Design of second-order sliding mode controllers for MR damper-embedded smart structures

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Abstract

This paper presents the design of second-order sliding mode controllers for semi-active control using magneto-rheological (MR) dampers. The approach can be useful in applications involving shock absorbers but here our main concern is the suppression of building vibrations induced by dynamic loadings such as earthquakes or strong winds. The MR dampers have been of increasing interest in structural control as they are inexpensive to manufacture and have attractive properties such as small energy requirements, reliability and stability in operations, as well as a fast response of milliseconds. Challenges of MR damper structural control rest with the system's high nonlinearity due to the force-velocity hysteresis, and the constraint of the magnetisation current, required to be between its zero and maximal values. A variety of control algorithms have been applied, including the decentralized bangbang control, modulated homogeneous friction algorithm, clipped optimal control, Lyapunov-based control, and also non model-based intelligent schemes. In these techniques, the currents are usually obtained from the damping force indirectly rather than directly from the controller output. For direct current control, in this paper we propose second-order sliding mode controllers, which can satisfy the control constraint, provide high accuracy, retain robustness and remove chattering. The effectiveness of the proposed direct current control technique is verified, in simulations, on a benchmark building model subject to excitation of various scaled earthquake records.

Introduction

Control devices and methodologies for suppression of high-rise building vibrations caused by a dynamic loading source can be classified as passive dampers requiring no input power to operate, active dampers requiring a great deal of power to generate counteracting forces, and semi-active combining features of passive and active damping (Datta, 2003; Symans & Constaninou, 1999; Yoshida *et al.*, 2004).

In structural control, active control devices require a certain amount of energy to drive the actuators to accomplish the control objective. On the other hand, semi-active control needs a relatively small amount of driving power and the actuators can also be operated in the passive mode. The philosophy adopted in these approaches is to effectively absorb the vibration energy by modifying the control device physical characteristics.

For semi-active structural control, the use of magneto-rheological (MR) dampers has been of increasing interest in smart civil structures as they are inexpensive to manufacture, have reliable, stable and fail-safe operations, small energy requirements, and a fast response of milliseconds.

Given the advantages of MR dampers and semi-active control strategies, a number of controller designs have been proposed for the building control problem. In most of MR damper controllers developed so far, the current supplied to the dampers is quite often derived, from the required damping force obtained as the control signal, via a secondary current-control loop. In this paper, the direct current control approach for MR-dampers is proposed using second-order sliding mode (SOSM) controllers. The idea is to control directly the magnetisation current of the semi-active device in order to drive to zero not only the sliding function of the state variables but also higher-order time derivatives of the sliding function. The SOSM approach retains strong robustness of the system in the sliding mode, at the same time removes the chattering effect, provides even higher accuracy in realisation, and is suitable for control signals subject to constraints. These features make it ideal for direct current control of the MR damper used in the smart structures.

The remainder of the paper is organized as follows. The system description and the design of the proposed SOSM controller are included in Section 2. Simulation results are given in Section 3 to verify the effectiveness of the proposed approach. Finally, a conclusion is drawn in Section 4.

Control Design

Consider pairs of MR dampers, placed in a differential configuration on the 1st ,… k^{th} ,… and n^{th} floors of a building, with the control current vector $\mathbf{i} = [i_1 \cdots i_k \cdots i_n]^{\mathbf{T}}$ whose entries are constrained between zero and the maximal values. By defining the system state $\mathbf{y} = [\mathbf{x}^{\mathbf{T}} \;\; \dot{\mathbf{x}}^{\mathbf{T}}]^{\mathbf{T}} \in \mathbb{R}^{2n}$, the state-space equation for the smart structure can be written as (Ha *et al.*, 2007):

