Stochastic processes for modelling bridge deterioration

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ABSTRACT: Traditionally, bridge management systems were designed using Markov chain models. Recently, researchers applied the gamma process successfully to structural deterioration problems. The stochastic process captures the temporal variability of degradation, and has been applied to a range of problems in structures. We report on a study for the modelling of the condition of bridges in the state of NSW. The study encompasses large amounts of data spanning more than 15 years. We argue for the applicability of the gamma process and other stochastic processes. While the gamma process has been adopted in the past decade on grounds of mathematical tractability and physical motivation, we also observe another distribution for the deterioration at different times. The finding promotes the stochastic process modelling direction taken in the past decade and brings forth new models for the time-dependent reliability analysis of bridges.

1 INTRODUCTION

In the past three decades, the maintenance of bridges and road infrastructures commanded a lot research in the modelling of deterioration. The maintenance optimization aspects and the visual inspection mode promoted the use of the Markov chain model. This model has restrictive assumptions. Frangopol et al. (2001) provide a view on the events and developments that shaped the Markov chain approach in bridge management systems. Other attempts were made at modelling the deterioration of structures. In recent times, considerable efforts have been put into the application of the gamma process to the maintenance of structures. van Noortwijk (2007) gives a comprehensive survey of the topic. The gamma deterioration process is a sophisticated statistical model that can capture the temporal variability of the deterioration of the elements of a structure. It is mathematically tractable and has been applied successfully to several maintenance problems. In this article we report on a study for the Roads and Traffic Authority of the state of New South Wales. In our research, we consider the gamma process in the development of a predictive model for the life of concrete and steel elements of bridges. The study makes use of large amounts of data spanning more than 15 years of condition history. In addition to the gamma process application, an observation is made on the distribution of deterioration at different times that leads to other stochastic processes for modelling time-dependent structural deterioration. The observation promotes the stochastic process approach of the past decade and brings forth new models for the analysis of bridge deterioration.

2 PROBLEM DESCRIPTION

The Roads and Traffic Authority (RTA) of the state of New South Wales, Australia, is faced with the dual aspects of public safety and maintenance cost in the management of bridges and road structures within its jurisdiction. The interest is on predicting element level of deterioration in a bridge, predicting the future condition of a bridge, assessing the performance of a group of bridges in a road network, estimating maintenance funding levels and predicting bridge network conditions for given levels of funding. Regular inspections are conducted and repair decisions are made based on the conditions of the elements. The inspections are visual for most elements and the condition states are discrete. A unit of quantity of an element, part of that element, is judged to be in one of n states, n=3,4 or 5, depending on the element. Condition state 1 represents the ‘as new’ condition, no-deterioration state, while condition states 2, ..., n mark increasing levels of deterioration. n depends on the type of element. The units of some elements are assessed using a 3 states system, while for other elements, the quantities are rated on a 1 to 4 scale, or 1 to 5 scale. The quantities for each element are measured either in square meters (m^2) if it is a surface, in meters (m) for some elements such as railing and joints, or units (ea) for timber elements. All quantities are recorded as integers. It is a practical approximation that facilitates the visual estimation of damage. A unit of quantity of an element is either $1m^2$, 1m or 1. There are 4,945 structures and 66 elements considered in the study, for which over 230,000 inspection records are analyzed. The records go back to 1989, up to the present time. Each record corresponds to an element of a particular structure on a scheduled inspection. Among the recorded entries are q, the total quantity of the element, q_1 the quantity in condition state 1, q_2 the quantity in state 2, …, to q_n the quantity in condition state n. For example, the concrete post-tensioned girder on some bridge, on a particular inspection, was found to have q_1=590m^2, q_2=2m^2, q_3=0m^2 and q_4=0m^2 for a total q= 592m^2. Most structures have about ten to twelve elements, with more than 30 in some cases. In the course of the years that the database spans, an element was inspected a number of times, roughly every two years. The elements fall into several categories; concrete elements, steel elements with lead based paint, steel elements with other protective treatment, timber elements, joints, bearings, railings and miscellaneous. Some of the more common types are concrete pre-tensioned girder, concrete reinforced prestressed pile, concrete deck slab, concrete culverts, steel rolled beams/I girders with lead based paint protective coating, timber beam/cross girder, pourable/cork joint seal, elastomeric bearing and metal bridge railing. Each element in each structure is classified as subject to one of three degrees of environmental stress; light, moderate and severe. The same element type can be classified as being in two different environmental stresses in two different structures. Approximately two thirds of the records show a moderate environmental stress, with most of the remaining third being light stress.

