# A BEHAVIORAL MODEL OF INVESTOR SENTIMENT IN LIMIT ORDER MARKETS

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ABSTRACT. This paper introduces a behavioral sentiment model to explore the stylized facts in limit order markets. Simulation results show that both the noise and sentiment trading can generate the absence of autocorrelation in returns, long memory in the absolute returns and bid-ask spread, and the hump shaped mean depth profile of the order book. However, sentiment trading plays a unique role in explaining the fat tails in the return distribution, long memory in the trading volume, an increasing and non-linear relationship between trade imbalance and mid-price returns, and also the diagonal effect or event clustering in order submission types, all of which cannot be explained by noise trading. Therefore, behavioral sentiment is an important driving force behind some of the well-documented stylized facts in limit order markets.

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## 1. INTRODUCTION

Recently, various stylized facts in limit order markets have been documented in market microstructure literature. According to surveys by Chakraborti, Toke, Patriarca and Abergel (2011a), Chen, Chang and Du (2012) and Gould, Porter, Williams, Fenn and Howison (2013), apart from the stylized facts in the time series of returns, including fat tails, the absence of autocorrelation in returns, volatility clustering, long memory in the absolute returns, the limit order has its own stylized facts, such as long memory in the bid-ask spread and trading volume, hump shapes in mean depth profiles of order books, non-linear relationships between trade imbalance and mid-price return, and diagonal effect or event clustering in order submission types, which are the most common and important statistical regularities in limit order markets. They have become the most important criteria to justify the explanatory power of financial market models, in particular agent-based models in limit order markets with a continuous double auction.

Among these agent-based models of market microstructure that are able to replicate some of the stylized facts, they are either zero-intelligence models or heterogenous agent models (see Chakraborti, Toke, Patriarca and Abergel (2011b), Chen et al. (2012) and Gould et al. (2013)). The zero-intelligence models assume that traders' behavior is very simple (without learning or strategy), and the stylized facts are generated by trading mechanism, instead of agents' strategic behavior. Some of them are able to generate fat tails, but only a few can generate volatility clustering (Raberto and Cincotti (2005)), and event clustering in order submission types (Ladley and Schenk-Hoppé (2009)). Different from the zero-intelligence models, the heterogenous agent models consider agents' strategic behaviors as potential explanations to the stylized facts. For example, chartist-fundamentalist models find that the chartist behavior contributes to fat tails, volatility clustering (see Anufriev and Panchenko (2009), Chiarella, Iori and Perellò (2009) and Chiarella, He and Pellizzari (2012)) and the diagonal effect in correlations of some order submission types (Kovaleva and Iori (2014)). However, replicating most of these stylized facts simultaneously remains very challenging for agent-based models.[1] As Gould et al. (2013) point out, "*no single model has yet been capable of simultaneously reproducing all of the statistical regularities, and there is no clear picture about how the stylized facts emerge as a consequence of the actions of many heterogeneous traders*".

Motivated by Chen et al. (2012) and in particular Gould et al. (2013), in this paper we explore the unique role played by behavioral sentiment in explaining some of the stylized facts in limit order markets. Different for the zero-intelligence models and heterogenous agent models (chartist-fundamentalist models), we show that behavioral sentiment plays an important role in explaining the stylized facts in limit order markets. In our model, traders are heterogeneous in their investment time horizons. Traders are bounded rational in the sense that although they observe changes in the fundamental value, they may underreact or overreact to those changes. More precisely, we model behavioral sentiment by following Barberis, Shleifer and Vishny (1998) (henceforth BSV98). The sentiment or *BSV* traders update their beliefs following a learning scheme using Baye's rule, however they believe that the mean growth rate of the observed fundamental value follows a Markov switching process, whereas the true process is a random walk. Furthermore, we compare the model with BSV traders to the one with *noise* traders who believe that the mean growth rate of the observed fundamental value is random. Our results show that certain stylized facts can only be generated in the market with BSV traders.

The modeling approach follows Chiarella et al. (2009), in which traders are utility maximizers and the order sizes are optimal given their submitted prices. There is a

[1]Most of agent-based financial models focus on daily frequency, instead of intra-day, see Chen et al. (2012).

risky asset and a risk-free asset and traders cannot short-sell both assets. The short-sale constraint puts an upper bound on the submission price at which a trader would sell all her current holdings of the risky asset, and also a lower bound at which a trader would use all her cash to purchase shares of the risky asset. Different from Chiarella et al. (2009) who assume the submission price is randomly chosen between the upper and lower bounds, we assume that the submission price is either equal to the upper bound or the lower bound and the probability of buy/sell depends on the distance between the upper/lower bound and the no-trade price, at which it is optimal not to trade at all.

Our main finding is that both noise and BSV trading can generate the absence of autocorrelation in returns, volatility clustering, long memory in the bid-ask spread, and the hump shape in the mean depth profile of the order book.[2] However, BSV trading leads to fat tails in the return distribution, long memory in the trading volume, an increasing and non-linear relationship between trade imbalance and mid-price return, and event clustering in order submission types, all of which cannot be explained by noise trading, which means that behavioral sentiment rather than noise trading is the driving force behind these stylized facts.

The rest of paper is organized as follows. The model is outlined in Section 2. Section 3 compares the stylized facts generated by in a market populated by BSV traders and in a market populated by noise traders using simulation analysis. Section 4 concludes.

