Random Vibration of Multistory buildings with Uncertainty

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Abstract
Seismic random vibration responses of multi-story buildings with interval parameters are investigated. A multi-story building is modelled as a shear beam structure, and the structural mass, stiffness and damping are considered as interval parameters. The earthquake inputs are random process ground acceleration in the horizontal direction. Using the interval factor method, an interval structural parameter can be expressed as the product of its midpoint value and interval factor. Structural natural frequencies, power spectral density and mean square value of structural seismic random displacements can then be expressed as functions of interval factors of structural parameters. The expressions for the lower bound, midpoint, upper bound, maximum width and interval change ratio of structural random responses are derived by means of the interval operations. The effects of the uncertainties of structural parameters on the random responses of a 10-story building are investigated and discussed in detail.

Key words: Multi-story buildings, Seismic random vibration, Shear beam structures, Interval analysis

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1. Introduction

The random motions such as wind loading, seismic waves and ocean waves etc., are often encountered in the design and manufacture of structures. Many structures are subjected to the action of non-stationary random loading, e.g., buildings subjected to earthquakes. Seismic response of structures and buildings has been widely investigated due to heavy loss of human life and property caused by earthquakes [1-3]. Uncertainty exists in most structures such as buildings, bridges, antennas, aircraft, vehicles, ships and aerospace structures which are essential parts of our modern lives. For example, the material properties of a real structure may vary considerably from the design values. Over the lifetime of a structure, the damaging effects associated with attacks from environmental aggressive agents such as a progressive deterioration of concrete and corrosion of steel usually lead to significant variations of system parameters. Therefore, the investigation of the problem of structures with uncertain parameters subject to random seismic excitation is of great significance in engineering practice.

The significant effects of inherent uncertainties on system behaviour have led the scientific community to recognize the importance of a stochastic approach to engineering problems. Probabilistic methods are most popular for analysis of systems with uncertainties in system parameters and inputs [4-6]. Under these circumstances, the mean value, variance and standard deviation of individual structural parameter and the correlation between different structural parameters are provided by the probabilistic information (probability density function and joint probability distribution function) of the structural parameters. However, the probabilistic methods are only applicable when information about an uncertain parameter in the form of a preference probability function is available. The interval methods can be used when the probability function is unavailable but the range of the uncertain parameter is known. The response quantities of interest will also be intervals. In the past decade, significant progresses in interval analyses of structures with bounded parameters have been achieved. Interval static response [7,8], natural frequencies/eigenvalues [9,10], dynamic response [11,12] and optimization [13] of structures with interval parameters have been investigated.

A shear beam structure is shown schematically in Fig. 1. This simple structural model can represent various types of engineering systems such as buildings, dams or soil profiles. The shear beam is widely used as the model of multistorey building [14-16]. Although the shear beam seems quite simple as a building model, it allows the analysis of complicated shear wave propagation effects which are very difficult to be explained by the traditional finite element approach commonly used in earthquake engineering. Chopra and Chintanapakee [17] showed that the shear beam was an effective structural model and it is independent of the type of seismic excitations. Sasani et al. [18] improved the shear beam wave propagation model to properly account for the dispersive type of damping.

To the authors' knowledge there has been very little research using interval technique to investigate the random response of structures with uncertainty. In this paper, the interval factor method (IFM) [8] is further developed to predict the random responses of shear beam structures with interval parameters subjected to stationary and non-stationary seismic excitations.

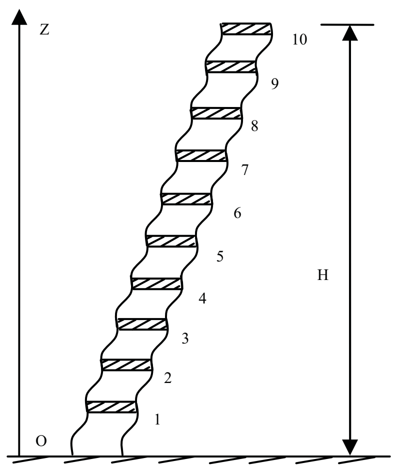


Fig. 1 Shear beam model of a multi-story building.