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}(\mathbf{y})\mathbf{i} + \mathbf{E} \quad , \qquad (1)$$

in which

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\overline{\mathbf{K}} & -\mathbf{M}^{-1}\overline{\mathbf{C}} \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} \mathbf{0}_n & \mathbf{0}_n & \mathbf{0}_n \\ -m_1^{-1}(\bar{c}_{1_1}\dot{x}_1 + \bar{k}_{1_1}x_1) & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & -m_k^{-1}(\bar{c}_{1_k}\dot{x}_k + \bar{k}_{1_k}x_k) & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & -m_n^{-1}(\bar{c}_{1n}\dot{x}_n + \bar{k}_{1_n}x_n) \end{bmatrix}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2, \qquad \mathbf{E}_1 = \begin{bmatrix} \mathbf{0}_n \\ -m_1^{-1}\bar{\alpha}_1 z_1 \\ \vdots \\ -m_k^{-1}\bar{\alpha}_k z_k \\ \vdots \\ -m_n^{-1}\bar{\alpha}_n z_n \end{bmatrix}, \qquad \mathbf{E}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi} \end{bmatrix} \ddot{x}_g,$$

where $\mathbf{A}$ is the system matrix, $\mathbf{B}$ is the gain matrix and $\mathbf{E}$ is the disturbances (earthquake excitation and model uncertainties) of appropriate dimensions with notation given in (Kwok et al., 2006).

To design a structural controller that can perform satisfactorily in the presence of disturbances and uncertainty, different approaches have been proposed such as the linear-quadratic-Gaussian (LQG) control, sliding mode control (SMC), or Lyapunov-based control. However, smart structures embedded with MR dampers require the control currents to be constrained between zero and maximal magnetisation value, which normally results in some quantisation scheme, and hence, would affect the system performance. In this regard, it is attractive to use higher-order sliding mode controllers (Levant & Alelishvili, 2007) as they allow for using rates of change of the current as the control signal while having the ability to remove chattering and also to retain a wide range of robustness.

With differential dampers installed on the k^{th} floor, the motion equation for this floor is as below:

$$m_k\ddot{x}_k + c_k\dot{x}_k + k_k x_k = -[\bar{c}_{d1_k}\dot{x}_k + \bar{k}_{d1k}x_k + (\bar{c}_{d2}\dot{x}_k + \bar{k}_{d2}x_k)i_k + \bar{\alpha}_k z_{dk}] + m_k\ddot{x}_g \qquad (2)$$

where $\dot{x}_k$ and $\ddot{x}_k$ are respectively the storey velocity and acceleration, i_k is the current supplied to the pair of dampers, and where $\bar{\alpha}_k = \bar{\alpha}_{10} + \bar{\alpha}_{11}i_k + \bar{\alpha}_{12}i_k^2$, $z_{d_k} = \tanh(\beta\dot{x}_k + \delta_k \,\mathrm{sign}(x_k))$, and $\delta_k = \delta_{k0} + \delta_{k1}i_k$ (Ha et al., 2007). From (2), we obtain:

$$\ddot{x}_k = -m_k^{-1}\bar{c}_k\dot{x}_k - m_k^{-1}\bar{k}_k x_k - m_k^{-1}(\bar{c}_{d2}\dot{x}_k + \bar{k}_{d2}x_k)i_k - m_k^{-1}\bar{\alpha}_k z_{d_k} + \ddot{x}_g\,, \qquad (3)$$

which has the general form of:

$$\ddot{x}_k = H_k(x_k, t) + G_k(x_k, t, i_k)\,. \qquad (4)$$

For this dynamic equation, let us define a sliding function

$$\sigma_k = \dot{x}_k + \lambda_k x_k,\ \ \lambda_k > 0 \tag{5}$$

with time derivatives $\dot{\sigma}_k = \ddot{x}_k + \lambda_k \dot{x}_k$ and $\ddot{\sigma}_k = \dddot{x}_k + \lambda_k \ddot{x}_k$. Hence,

$$\ddot{\sigma}_k = \frac{dH_k}{dt} + \frac{dG_k}{dt} + \lambda_k (H_k + G_k)\,, \tag{6}$$

where

$$\frac{dH_k}{dt} = -m_k^{-1}\bar{c}_k \ddot{x}_k - m_k^{-1}\bar{k}_k \dot{x}_k + \dddot{x}_g$$

