the *NC error probability*, is defined as follows.

$$P_e^{NC} \triangleq \Pr\{\mathbf{u} \neq \hat{\mathbf{u}}\} \quad (12)$$

There are various NC generator vectors satisfying the Full-Rank Condition. It is important to note that the selection of $\mathbf{v}_l$ is critical for the NC error probability performance.

Remark 3: In practice, since the NC generator vectors only need to be selected once for each channel realization, the resultant overhead is negligible for a slow fading channel of a reasonably large channel coherence time. We assume that the selected generator vectors are delivered to all the three users via reliable links.

Remark 4: For the schemes where the CSI is assumed to be known to all transmitters [5] [6], joint precoding is usually employed to align the signal directions of the two users. However, the strict signal direction alignment requires very accurate CSI feedback from the relay and it is very difficult to realize in practice due to the CPO. In addition, the delivering of the continuous-valued full CSI from the relay to the users requires a much larger overhead than that in our proposed scheme, which only sends the index of the NC generator vectors.

C. Downlink

In the downlink phase, the relay modulates $\hat{\mathbf{u}}$ and broadcasts it to all users. Suppose that messages u_1 and u_2 are generated and delivered to user l correctly.

User A first removes its own messages $\mathbf{w}_A$ from $\mathbf{u}$, and obtains the resultant message vector

$$\mathbf{u} \ominus (\mathbf{v}_A \otimes w_A) = (\mathbf{v}_B \otimes w_B) \oplus (\mathbf{v}_C \otimes w_C),$$

which forms a set of linear equations for w_B and w_C. Recall that matrix $\begin{bmatrix} \mathbf{v}_B & \mathbf{v}_C \end{bmatrix}$ is of full rank from the Full-Rank Condition.

By solving the linear equations, user A can recover the desired messages from user B and user C. User B and user C resolve their intended messages in a similar way.

Remark 5: In this paper we focus on the NC error performance for the uplink phase as the end-to-end error performance is subject to the NC error probability in the uplink phase. This is because the downlink phase is a standard point-to-point transmission and its error performance remains the same for any NC generator vectors selected at the relay. Also, from the PNC design point of view, we are most interested in the NC error performance in the uplink phase.

IV. DESIGN OF LINEAR PHYSICAL-LAYER NETWORK CODING

A. Problem formulation

This paper aims at designing the NC generator vectors $\mathbf{v}_l$ that can achieve the optimal NC error performance. The problem can be formulated as

$$\mathbf{v}_A^{opt}, \mathbf{v}_B^{opt}, \mathbf{v}_C^{opt} = \arg \min_{\mathbf{v}_A, \mathbf{v}_B, \mathbf{v}_C} P_e^{NC}, \text{ s.t. Condition1} \quad (13)$$

Consider a given channel realization of h_A, h_B and h_C. Define

$$w_s \triangleq h_A w_A + h_B w_B + h_C w_C, \quad (14)$$

which is referred to as a *superimposed* (SI) *symbol*. Note that a SI symbol w_s is in the real number field. Let w_s and w_s' be two different SI symbols, and the squared Euclidean distance between them is

$$d = \|w_s - w_s'\|^2. \quad (15)$$

Denote the minimum distance between any two SI symbols by d_1. We next define a minimum set-distance (MSD) between two SI symbols w.r.t. different NC codewords as

$$d_{MSD} \triangleq \min_{\substack{\mathbf{w}_s, \mathbf{w}_s' \\ (\mathbf{v}_A \otimes w_A) \oplus (\mathbf{v}_B \otimes w_B) \oplus (\mathbf{v}_C \otimes w_C) \\ \neq (\mathbf{v}_A \otimes w_A') \oplus (\mathbf{v}_B \otimes w_B') \oplus (\mathbf{v}_C \otimes w_C')}} d. \quad (16)$$

We see that $d_{MSD} \geq d_1$. It is important to note that at a high SNR, the NC error probability P_e^{NC} is dominated by the minimum set-distance d_{MSD} between two SI symbols whose underlying NC codewords $\mathbf{u}$ are different [13]. The problem in (13) now becomes

$$\mathbf{v}_A^{opt}, \mathbf{v}_B^{opt}, \mathbf{v}_C^{opt} = \arg \max_{\mathbf{v}_A, \mathbf{v}_B, \mathbf{v}_C} d_{MSD}, \text{ s.t. Condition1} \quad (17)$$

We next investigate the solution to this problem.