3 MODELLING BRIDGE DETERIORATION

Effort went into the design and implementation of pavement and bridge management systems (BMS) due to ageing road infrastructure. In the United States, more than 70% of the bridges were built prior to 1935, and a large percentage of the United Kingdom's current bridge stock was built between the late 1950s and early 1970s. In the state of New South Wales, Australia, around 70% of the operating bridges were built before 1985, with a significant proportion before the 1940's. The near completion of most of the road networks and the ageing of the bridge stocks shifted the emphasis from developing new networks to the maintenance, repair, rehabilitation and replacement of existing road structures. Bridge management systems were developed as software tools for gathering and analyzing bridge condition data. The goal of a BMS is to predict conditions for bridge stocks and estimate maintenance funding. Pontis (Golabi & Shepard 1997) is one of the most widely used BMS. Pontis was designed and developed at the request of the United States Federal Highway Administration with the collaboration of six states and two national agencies. Bridge inspection data in Pontis are collected in the same fashion as for the data of this study.

3.1 *The Markov Model*

The decision model of Pontis, and similar BMS such as the BRIDGIT bridge management system (Hawk & Small 1998), include a Markov chain as a state-based deterioration model that drives the prediction and maintenance optimization. The Markov chain model can be found in the maintenance and repair problems since the early 60's. It is introduced to the maintenance of road infrastructure by Golabi et al. (1982). The model is appealing for its handling of uncertainty and its formulation of solvable maintenance optimization problems. A number of criticism points have been made against the usefulness of the model (Frangopol et al. 2001); bridge element performance is not addressed from a reliability viewpoint, maintenance and repair are driven by economic reasons, and the Markovian assumption does not take into account the history of the bridge deterioration. Time can be used in the assessment of the transition probabilities of the Markov chain, but the model does not explicitly capture the deterioration process. Madanat et al. (1995) make a number of observations. They point to the fact that the methods used in estimating these probabilities are ad-hoc and suffer from important methodological limitations; (i) the change in condition from one inspection to the next is not modeled explicitly, failing to capture the structure of the deterioration process, (ii) consequently, the model fails to capture the inherent non-stationarity of the deterioration process, and (iii) the approach does not recognize the latent nature of deterioration. Since deterioration is an unobservable process, it is not the state of the observable condition that should be modeled, but rather the process that generates these conditions. A practical difficulty with the estimation of the transition probabilities is the lack of data for some condition states. The severe conditions of elements of a bridge are rarely observed, due to the maintenance effort. Only the first states transition probabilities are estimated properly. Change in condition from one point in time to another is not explicitly modeled, and the Markov approach does not capture the inherent non-stationarity nature of the deterioration process.

3.2 *Time-Based Models*

Time based models consider the time between condition changes. If the condition states are discrete, the transition times can vary from one structure to another. The Markov chain, a state-based deterioration model, assumes that the time of transition from one state to another is exponentially distributed. This assumption permits the application of the memorylessness property by which an old unit of element and a newer one are equally likely to move to the next condition state (Golabi & Shepard 1997). A departure from the Markov assumption is the reliability based approach of Estes & Frangopol (1999). They proposed a system reliability based approach for optimising repair strategies for bridges, consisting of identifying relevant failure modes of the bridge, developing a system model of the overall bridge as a series-parallel combination of individual failure modes and computing the system reliability of the bridge. The idea is carried over by Yang et al. (2004). This approach applies mainly to bridges where the times to reach severe condition states can be estimated. In addition, the assumption of independence of the lifetimes of elements of bridges is a hard one to make, while modelling correlation is a harder task. It was shown that the optimum maintenance cost is strongly dependent on the system model used to represent the structure.