$$\frac{dG_k}{dt} = -m_k^{-1}\,\mathrm{sech}^2(\beta\dot{x}_k + \delta_k\,\mathrm{sign}(x_k))\bar{\alpha}_k \ddot{x}_k - m_k^{-1}(\bar{c}_{d2}\ddot{x}_k + \bar{k}_{d2}\dot{x}_k)i_k$$

$$-\, m_k^{-1}\left[\bar{c}_{d2}\dot{x}_k + \bar{k}_{d2}x_k + (\bar{\alpha}_{11} + 2\bar{\alpha}_{12}i_k)z_{d_k} + \bar{\alpha}_k\,\mathrm{sech}^2(\beta\dot{x}_k + \delta_k\,\mathrm{sign}(x_k)).\delta_{k1}\,\mathrm{sign}(x_k)\right]\frac{di_k}{dt}.$$

Therefore, by denoting $u_k = di_k / dt,$ we can obtain the form:

$$\ddot{\sigma}_k = h_k(t, x_k, i_k) + g_k(t, x_k, i_k)u_k,\ \ h_k = \ddot{\sigma}_k\big|_{u_k=0},\ \ g_k = \frac{\delta}{\delta u_k}\ddot{\sigma}_k \neq 0 \tag{7}$$

where $h_k(t, x_k, i_k) = \dfrac{dH_k}{dt} + \lambda_k(H_k + G_k) - m_k^{-1}\,\mathrm{sech}^2(\beta\dot{x}_k + \delta_k\,\mathrm{sign}(x_k))\bar{\alpha}_k\ddot{x}_k - m_k^{-1}(\bar{c}_{d2}\ddot{x}_k + \bar{k}_{d2}\dot{x}_k)i_k$

$g_k(t, x_k, i_k) = -m_k^{-1}[\bar{c}_{d2}\dot{x}_k + \bar{k}_{d2}x_k + (\bar{\alpha}_{11} + 2\bar{\alpha}_{12}i_k)z_{d_k} + \bar{\alpha}_k\,\mathrm{sech}^2(\beta\dot{x}_k + \delta_k\,\mathrm{sign}(x_k)).\delta_{k1}\,\mathrm{sign}(x_k)].$

Now, if we impose two conditions:

$$0 < k_{mk} \leq g_k(t, x_k, i_k) \leq k_{M_k},\ \text{ and }\ \left|h_k(t, x_k, i_k)\right| \leq C_k\,, \tag{8}$$

then according to (Levant, 2007), there exists a SOSM controller for $u_k(t)$ to drive σ_k and $\dot{\sigma}_k$ asymptotically to zero.

Assume now that (7) holds globally. Then (7) and (8) imply the differential inclusion

$$\ddot{\sigma}_k \in [-C_k, C_k] + [K_{m_k}, K_{M_k}]u_k\,, \tag{9}$$

where C_k, K_{m_k} and K_{M_k} are constants depending on the damper-embedded structure parameters defined in (7). Most SOSM controllers, for example (Levant, 2007; Polyakov & Poznyak, 2008; Levant & Pavlov, 2008, and Boiko et al., 2007), may be considered to steer $\sigma_k, \dot{\sigma}_k$ to 0 in finite time, which is essential for mitigation of quake-induced vibrations in structural control. Since inclusion (9) is not explicitly related to system (3), such controllers are obviously robust with respect to any perturbations, preserving (7). Hence, the problem is now to find a feedback control

$$u_k = \varphi_k(\sigma_k, \dot{\sigma}_k)\,, \tag{10}$$

such that all the trajectories of (9), (10) converge in finite time to the origin $\sigma_k = \dot{\sigma}_k = 0$ of the phase plane.

Differential inclusions (9), (10) are understood here in the Filippov sense (Filippov, 1988), which means that the right-hand set is enlarged in certain convexity and semi-continuity conditions. The function φ_k is assumed to be a locally bounded Borel-measurable function, which is physically true due to inertia of the magneto-rheological fluid. Indeed, in the smart structure control system, it represents the time rate of change of the magnetisation current to the MR dampers. A solution can therefore take any absolutely continuous vector function $(\sigma_k(t), \dot{\sigma}_k(t))$ satisfying (9), (10) for almost all t.