B. Solution to (13)

1) $T = 1$: For two different SI symbols w_s and w_s', let $\delta_A = w_A - w_A'$, $\delta_B = w_B - w_B'$, $\delta_C = w_C - w_C'$ and vector $\boldsymbol{\delta} = \begin{bmatrix} \delta_A & \delta_B & \delta_C \end{bmatrix}^T$ is referred to as the *difference vector* (DV) of w_s and w_s'. The squared Euclidean distance between w_s and w_s' is

$$d = \left\| \begin{bmatrix} h_A & h_B & h_C \end{bmatrix} \boldsymbol{\delta} \right\|^2. \quad (18)$$

In the light of the NC method in [13], we first find the DV $\boldsymbol{\Delta}$ that corresponds to the minimum distance of SI symbols, i.e.,

$$\boldsymbol{\Delta} = \arg \min_{|\delta_A| \neq 0, |\delta_B| \neq 0, |\delta_C| \neq 0} \left\| \begin{bmatrix} h_A & h_B & h_C \end{bmatrix} \boldsymbol{\delta} \right\|^2. \quad (19)$$

Then we obtain the solution to (13) as follows.

Theorem 1: As $\rho \to \infty$, the NC generator vectors $\mathbf{v}_A, \mathbf{v}_B, \mathbf{v}_C$ that minimizes the NC error probability satisfies

$$\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B & \mathbf{v}_C \end{bmatrix} \otimes \mathrm{mod}(\boldsymbol{\Delta}, q) = 0. \qquad (20)$$

Proof. In general, there exists only one vector $\boldsymbol{\Delta}$ that corresponds to the minimum distance d_1. (Here, $\boldsymbol{\Delta}$ and $-\boldsymbol{\Delta}$ are viewed as the same vector.). Using the distance clustering method in [13], let the NC generator vectors $\mathbf{v}_A, \mathbf{v}_B, \mathbf{v}_C$ satisfy

$$\mathrm{mod}(\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B & \mathbf{v}_C \end{bmatrix} \otimes (\begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} \pm \boldsymbol{\Delta}), q)$$

$$= \mathrm{mod}(\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B & \mathbf{v}_C \end{bmatrix} \otimes \begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix}, q), \qquad (21)$$

or equivalently

$$\begin{bmatrix} \mathbf{v}_A & \mathbf{v}_B & \mathbf{v}_C \end{bmatrix} \otimes \mathrm{mod}(\boldsymbol{\Delta}, q) = 0. \qquad (22)$$

This will lead to that any two SI symbol vectors with distance d_1 are *clustered*, and they will correspond to the same linear NC codeword. Therefore, the minimum set-distance satisfies $d_{MSD} > d_1$.

On the other hand, it can be shown that any choice of NC generator vectors not satisfying (22) will cause that any two SI symbol vectors with distance d_1 are *not clustered*, and in this case, d_{MSD} strictly equals to d_1. Therefore, we see that d_{MSD} is maximized through (22). This completes the proof. ∎

2) $T = 2$: Next, we present solutions to the two options described in Section III.A.2) respectively as follows.

a) Option 1: Separate Transmission: In this case, the minimum set-distance d_{MSD} can be maximized by properly clustering the superimposed symbols of user B's and user C's messages.

First, the classic maximum-likelihood single-user detection can be performed based on the received signal in the first time-slot y_1. Then, the linear NC method for TWRCs [13] can be performed based on the received signal in the second time-slot y_2. This method has been shown to maximize the d_{MSD} between the superimposed symbols of two users' messages. Denote the selected NC coefficient for user B and user C by β and γ respectively, $\beta, \gamma \in GF(q)$, $\beta \neq 0$, $\gamma \neq 0$. Then, we have the NC generator vectors as follows.

$$\mathbf{v}_A = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \mathbf{v}_B = \begin{bmatrix} 0 & \beta \end{bmatrix}^T, \mathbf{v}_C = \begin{bmatrix} 0 & \gamma \end{bmatrix}^T. \quad (23)$$

It can be easily verified that they satisfy the Full-Rank Condition.