3.3 *Stochastic Deterioration Process*

Many factors contribute to the deterioration of bridge components; the quality and time of construction, traffic load, environmental stress. There are inherent deterioration processes in the materials of bridges that can be caused by chloride ingress, carbonation, a large sulphate attack, freeze-thaw action or alkali silica reaction (Gall 2004). Gaal et al. (2002) relate the deterioration of concrete bridges in the Netherlands to the use of de-icing salt resulting in chloride induced corrosion of the reinforcement. The work is cited by van Beek et al. (2003) in their study of the deterioration of concrete structures. A stochastic process that considers non-linear rates of deteriorations is more appropriate to model the deterioration of structures (Pandey et al. 2005). The stochastic process can capture the temporal variability of degradation. The argument is made in a series of papers by Pandey & van Noortwijk (2004), van Noortwijk et al. (2005) and Pandey at al. (2007). A particularly convenient stochastic process is the gamma process. Empirical studies showed that the expected deterioration in some cases followed the power law at^b, t being the time. This function of time is incorporated into the gamma process, along with a time-dependent variance $\sigma^2 t$, and used to model structural deterioration (van Noortwijk 2007). The gamma process was first applied by the Australian scientist Moran in the 1950's to model water flow into a dam (Moran 1954). In recent times, Abdel-Hameed (1975) was the first to propose the gamma process as a deterioration model. The gamma process is suitable for modelling gradual wear and degradation in cumulative amounts. This is recognized and applied in many structural studies, for example Nicolai et al. (2007) and van Noortwijk et al. (2007).

4 THE DETERIORATION DATA

Deterioration is most likely to suit the measuring model of a continuous variable, except possibly in the case of some timber elements. In this study, the condition data ($q_1, \ldots, q_n$) of an element is converted to a univariate measure C, using the notion of a 'Condition Index'. The percentage of the undamaged quantity of the element is q_1/q 100%. $C=100q_1/q$ is a possible condition index formulation. This index provides a first level of information on the condition of the element. Usually, the severity of the defects is introduced in an additive manner into the equation. Since $q_2, .., q_n$ represent the quantities in various damaged states, using carefully chosen parameters $\alpha_{2, \ldots,} \alpha_n$, $C=100/(1-(\alpha_2 q_2 + \ldots + \alpha_n q_n)/q\alpha_n)$ is a possible condition index formulation. Another formulation may consider introducing a severity factor such as $Sev=1/(n-1)\ q_3/q_d+\ldots+(n-2)/(n-1)\ q_n/q_d$, where the quantity of deteriorated element is $q_d=q_2+\ldots+q_n$. The index is $C= 100/(1-(q_d+\lambda Sev)/q)$ with parameter λ. Sensitivity analysis showed that these indices display similar trends in the element condition in most cases, and differ only in scale. With a bit of mathematical manipulation, these indices can be related to the California bridge health index (Shepard & Johnson 1999). Often, several types of measures were tried out until one was adopted, as in the case of the California bridge health index, a ranking system that takes values in [0,100]. The California Department of Transportation was involved in the development and implementation of Pontis. A condition index has two functions; (1) its use in a cost/benefit analysis where the condition history of a structure can be estimated with the inspection data of its elements, through the use of a weighted sum of the conditions of the elements (Shepard & Johnson 1999), and (2) its use in the study of deterioration by modelling the univariate measure as a stochastic process in time.