Design of SOSM controllers is greatly facilitated in the 2-dimensional phase plane with coordinates $\sigma_k, \dot{\sigma}_k$ by the simple geometry of any smooth curve that locally divides the plane into two regions. A number of known SOSM controllers may be considered as particular cases of a generalized 2-sliding homogeneous controller (Levant & Pavlov, 2008):

$$u_k = -r_{1_k} sign(\mu_{1_k}\dot{\sigma}_k + \lambda_{1_k}|\sigma_k|^{1/2} sign\sigma_k) - r_{2_k} sign(\mu_{2_k}\dot{\sigma}_k + \lambda_{2_k}|\sigma_k|^{1/2} sign\sigma_k),\ r_{1k}, r_{2k} > 0 \quad (11)$$

Drawing the two switching lines $\mu_{ik}\dot{\sigma}_k + \lambda_{ik}|\sigma_k|^{1/2} sign\sigma_k = 0,\ \mu_{ik}, \lambda_{ik} \geq 0,\ i = 1,2,\ \mu_{ik}^2 + \lambda_{ik}^2 > 0,$ $\mu_{1k}^2 + \mu_{2k}^2 > 0,\ \lambda_{1k}^2 + \lambda_{2k}^2 > 0$, in the phase plane, and considering various possible cases, one can readily check that it is always possible to choose r_{1k}, r_{2k} such that controller (11) yields finite-time stable responses. Indeed, if, for example, $\mu_{1k}, \lambda_{1k} > 0,$ then a 1-sliding mode can easily be induced on the line $\mu_{1k}\dot{\sigma}_k + \lambda_{1k}|\sigma_k|^{1/2} sign\sigma_k = 0$. If for each i one of the coefficients is zero, the twisting controller

$$u_k = -r_{1_k} sign(\sigma_k) - r_{2_k} sign(\dot{\sigma}_k) \quad (12)$$

is obtained with its convergence condition (Polyakov & Poznyak, 2008):

$$(r_{1k} + r_{2k})K_{mk} - C_k > (r_{1k} - r_{2k})K_{Mk} + C_k,\ (r_{1k} - r_{2k})K_{mk} > C_k. \quad (13)$$

Controller (11) may be considered as a generalization of the twisting controller, when the switching takes place on parabolas $\mu_{1k}\dot{\sigma}_k + \lambda_{1k}|\sigma_k|^{1/2} sign\sigma_k = 0$ instead of the coordinate axes.

An important class of SOSM controllers comprises the so-called quasi-continuous controllers, featuring control continuous everywhere except the SOSM $\sigma_k = \dot{\sigma}_k = 0$ itself. Since the 2-sliding condition is of dimension 2, the trajectory in general never hits the 2-sliding manifold. Hence, the control signal, or the time derivative of the damper magnetisation current in (7), remains a time-continuous function all the time. As a result, chattering is significantly reduced. In this paper, we select the following SOSM controller from such a family, as given in (Levant, 2007):

$$u_k = -\alpha_k \frac{\dot{\sigma}_k + \beta_k|\sigma_k|^{1/2} sign\sigma_k}{|\dot{\sigma}_k| + \beta_k|\sigma_k|^{1/2}}. \quad (14)$$

This controller is continuous everywhere except of the origin and vanishes on the parabola $\dot{\sigma}_k + \beta_k|\sigma_k|^{1/2} sign\sigma_k = 0$. With sufficiently large α_k there are such numbers ρ_{1k}, ρ_{2k}, where $0 < \rho_{1k} < \beta_k < \rho_{2k}$, that all the trajectories enter the region between the curves $\dot{\sigma}_k + \rho_{1_k}|\sigma_k|^{1/2} sign\sigma_k = 0$ and remain there.