Remark 6: Option 1 simplifies the PNC problem of three users to that of two users, and utilizes existing PNC work for TWRCs. Its computation complexity is in the order of q^4.

b) Option 2: TDD-TWRC: We can apply two types of relay operations for Option 2 as follows.

- Separate Processing

Since the received signal at each time-slot is a superposition of two user's signals, we can apply the networking method (for example, [13]) for TWRCs respectively for each time-slot.

Denote the optimized NC coefficients by α and β, α' and γ respectively, $\alpha, \beta, \alpha', \gamma \in GF(q)$, $\alpha \neq 0$, $\beta \neq 0$, $\alpha' \neq 0$, $\gamma \neq 0$. The NC generator vectors are

$$\mathbf{v}_A = \begin{bmatrix} \alpha & \alpha' \end{bmatrix}^T, \mathbf{v}_B = \begin{bmatrix} \beta & 0 \end{bmatrix}^T, \mathbf{v}_C = \begin{bmatrix} 0 & \gamma \end{bmatrix}^T, \qquad (24)$$

and they also satisfy the Full-Rank Condition.

Remark 7: Similar to Option 1, the separate processing for Option 2 also simplifies the PNC problem of three users to that of two users. Its computation complexity is also in the order of q^4. However, the computation expense is at least twice that of Option 1, as the linear NC method for TWRCs have to be performed twice for Option 2.

- Improvement using Joint Processing

We can also jointly process the the received signal in two time-slots. Similar to Theorem 1, we first find the DV vector that corresponds to the minimum distance of SI symbols, i.e.,

$$\boldsymbol{\Delta} = \underset{|\delta_A| \neq 0, |\delta_B| \neq 0, |\delta_C| \neq 0}{\arg \min} \left\| \begin{bmatrix} \frac{\sqrt{2}}{2} h_A & h_B & 0 \\ \frac{\sqrt{2}}{2} h_A & 0 & h_C \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \\ \delta_C \end{bmatrix} \right\|. \qquad (25)$$

Then, we determine the NC coefficients by solving (20).

Remark 8: The joint processing for Option 2 utilizes the benefits of joint processing and Theorem 1. However, its computation complexity is in the order of q^6.

We will compare the error performances for these options in Section VI via simulations.

V. PERFORMANCE ANALYSIS OF THE PROPOSED SCHEME

In this section, we present a lower bound of the average (over all channel realizations) error probability of our proposed scheme at a high SNR when $T = 1$.

Consider a single user scheme where only user l transmits to the relay, i.e.,

$$y = h_l x_l + n. \qquad (26)$$

and denote the average error probability by $P_e^l \triangleq \{\widehat{w}_l \neq w_l\}$.

Consider a two-user scheme where two users l and k transmit to the relay simultaneously, i.e.,

$$y = h_l x_l + h_k x_k + n. \qquad (27)$$

and denote the average error probability by $P_e^{lk} \triangleq \{[\widehat{w}_l, \widehat{w}_k] \neq [w_l, w_k]\}$.

With P_e^l and P_e^{lk}, enlightened by the concept of cut-set bound, we have a lower bound of the error probability for the proposed scheme as

$$P_e^{LB} = P_e^A + P_e^B + P_e^C + P_e^{AB} + P_e^{AC} + P_e^{BC}. \qquad (28)$$

Theorem 2: As $\rho \to \infty$, the average error probability of the proposed scheme approach its lower bound using the PNC method in Theorem 1, i.e.,