2. Interval seismic vibration of shear beam structures

The equation of motion of a uniform cantilever shear beam under horizontal seismic excitations takes the following form [19]

$$m\frac{\partial^2 u(z,t)}{\partial t^2} - c\frac{\partial^3 u(z,t)}{\partial t \partial z^2} - k\frac{\partial^2 u(z,t)}{\partial z^2} = -m\ddot{x}_g(t) \quad (1)$$

where m, c and k are the mass per unit length, damping and shear stiffness respectively. $\ddot{x}_g(t)$ is the ground acceleration.

In Eq.(1), the input ground acceleration can be described by the modulated non-stationary random process as follows

$$\ddot{x}_g(t) = A(\omega,t) f(t) \tag{2}$$

$A(\omega,t)$ is a given envelope (or modulation) function, and $f(t)$ is a stationary random Gaussian process. The input ground acceleration $\ddot{x}_g(t)$ becomes a stationary random force and $\ddot{x}_g(t) = f(t)$ when $A(\omega,t) = 1\,(-\infty < t < \infty)$.

The natural frequencies ω_j and mode shapes ϕ_j of mode $j\,(j = 1,2,\cdots)$ can be written in closed form solutions

$$\omega_j = (2j-1)\frac{\pi}{2H}\sqrt{\frac{k}{m}} \tag{3}$$

$$\phi_j(z) = \sin\left(\omega_j z \middle/ \sqrt{\frac{k}{m}}\right) \tag{4}$$

where H is the height of the shear beam shown in Fig. 1.

Considering the structural shear stiffness k, mass m and damping c as interval variables, by means of the interval factor method [8], they can be respectively expressed as

$$k^I = k_F^I \cdot k^c\,,\quad m^I = m_F^I \cdot m^c\,,\quad c^I = c_F^I \cdot c^c \tag{5}$$

where k_F^I, m_F^I and c_F^I are interval factors of interval variables k^I, m^I and c^I respectively. k^c, m^c and c^c are midpoints values of k^I, m^I and c^I respectively.

Substituting Eq.(5) into Eq.(3) yields

$$\omega_j^I = \sqrt{\frac{k_F^I}{m_F^I}} \cdot (2j-1)\frac{\pi}{2H}\sqrt{\frac{k^c}{m^c}} \tag{6}$$

Substituting Eqs.(5) and (6) into Eq.(4) yields

$$\phi_j(z) = \sin\left(\sqrt{\frac{k_F^I}{m_F^I}} \cdot (2j-1)\frac{\pi}{2H}\sqrt{\frac{k^c}{m^c}} \cdot z \middle/ \sqrt{\frac{k_F^I}{m_F^I}} \cdot \sqrt{\frac{k^c}{m^c}}\right) = \sin\left((2j-1)\frac{\pi z}{2H}\right) \tag{7}$$

The power spectral density (PSD) $S_u(t,\omega)$ of displacement response in frequency domain is given by [5,14]

$$S_u(t,\omega)=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j\,a_i\,\phi_j(z)\phi_i(z)A(\omega,t_1)h_j(\omega)S_f(\omega)A(\omega,t_2)h_i^*(\omega) \tag{8}$$

where $S_f(\omega)$ is the PSD of $f(t)$. $i=\sqrt{-1}$ is the complex number. $h_j^*(\omega)$ is the complex conjugate of $h_j(\omega)$. $h_j(\omega)$ is the interval frequency response function and can be expressed as

$$h_j(\omega)^I=\frac{1}{\frac{k_F^I}{m_F^I}(2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^c}-\omega^2+i\cdot c_F^I\cdot c^c\cdot\sqrt{\frac{1}{m_F^I}}\cdot\sqrt{\frac{1}{m^c}}\cdot\omega} \tag{9}$$

After integrating $S_u(t,\omega)$ within the frequency domain, the interval mean square displacements along the height z of the shear beam are given by

$$\psi_{uZ}^{2\ I}=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j\,a_i\,\phi_j(z)\phi_i(z)\int_{-\infty}^{\infty}A(\omega,t_1)h_j(\omega)^I\,S_f(\omega)A(\omega,t_2)h_i^*(\omega)^I\,d\omega \tag{10}$$