4.1 *Deterioration Information*

An element can be in one of 11 states at an inspection time. The element condition is a value between 0 and 100%, 100% being the as new condition (no deterioration) state. The 11 states are (S_1) the element condition is C=100 and C_p=100 at the previous inspection, (S_2) 0<C<100 and C_p=100, (S_3) C=0 and

C_p=100, (S_4) 0<C=C_p<100, (S_5) 0<C_p<C<100, (S_6) C=100 and 0<C_p<100, (S_7) 0<C<C_p<100, (S_8) C=0 and 0<C_p<100, (S_9) C=C_p=0, (S_{10}) 0=C_p<C<100, and (S_{11}) C_p=0 and C=100. The 11 states are mutually exclusive and exhaustive events, defined using two consecutive element conditions. In the study of a stochastic process, we look for points of renewal in which the process is restarted, hence using the information from the previous inspection. For descriptive purposes, transition frequencies for these 11 states were estimated from the data. These statistics provide information as to how the element is coping in a set of structures. The deterioration is defined as Z=100-C. Z_1, Z_2, Z_3, Z_7 and Z_8 are the deterioration corresponding to states S_1, S_2, S_3, S_7 and S_8. These variables are of relevance when studying the deterioration process, along with their inter-inspection times. The time at which a deterioration increase, dZ in a time interval dt, occurs is also important. The pairs {(dt,dZ)}, for $dZ \geq 0$, and the time at which dt starts is sufficient information to estimate the parameters of stochastic deterioration.

4.2 *Time Pattern*

To study the distribution of variables such as Z_2 and dZ_7, the increase associated with Z_7, we need data that occur at the same time. In practice, inspections are not always strictly periodic. However, the inter-inspection times can be adjusted so that they are grouped together, making it possible to study the probabilistic behaviour of deterioration as a random variable at different times. The adjustment was made through rounding off the inspection times. The pattern showed that the inspection times were meant to be 2 years apart. Figure 1 shows the conditions C=100-Z_2 plotted in dots, of some concrete element type chosen as an example, as they occur in time from a renewal, on the structures the element belongs to. The adjusted conditions are plotted as stars.

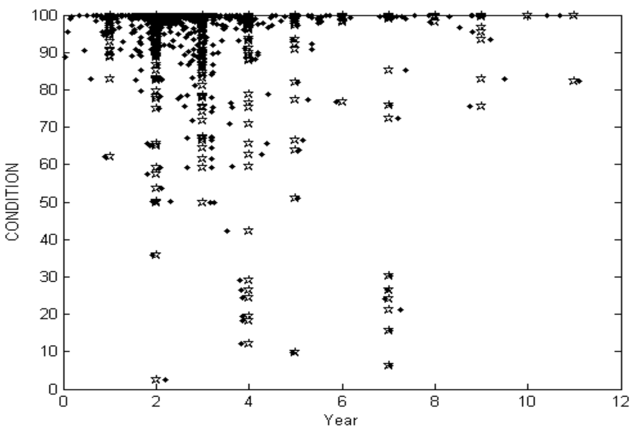

Figure 1. Condition data of a concrete element type.

Judgement was used in eliminating data for inspection time intervals that exceed 4 years. Either special inspections were applied to the element in these cases, or inspections were conducted within the time period but not reported. In addition, beyond 4 years, the data start thinning, which is not helpful in this part of the study. From the original 913 conditions, 836 adjusted are plotted in Figure 2. Figure 3 displays simulated condition data using a gamma process with a nonlinear rate of deterioration. It shows the probabilistic behaviour of the theoretical model at different points in time. In looking for the validation of similar behaviour in our data, we used a least squares approach in fitting the data of Figure 2 to a nonlinear deterioration curve $100\text{-}\mu t^q$, with parameters μ and q. Figure 2 displays the least squares results using (i) the condition means at times 1,2,3 and 4, (ii) all the adjusted data, and (iii) the actual data. q was found to be 2.7, 2.4 and 2.4 respectively, and μ=0.29, 0.404 and 0.408, showing again that the adjustment to the data was minimal.