$$Pe^{NC} \to P_e^{LB}. \qquad (29)$$

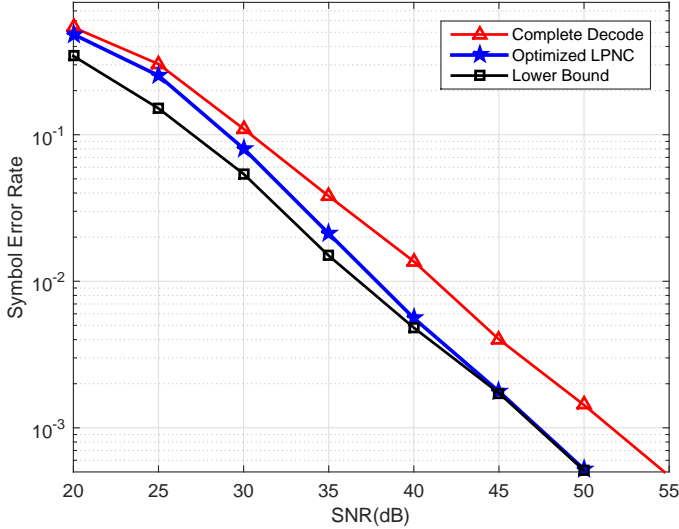We do not show the proof here due to the space limitation of the paper.

Fig. 3. Error performance of one Time-slot case in the uplink for Y-Channel.

Fig. 4. Error performance of two Time-slots case in the uplink for Y-Channel.

VI. NUMERICAL RESULTS

In this section, we present numerical results for the error-rate performance of the proposed scheme. The results are obtained by averaging over more than 1,000,000 channel realizations, where the fading channel coefficients follow i.i.d. Rayleigh distribution.

Fig. 3 shows the error-rate performance when one time-slot is allocated to the uplink phase and $q = 3$ (9-QAM for each user). For comparison purpose, we also show the performance of a complete decoding scheme, where the relay completely and jointly decodes all users' messages. We also include the numerical results of the lower bound, as discussed in Section V. We observe that our proposed scheme exhibits a 3 dB improvement over complete decoding scheme and a non-optimized at the error rate of 10^{-3}. In addition, we observe that at a sufficiently high SNR, the proposed linear PNC scheme achieves the lower bound. This agrees with Theorem 2.

Fig. 4 shows the error-rate performance of Option 1 and Option 2 as discussed in Section IV.C, where two time-slots are allocated to the uplink phase and $q = 3$ (9-QAM for each user). We can see from the numerical results that for Option 2, joint processing shows 1.5 dB improvement over separate processing. This agrees with our expectation that joint processing should behave better. On the other hand, we also see that the performance of Option 1 is almost as good as the joint processing for Option 2. Considering that the computation complexity for Option 1 is in the order of q^4 while that for Option 2 is in the order of q^6, as mentioned in Section IV.C, the separate transmission in Option 1 is a more practical choice. This reminds us that TDD-TWRC is not always the better choice for Y-channel scheme without CSIT if more time-slots in the uplink are considered.

VII. CONCLUSIONS

In this paper, we proposed a practical Y-channel PNC scheme consisting of three users and one relay where the transmitters have no CSI and each node has a single antenna. We presented an explicit solution for the generator vectors that minimize the
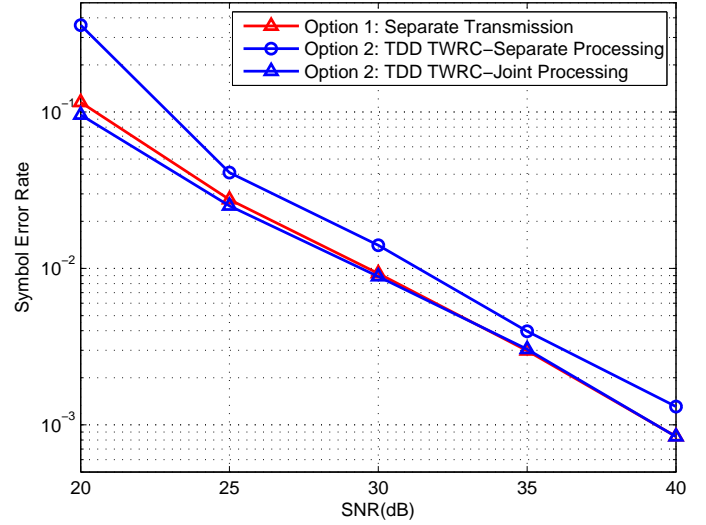
error probability at a high SNR, and analyzed the lower bound of the error performance at the relay. Numerical results showed that our proposed method obtained about 3 dB over conventional schemes at a high SNR. For future work, the problem of optimal PNC design when the nodes are equipped with multiple antennas will be an interesting topic .

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