3. Interval random responses of shear beam structures

From Eq.(9) and by using the interval operations [8], the midpoint value and maximum width of frequency response function can be expressed as

$$h_j(\omega)^c=\frac{1}{(2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^c}-\omega^2+i\cdot c^c\cdot\sqrt{\frac{1}{m^c}}\cdot\omega} \tag{11}$$

$$\begin{aligned}\Delta h_j(\omega)=\Bigg\{&\left(-\frac{-(2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^{c2}}-i\cdot c^c\cdot\sqrt{\frac{1}{m^{c3}}}\cdot\omega}{\left((2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^c}-\omega^2+i\cdot c^c\cdot\sqrt{\frac{1}{m^c}}\cdot\omega\right)^2}\cdot\Delta m_F\cdot m^c\right)^2\\&+\left(-\frac{-i\cdot\sqrt{\frac{1}{m^c}}\cdot\omega}{\left((2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^c}-\omega^2+i\cdot c^c\cdot\sqrt{\frac{1}{m^c}}\cdot\omega\right)^2}\cdot\Delta c_F\cdot c^c\right)^2\\&+\left(-\frac{(2j-1)^2\frac{\pi^2}{4H^2}\frac{1}{m^c}}{\left((2j-1)^2\frac{\pi^2}{4H^2}\frac{k^c}{m^c}-\omega^2+i\cdot c^c\cdot\sqrt{\frac{1}{m^c}}\cdot\omega\right)^2}\cdot\Delta k_F\cdot k^c\right)^2\Bigg\}^{1/2}\end{aligned} \tag{12}$$

Similarly, the midpoint and maximum width of the PSD of structural random displacement response can be obtained

$$S_u(t,\omega)^c = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) A(\omega,t_1) h_j(\omega)^c S_f(\omega) A(\omega,t_2) h_i^*(\omega)^c \tag{13}$$

$$\Delta S_u(t,\omega) = \left\{\left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) A(\omega,t_1)\Delta h_j(\omega) S_f(\omega) A(\omega,t_2) h_i^*(\omega)^c\right)^2 + \left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) A(\omega,t_1) h_j(\omega)^c S_f(\omega) A(\omega,t_2)\Delta h_i^*(\omega)\right)^2\right\}^{1/2} \tag{14}$$

Then, the interval values of mean square value of seismic random response can be written as

$$\psi_{uZ}^{2\ c} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) \int_{-\infty}^{\infty} A(\omega,t_1) h_j(\omega)^c S_f(\omega) A(\omega,t_2) h_i^*(\omega)^c \tag{15}$$

$$\Delta\psi_{uZ}^2 = \left\{\left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) \int_{-\infty}^{\infty} A(\omega,t_1)\Delta h_j(\omega) S_f(\omega) A(\omega,t_2) h_i^*(\omega)^c\, d\omega\right)^2 + \left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_j a_i \phi_j(z)\phi_i(z) \int_{-\infty}^{\infty} A(\omega,t_1) h_j(\omega)^c S_f(\omega) A(\omega,t_2)\Delta h_i^*(\omega)\, d\omega\right)^2\right\}^{1/2} \tag{16}$$

Therefore, the lower bound $\underline{\psi_{uZ}^2}$, upper bound $\overline{\psi_{uZ}^2}$ and interval change ratio $\Delta\psi_{uZF}^2$ of mean square seismic displacement are given by

$$\underline{\psi_{uZ}^2} = \psi_{uZ}^{2\ c} - \Delta\psi_{uZ}^2,\quad \overline{\psi_{uZ}^2} = \psi_{uZ}^{2\ c} + \Delta\psi_{uZ}^2,\quad \Delta\psi_{uZF}^2 = \frac{\Delta\psi_{uZ}^2}{\psi_{uZ}^{2\ c}} \tag{17}$$

4. Numerical example

The shear beam shown in Fig. 1 is used as an example to investigate the seismic random response of multi-storey buildings with uncertainty. The height of shear beam is $H = 35m$. The midpoint values of the structural shear stiffness, mass and damping ratio are $k^c = 8.1\times10^8 kg\cdot m/s^2$, $m^c = 2.2\times10^5 kg/m$ and $c = 18.5\times10^3 kgm/s$ respectively.