As described in (7), since the SOSM control is the derivative of the damper current, the current itself is obtained by integration:

$$i_k = \int u_k dt = \int -\alpha_k \frac{\dot{\sigma}_k + \beta_k|\sigma_k|^{1/2} sign\sigma_k}{|\dot{\sigma}_k| + \beta_k|\sigma_k|^{1/2}} dt. \quad (15)$$

Simulation Results

For illustration, a 3-storey structure is considered in which differential dampers are placed on the first floor. A block diagram of the structure is depicted in Fig. 1. The parameters for the smart structure are as below

$$\mathbf{M} = \begin{bmatrix} 98.3 & 0 & 0 \\ 0 & 98.3 & 0 \\ 0 & 0 & 98.3 \end{bmatrix}(kg),\ \mathbf{C} = \begin{bmatrix} 175 & -50 & 0 \\ -50 & 100 & -50 \\ 0 & -50 & 50 \end{bmatrix}(Ns/m),$$

$$\mathbf{K} = \begin{bmatrix} 12.00 & -6.84 & 0 \\ -6.84 & 13.80 & -6.84 \\ 0 & -6.84 & 6.84 \end{bmatrix} \times 10^5 (N/m)$$

$$i_1 = i,\ i_2 = i_3 = 0,\ \overline{\alpha}_1 = 64 + 1836i - 488i^2,\ \overline{\alpha}_2 = \overline{\alpha}_3 = 0,$$
$$z_{d1} = \tanh(100\dot{x}_1 + (0.58 + 0.30i)\,\text{sign}(x_1)),\ z_{d2} = z_{d3} = 0.$$

A. Controlled responses

Firstly, a number of excitations including step, random, sinusoidal and square waveforms are considered. The responses of the structure, for example to random and sinusoidal excitations are illustrated in Figs. 2 and 3, respectively. According to the figures, the proposed controller can mitigate effectively the affect of the external disturbances by directly controlling the damper current.

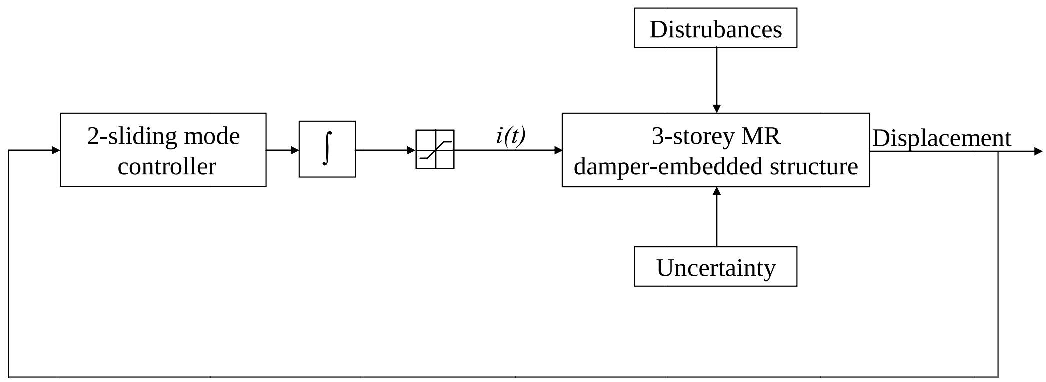
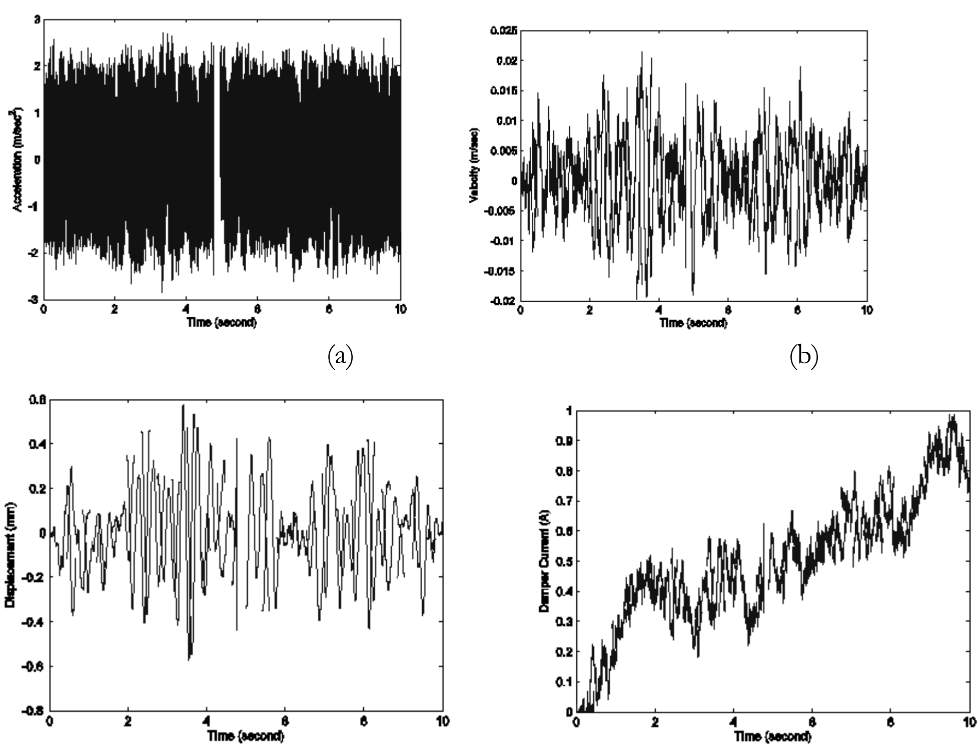