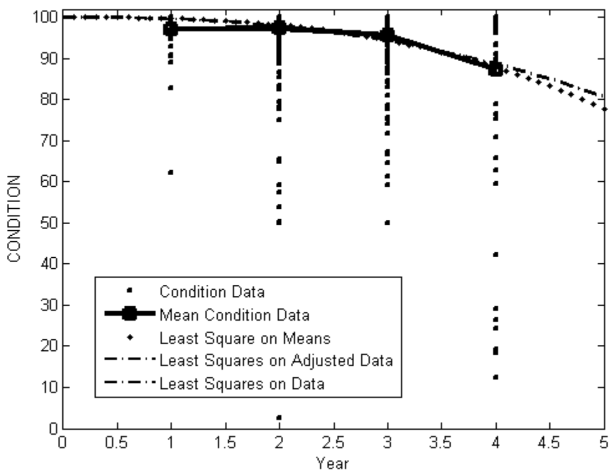

Figure 2. Adjusted condition data.

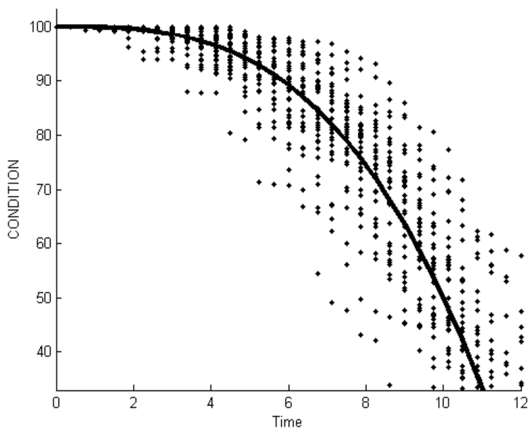

Figure 3. Gamma process with a nonlinear rate.

The time pattern of the deterioration of the element, shown through the zoomed view of Figure 4, fits the behaviour of the gamma process

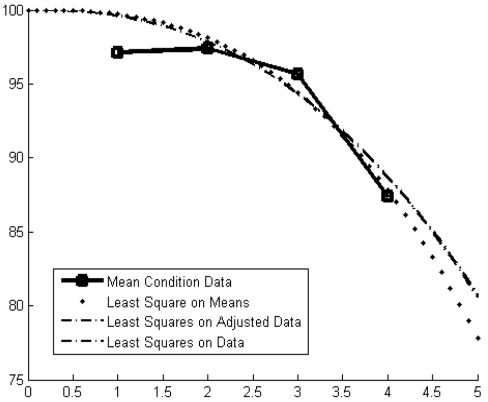

Figure 4. Least squares estimation of the deterioration curve.

4.3 *Probability Distributions*

The distributions of Z_2 and dZ_7 were found to adhere to the gamma distribution. This concords with the properties of the gamma deterioration process, where the distributions of incremental deteriorations are gamma distributed (van Noortwijk 2007). The fit to the gamma distribution for all incremental intervals was good when abundant data are available, as in the case of Figure 5, and not rejected otherwise. This was observed for the concrete and steel elements, as well as some railing, joints and other elements. In all cases, the lognormal was also found to be a better fit. The fit was also observed within classes of similar structures. At this point, the data has not been stratified according to influencing factors such as traffic load, age of bridge and location, except in some examples. The observations of distribution fit held in those cases. A formal study of the probabilistic behaviour of the deterioration using model selection methods is currently being conducted. For the purpose of building a statistical model, the findings promote the use of a stochastic deterioration process such as the gamma process with mean deterioration function μt^q that includes the stationarity case of q=1.

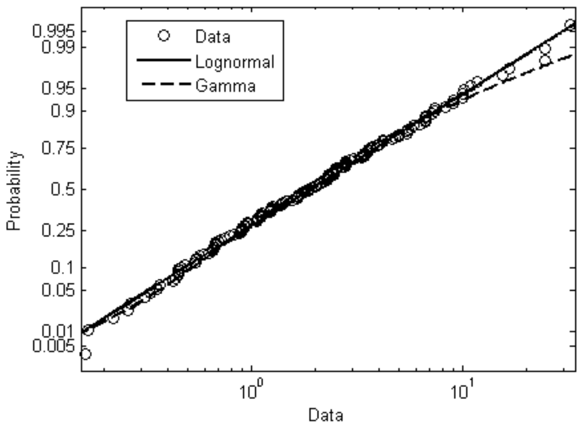

Figure 5. Probability distribution fitting.