For the stationary random ground acceleration, let $A(\omega,t) = 1$ and the power spectral density of $f(t)$ is described by the modified version of the Kanai-Tajimi model [20]

$$S_f(\omega) = \frac{\omega_g^4 + 4\omega^2\omega_g^2\zeta_g^2}{\left(\omega^2-\omega_g^2\right)^2 + 4\omega^2\omega_g^2\zeta_g^2}\,\frac{\omega^4}{\left(\omega^2-\omega_f^2\right)^2 + 4\omega^2\omega_g^2\zeta_f^2}\,S_0 \tag{18}$$

where S_0 is the ordinate of the PSD of the bedrock acceleration. ω_g and ζ_g are the natural frequency and critical damping ratio of soil layer, ω_f and ζ_f are meters of a second filter which is introduced to assure a finite power for the ground displacement. In this example, $\omega_g = 15.0\,rad/s$, $\zeta_g = 0.6$, $S_0 = 2.0\,cm^2/s^3$, $\omega_f = 1.5\,rad/s$ and $\zeta_f = 0.6$.

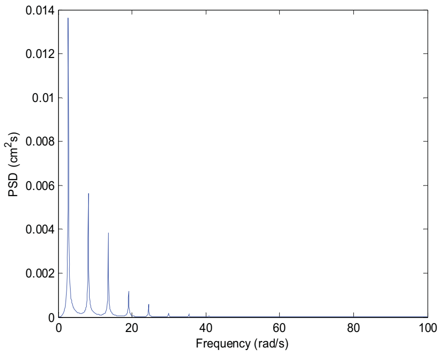
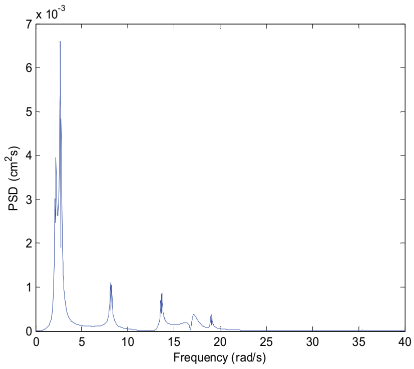
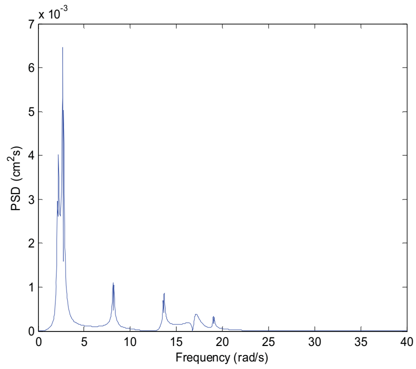


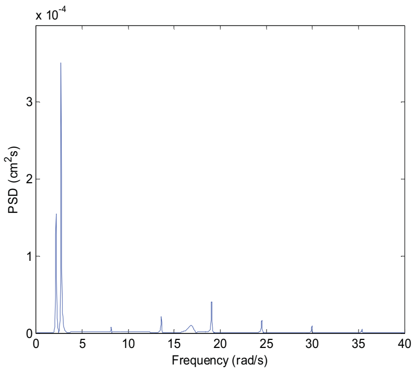
Fig. 2 Power spectral density of $S_{u10}(t,\omega)^c$ ($\Delta k_F = \Delta m_F = \Delta\zeta_{jF}$ =0).



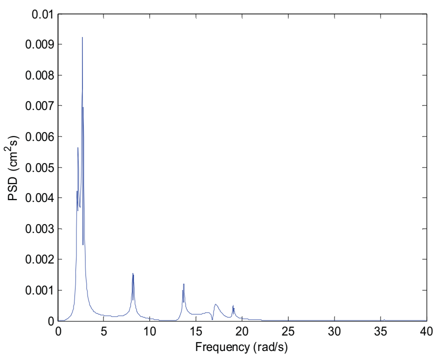
(a) Δk_F =0.2, $\Delta m_F = \Delta\zeta_{jF}$ =0



(b) Δm_F =0.2, $\Delta k_F = \Delta\zeta_{jF}$ =0



(c) $\Delta\zeta_{jF}$ =0.2, $\Delta k_F = \Delta m_F$ =0



(d) $\Delta k_F = \Delta m_F = \Delta\zeta_{jF}$ =0.2

Fig. 3 Power spectral density of $\Delta S_{u10}(t,\omega)$.