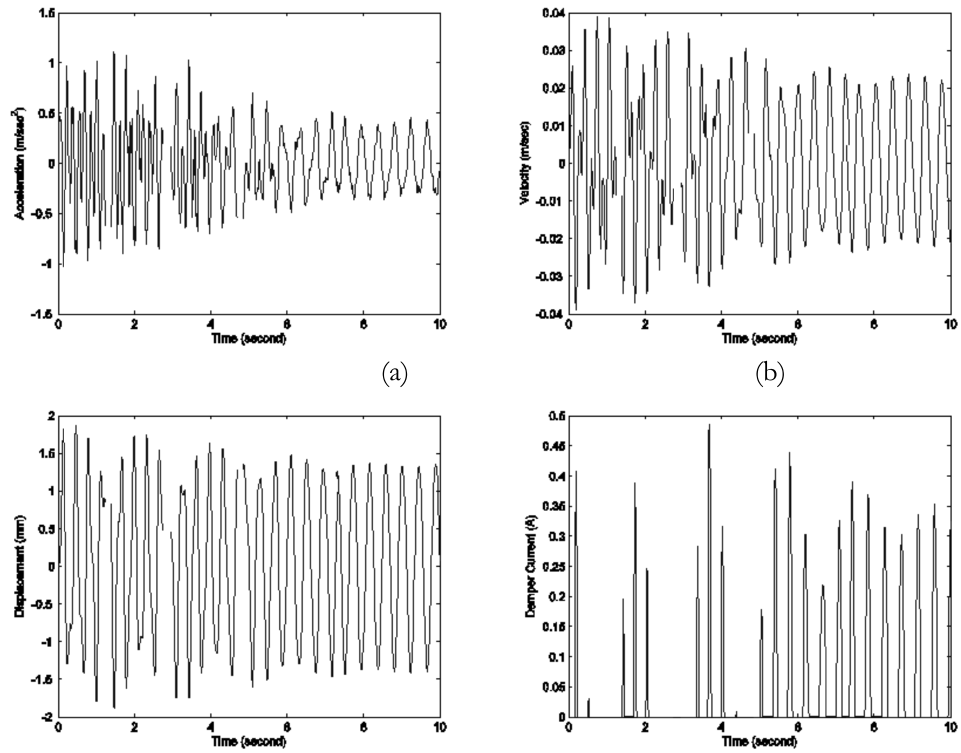
Figure 1. Second order sliding mode controlled 3-storey MR damper-embedded structure



(a) (b)

(c) (d)

Figure 2. Random excitation responses: (a) 1st floor Acceleration, (b) 1st floor Velocity, (c) 1st floor Displacement, and (d) Damper Current



(a) (b)

(c) (d)

Figure 3. Sinusoidal excitation responses: (a) 1st floor Acceleration, (b) 1st floor Velocity, (c) 1st floor Displacement, and (d) Damper Current.