5 STOCHASTIC PROCESSES

The gamma process is defined by van Noortwijk (2007) in the context of the deterioration of structures. We experimented with the process using simulation, estimating the parameters with the maximum likelihood approach. With few data points, the model captures the deterioration process efficiently. The lower the data points are along the curve, the better the estimation. As an illustration of the methodology, the process is applied to a concrete element on two different bridges. We estimated two different rates $\hat{q}$=3.3 and $\hat{q}$=2. Two data points are used in each example; (15.81,99.69) and (17.75,95.46) for the first case, with the renewal at year 13.92, and [(15.09,99.26);(17.12,98.15)] with the renewal at 13.6 years since the start of the database. These two examples were chosen for the nonlinear deterioration rates they display, the second one being less pronounced. In many cases, the bridge elements show lower rates. Figure 6 displays the estimated process mean where the maximization of the likelihood function is conducted over (μ,σ,q). The estimated curve is the doted line, chosen out of the plotted straight lines where the maximization is conducted over (μ,σ) for a fixed q. The thick line is a condition path simulated using the maximum likelihood estimates.

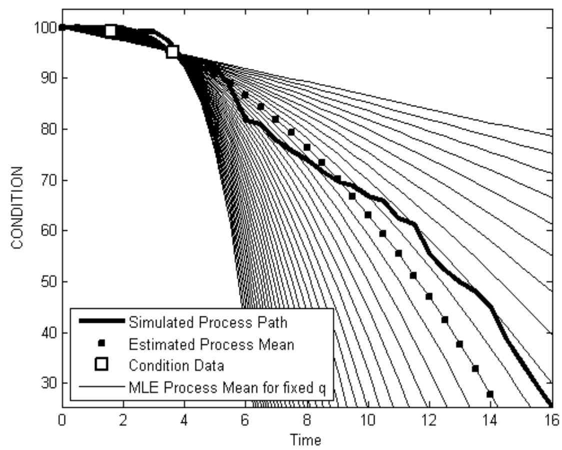

Figure 6. Gamma process estimation, $\hat{q}$ =2.

Often, the data for each individual bridge aren't enough to estimate the parameters properly. In such cases, the deterioration path does not extend beyond the first reported positive deterioration. The advantage of using the gamma process is to aggregate the data by dividing the bridges into similarity classes. Then within a class, a second stratification occurs according to influencing factors, such as traffic load, age, location and environmental stress. The estimation of the parameters within the class of structures results in better assessment and prediction. Due to the independent increments property and with the assumption of independence between elements on different structures, the likelihood function can be written and maximized (Nicolai et al. 2007). Most condition paths starting with a renewal have a variable number of as new condition state inspection intervals before the deterioration is advanced enough to be noticed. This element behaviour has been corroborated by bridge engineers. We can use the gamma process for the positive deterioration part of the process, while a statistical model can capture the amount of time an element remains in full condition. Another approach is to include both parts in the same model with a stochastic process that has a flatter mean condition path before curving downwards. A candidate is the lognormal diffusion process with exogenous factors (Gutierrez et al. 2001). Used in statistical modelling, it has been mainly confined to the analysis of economic data. Replacing the process mean μt^q of the gamma process with a trend $\exp(\lambda t^q)$-1 in the lognormal diffusion process can capture the whole deterioration process. These approaches are being investigated.

6 CONCLUSIONS

A study of bridges in the state of New South Wales, Australia, using data spanning more than 15 years of condition data, promotes the use of stochastic processes for capturing the non-stationarity in a deterioration process. We encounter a deterioration process with a part in it that leads itself to the use of the gamma process which offers a mathematically tractable solution. We also observe the lognormal distribution for the deterioration at different times. This observation leads to the consideration and development of other stochastic deterioration processes.

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