The midpoint value of the PSD of the random displacement at Level 10 of the building ($z = 35m$) $S_{u10}(t,\omega)^c$ is shown in Fig. 2. The effect of each of vibration modes on the structural response can be seen from this figure. The lower order natural frequencies and mode shapes contribute more to the structural responses, especially for the first five modes. Therefore, the structural responses could be sufficiently accurate when the integration region of the frequency of the ground acceleration excitation is selected as [0,40] rad/s, which includes the dominant vibration modes of the structure.

To determine the effect of the change of interval parameters variables on the structural responses, the values of interval change ratios of interval structural parameters are taken as different groups. The effect of the uncertainty of structural stiffness, mass and damping on the maximum width of the PSD of the random displacement of Level 10 $\Delta S_{u10}(t,\omega)^c$ can be examined from Figures 3(a)-(d). In order to show the peaks of the response PSDs more clearly, the frequency region of earthquake excitation is selected as [0,40] rad/s. The peaks of the response PSDs in Fig. 2 are very sharp, however, they have ranges as shown in Fig. 3 because each natural frequency has its own change ranges depending on the interval change ratios of structural parameters. Therefore, the resonant response at any natural frequency also has a change region. It can be seen that the stiffness and mass produce the similar effects on the uncertainty of structural random displacement because they produce the similar effect on structural natural frequencies.

Computational results of the midpoint value $\psi_{uZ}^{2\ c}$, maximum width $\Delta\psi_{uZ}^2$ and interval change ratio $\Delta\psi_{uZF}^2$ of the mean square displacement response $\psi_{uZ}^{2\ I}$ $(z = 3.5, 7, ..., 35m)$ of each level of the building when $\Delta k_F = \Delta m_F = \Delta\zeta_{jF}$ =0.1 are given in Table 1. The midpoint values increase along the height of the building, that is, higher levels have larger displacement responses. Similarly, it can be seen from Table 1 that the maximum widths of the random displacement at higher levels are also bigger. Generally speaking, the uncertainties of structural parameters will produce greater effect on the structural responses of higher levels. It is hard to say they will produce the biggest effect on the highest level because its corresponding interval change ratios may not be the biggest one.

Table 1. Interval values of mean square random responses

Z(m)	$\psi_{uZ}^{2\ c}$ $(\times 10^{-2} cm^2)$	$\Delta\psi_{uZ}^2$ $(\times 10^{-2} cm^2)$	$\Delta\psi_{uZF}^2$
3.5 (level 1)	1.03	0.05	0.0486
7.0 (level 2)	2.08	0.21	0.1009
10.5 (level 3)	2.55	0.36	0.1398
14.0 (level 4)	2.83	0.49	0.1738
17.5 (level 5)	3.19	0.60	0.1879
21.0 (level 6)	3.56	0.76	0.2134
24.5 (level 7)	3.88	0.95	0.2439
28.0 (level 8)	4.36	1.08	0.2473
31.5 (level 9)	5.43	1.17	0.2152
35.0 (level 10)	6.52	1.20	0.1843

For the non-stationary random ground excitation, the following envelop function of the input acceleration are considered [21]