We consider next responses to the scaled records of known earthquakes. The first-storey time responses for the Kobe earthquake are shown in Fig. 4. The responses display significant reductions in displacement, velocity and acceleration. Similar responses can be obtained for scaled records of the El-Centro, Northridge, and Hachinohe. The SOSM controller indicates the system stability in most of earthquake period except where the magnitude is too large. However, the derivative returns to negative and the building structure under control becomes stable.

Performance Evaluation

Apart from six performance criteria given in (Ha et al., 2007), we consider here with reference to (Spencer et al., 1999) further four evaluation criteria, two for peak responses and two for RMS responses. They are:

Peak inter-storey drift ratio

$$J_7 = \max\left\{ \frac{\max\left\{\left|\bar{x}_{k,c}(t)\right|\right\}}{\max\left\{\left|x_{k,u}(t)\right|\right\}} \right\}, \qquad (16)$$

whereby the maximum drifts are normalized with respect to the uncontrolled peak displacement, subscript $k = 1,\ldots,3$ stands for the storey index and subscripts c, u denote controlled and uncontrolled cases.

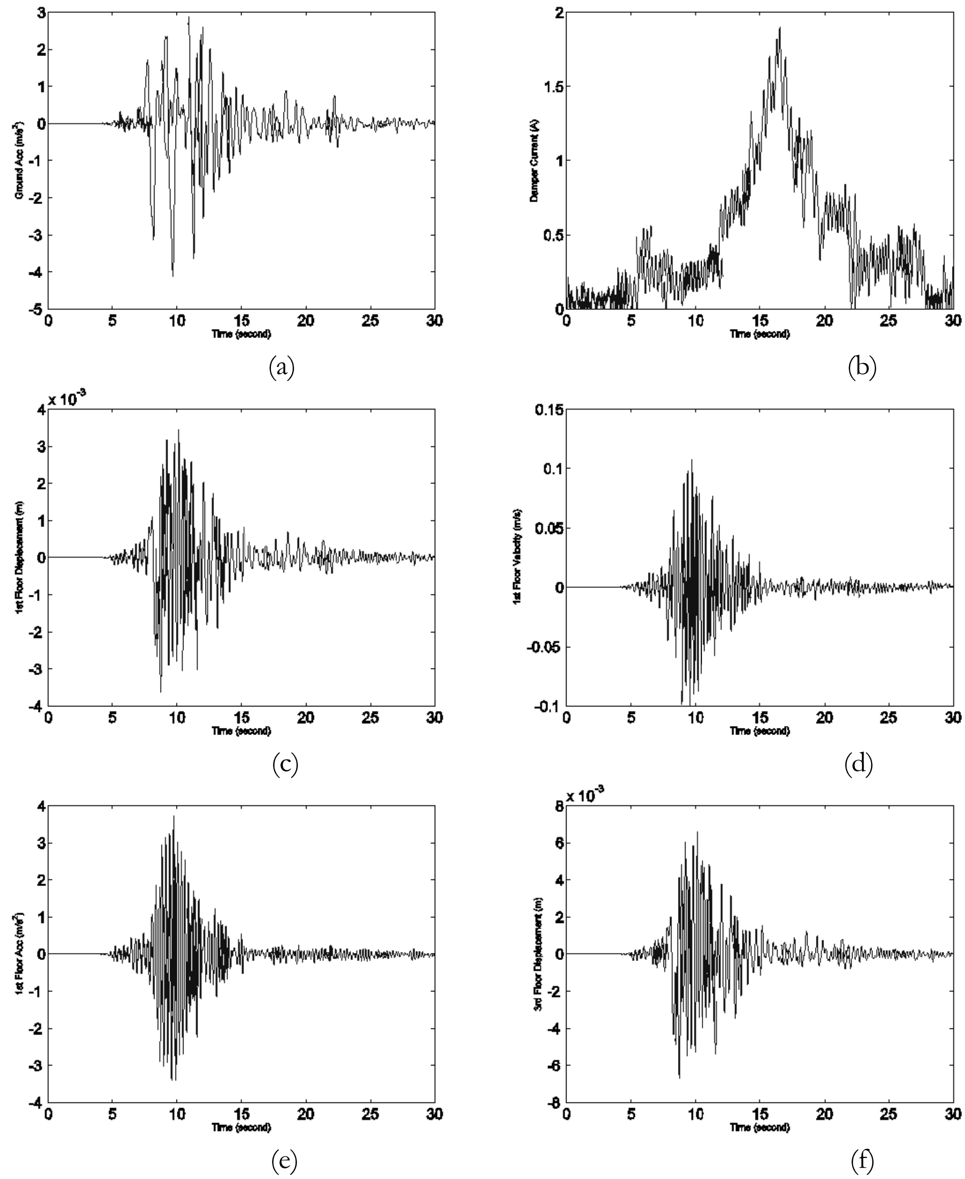


Figure 4. *Kobe* Earthquake Responses: (a) Excitation, (b) Damper Current, (c) 1st floor Displacement, (d) 1st floor Velocity, (e) 1st floor Acceleration, and (f) 3rd floor Displacement

Peak storey acceleration ratio

$$J_8 = \max\left\{\frac{\max\{|\ddot{x}_{k,c}(t)|\}}{\max\{|\ddot{x}_{k,u}(t)|\}}\right\}, \quad (17)$$

whereby the accelerations are normalized by the peak uncontrolled acceleration.

Maximum RMS inter-storey drift ratio

$$J_9 = \max\left\{\frac{\tilde{\tilde{x}}_{k,c}(t)}{\tilde{x}_{k,u}(t)}\right\}, \quad (18)$$

which evaluates the ability of minimizing the maximum RMS inter-storey drift due to all admissible ground motions. The notation $\tilde{x} = \sqrt{T^{-1} \sum \{\delta_t x_k^2(t)\}}$ is for the root-mean-square (RMS) values, δ_t is the sampling time, and T is the total excitation duration.

Maximum RMS storey acceleration ratio