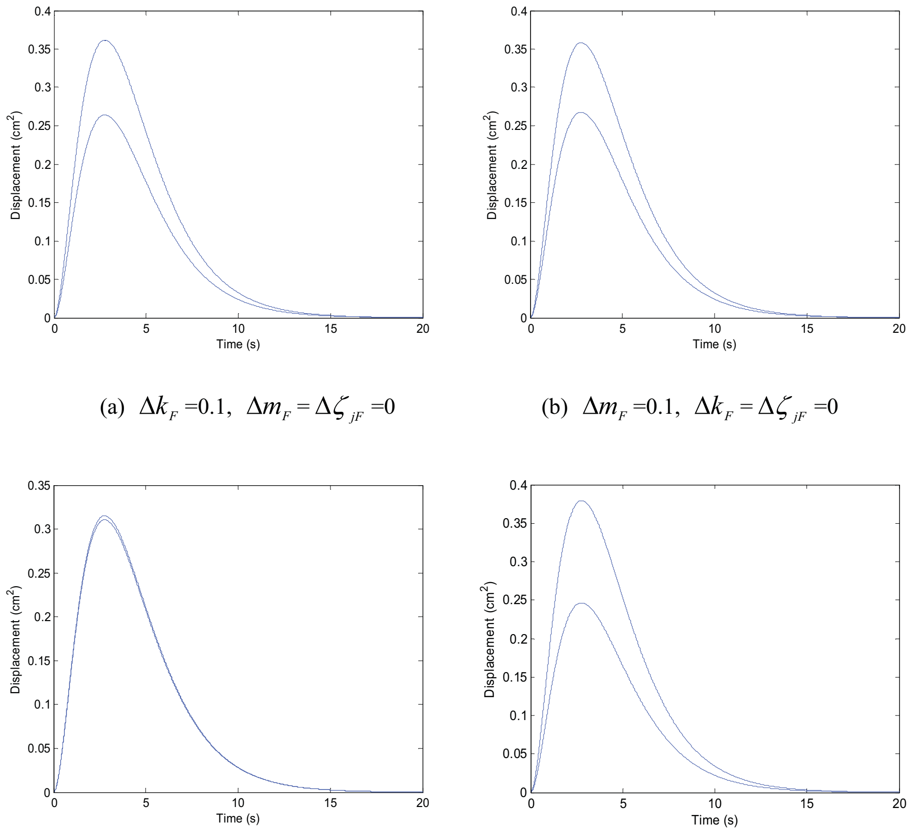
$$S_f(\omega) = \frac{\omega_g^4 + 4\omega^2\omega_g^2\zeta_g^2}{(\omega^2 - \omega_g^2)^2 + 4\omega^2\omega_g^2\zeta_g^2} \frac{\omega^4}{(\omega^2 - \omega_f^2)^2 + 4\omega^2\omega_g^2\zeta_f^2} S_0 \tag{19}$$

$$A(\omega,t)=\frac{e^{-at}-e^{-bt}}{\max[e^{-at}-e^{-bt}]}\left\{\frac{\omega_g^4+4\omega^2\omega_g^2\zeta_g^2}{\left(\omega^2-\omega_g^2\right)^2+4\omega^2\omega_g^2\zeta_g^2}\right\}^{1/2} \tag{20}$$

$$S_f(\omega)=S_0 \tag{21}$$

where $a=0.25$, $b=0.5$, $\omega_g=15.0\,rad/s$, $\zeta_g=0.25$ and $S_0=2.0\,cm^2/s^3$.

For non-stationary seismic responses, curves of the lower and upper bounds of the mean square displacement response at Level 10 are shown in Fig. 4. It can be observed that the uncertainties of the structural non-stationary random responses are quite similar to those of the structural stationary random responses. The changes of the stiffness and mass have similar effect on the uncertainty of structural non-stationary seismic displacement, and the uncertainty of the structural non-stationary mean square random displacement is very obvious when the uncertainties of all structural parameters are taken into account.



(a) Δk_F =0.1, $\Delta m_F=\Delta\zeta_{jF}$ =0

(b) Δm_F =0.1, $\Delta k_F=\Delta\zeta_{jF}$ =0

(c) $\Delta\zeta_{jF}$ =0.1, $\Delta k_F=\Delta m_F$ =0

(d) $\Delta k_F=\Delta m_F=\Delta\zeta_{jF}$ =0.1

Fig. 4 Non-stationary random displacement response $\psi_{uZ}^{2\,I}$ $(z=35m)$.

5. Conclusions

In this paper, the effects of bounded structural parameters on the change of the natural frequencies, power spectral density of structural random responses and mean square displacement shear beams are presented. Expressions for the midpoint value, lower bound, upper bound, interval maximum width, interval change ratio for the mean square value of seismic responses of shear beams under the stationary and non-stationary random earthquake excitations are developed. The dynamic responses of shear beams with uncertainty under random seismic excitations can be obtained expediently.