$$J_{10} = \max \left\{ \frac{\tilde{\ddot{x}}_{k,c}(t)}{\tilde{\ddot{x}}_{k,u}(t)} \right\} \tag{19}$$

that is given in terms of the maximum RMS absolute acceleration with respect to the uncontrolled case.

Table I below summarizes all the criteria evaluated using the simulated responses with the proposed SOSM controller, typically, for the second floor. As can be seen, all the corresponding ratios using the SOSM controller are much smaller than those obtained in the uncontrolled case and further improved from the Lyapunov-based method (Ha et al., 2007).

Table I. Response ratios:

2nd Floor	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
El-Centro	0.148	0.132	0.145	0.040	0.069	0.076	0.068	0.078	0.126	0.555
Kobe	0.152	0.127	0.146	0.043	0.0620	0.079	0.061	0.081	0.127	0.454
Hachinohe	0.103	0.061	0.099	0.026	0.041	0.051	0.042	0.055	0.059	0.334
Northridge	0.130	0.103	0.125	0.032	0.047	0.060	0.047	0.070	0.101	0.307

Conclusions

We have presented an effective scheme for semi-active control of smart structures embedded with pairs of MR dampers. Using the proposed control system, the building structures are shown to be capable of effectively suppressing vibrations due to earthquakes by directly controlling the damper magnetization currents. Differential configuration for the dampers is used to remove the problem arising from damper offset forces. For this semi-active structural control system, a second-order sliding mode controller is proposed to obtain the time rate of change of the supplied currents to the dampers. These magnetisation currents, after integration, can efficiently control the fluid to yield required damping forces for the structures with provident power consumption and improved control performance. Simulation results for a three-floor building model are evaluated using a set of performance criteria. The results obtained demonstrate the effectiveness of the proposed scheme under constraints of the control signals in mitigation of seismic vibrations of MR damper embedded smart structures.

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