References

(1) Takewaki I, Resonance and criticality measure of ground motions via probabilistic critical excitation method, Soil Dynamics and Earthquake Engineering, Volume 21(8), 2001, pp.645-659.

(2) Su L, Dong SL and Kato S, A new average response spectrum method for linear response analysis of structures to spatial earthquake ground motions, Engineering Structures, Volume 28(13), 2006, pp.1835-1842.

(3) Du H and Zhang N, H_∞ control for buildings with time delay in control via linear matrix inequalities and genetic algorithms, Engineering Structures, Volume 30(1), 2008, 81-92.

(4) Beck AT, Melchers RE, On the ensemble crossing rate approach to time variant reliability analysis of uncertain structures, Probabilistic Engineering Mechanics, Volume 19, 2004, pp. 9-19.

(5) Gao W, Stochastically optimal active control of a smart truss structure under stationary random excitation, Journal of Sound and Vibration, Volume 290, 2006, pp.1256-1268.

(6) Stewart MG, Suo QH, Extent of spatially variable corrosion damage as an indicator of strength and time-dependent reliability of RC beams, Engineering Structures, Volume 31, 2009, pp.198-207.

(7) Chen SH, Yang XW, Interval finite element method for beam structures, Finite Elements in Analysis and Design, Volume 34, 2000, pp.75-88.

(8) Gao W, Interval finite element analysis using interval factor method, Computational Mechanics, Volume 39, 2007, pp.709-717.

(9) Qiu ZP, Wang XJ, Friswell MI, Eigenvalue bounds of structures with uncertain-but-bounded parameters, Journal of Sound and Vibration, Volume 282, 2005, pp.297-312.

(10) Gao W, Natural frequency and mode shape analysis of structures with uncertainty, Mechanical Systems and Signal Processing, Volume 21, 2007, pp.24-39.

(11) Zhang XM, Ding H, Chen SH, Interval finite element method for dynamic response of closed-loop system with uncertain parameters, International Journal for Numerical Methods in Engineering, Volume 70, 2007, pp.543-562.

(12) Wang XJ, Qiu ZP, Nonprobabilistic interval reliability analysis of wing flutter, AIAA Journal, Volume 47, 2009, pp.743-748.

(13) Maiumder L, Rao SS, Interval-based multi-objective optimization of aircraft wings under gust loads, AIAA Journal, Volume 47, 2009, pp.563-575.

(14) Zembaty Z, Non-stationary random vibration of a shear beam under-high frequency seismic effects, Soil Dynamics and Earthquake Engineering, Volume 27(11), 2007, pp.1000-1011.

(15) Todorovska MI and Lee VW, Seismic waves in buildings with shear walls or central core, Journal of Structural Engineering ASCE, Volume 115(12), 1989, pp.2669-2686.

(16) Safak E, Wave-propagation formulation of seismic response of multistory buildings, Journal of Structural Engineering ASCE, Volume 125(4), 1999, pp.426-437.

(17) Chopra AK and Chintanaapakdee C, Drift sepectrum vs. modal analysis of structural response to near fault ground motions, Earthquake Spectra, Volume 17(2), 2001, pp.221-234.

(18) Sasani M, Makris N and Bolt BA, Damping in shear beam structures and estimation of drift response, Journal of Engineering Mechanics ASCE, Volume 132(9), 2006, pp.851-858.

(19) Fang T, Leng XL, Sun MN and Li JQ, Evolutionary random response of liner random structures, Journal of Sound and Vibration, Volume 256(1), 2002, pp.189-195.

(20) Li QS, Zhang YH, Wu JR, Lin JH, Seismic random vibration analysis of tall buildings, Engineering Structures, Volume 26(12), 2004, pp.1767-1778.

(21) Leung AYT, Zhang HT, Cheng CC and Lee YY, Particle swarm optimization of TMD by non-stationary base excitation during earthquake, Earthquake Engineering & Structural Dynamics, Volume 37(9), 2008, pp.1223-1246.

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