## 2. The Model

We consider a limit order market with many heterogeneous traders who arrive the market and submit orders with different trading time horizons. Trader $i$ with a trading time horizon $\tau^i$ has a probability $1/\tau^i$ of entering the market at the start of each period,

[2]The differences are that BSV trading leads to significantly stronger serial correlations in the bid-ask spread and a hump share closer to the best quotes in the mean depth profile of the order book shape, comparing to the noise trading.

which implies that trader $i$ is expected to arrive at the market every $\tau^i$ periods and each period corresponds to a short time interval such as one minute. We assume that $\tau^i$ follows a uniform distribution between the shortest horizon $\underline{\tau}$ and the longest horizon $\bar{\tau}$, and restrict $\tau^i$ to be positive integers. To simplify the analysis, we assume that the fundamental price $F_t$ follows

$$\ln(F_{t+1}) = \ln(F_t) + \sigma\epsilon_{t+1}, \qquad \epsilon_{t+1} \overset{i.i.d}{\sim} \mathcal{N}(0,1), \tag{2.1}$$

which means that the log fundamental price is a martingale with $\mathbb{E}_t[\ln(F_{t+\tau})] = \ln(F_t)$ for $\tau \geq 1$ or the growth rate of the fundamental value is a random walk, where the volatility per period is measured by $\sigma$. Traders do not monitor the market continuously. When trader $i$ enters the market at time $t$, she knows the fundamental value of the current period $F_t$, and the historical fundamental values every $\tau^i$ periods, but she does not know the fundamental value process (2.1). Her information set is given by $I_t^i \equiv \{F_t, F_{t-\tau^i}, \cdots, F_{t-N^i\tau^i}\}$, where $N^i$ measures the length of her observations.[3]

2.1. **Behavioral Sentiment.** We follow BSV98 and assume that traders have behavioral sentiment in their beliefs of the fundamental value, and are thus called *BSV traders*. More precisely, trader $i$ believes that the log fundamental price $\ln(F_t)$ follows

$$\ln(F_{t+\tau^i}) = \ln(F_t) + \theta_{t+\tau^i} + \sigma\epsilon_{t+\tau^i}, \tag{2.2}$$

[3]Traders also have information about past transaction prices. If no transaction occurs at a given time $t$, the mid-price of the best ask price $a_t$ and the best bid price $b_t$, that is $p_t = \frac{1}{2}(a_t + b_t)$. If there are no bids or asks in the book, then the previous transaction price is used as a proxy.

where $\epsilon_{t+\tau^i} \overset{i.i.d}{\sim} \mathcal{N}(0, \sqrt{\tau^i})$ and the mean growth rate $\theta_{t+\tau^i}$ follows a two-state Markov chain with transition matrix among two states $\{-\theta^i, \theta^i\}$,

$$\begin{array}{|c|cc|} \hline & \theta_{t+\tau^i} = \theta^i & \theta_{t+\tau^i} = -\theta^i \\ \hline \theta_t = \theta^i & \pi_{t+\tau^i} & 1 - \pi_{t+\tau^i} \\ \theta_t = -\theta^i & 1 - \pi_{t+\tau^i} & \pi_{t+\tau^i} \\ \hline \end{array} \tag{2.3}$$

where $\theta^i = \sigma\sqrt{\tau^i}$. Therefore, trader $i$ believes that there is a good state $\theta^i$ and a bad state $-\theta^i$ in which the mean growth rate of the fundamental price is positive and negative, respectively. Given the current state, the probability of staying in the same state is given by $\pi_{t+\tau^i}$. When $\theta^i$ is different from zero, trader $i$ exhibits behavioral sentiment in the same spirit as in BSV98, believing that future growth rate of the fundamental value is predcitable[4]. Furthermore, as in BSV98, trader $i$ believes that the transition probability $\pi_{t+\tau^i}$ also follows a Markov chain of two states $\{\pi_L, \pi_H\}$ with transition matrix,

$$\begin{array}{|c|cc|} \hline & \pi_{t+\tau^i} = \pi_L & \pi_{t+\tau^i} = \pi_H \\ \hline \pi_t = \pi_L & 1 - \lambda_1 & \lambda_1 \\ \pi_t = \pi_H & \lambda_2 & 1 - \lambda_2. \\ \hline \end{array} \tag{2.4}$$

meaning that trader $i$ believes that there is one state $\pi_t = \pi_L$ in which the mean growth rate is more likely to remain the same as the last period and a state ($\pi_t = \pi_H$) in which the mean growth rate is more likely to switch from one state to another, in which $\lambda_1$ and $\lambda_2$ measure the switching intensities.

Traders do not observe the mean growth rate $\theta_t$ and they update their probability beliefs about $\theta_t$ and $\pi_t$ using Baye's rule. More explicitly, let

$$q^i_{\theta,t} \equiv \mathbb{P}(\theta_t = \theta^i | I^i_t), \qquad q^i_{\pi,t} \equiv \mathbb{P}(\pi_t = \pi_L | I^i_t),$$

[4]When $\theta^i = 0$, trader $i$ becomes fully rational believing the true fundamental value process (2.1).

where $I_t^i \equiv \{F_t, F_{t-\tau^i}, \cdots, F_{t-N\tau^i}\}$. Let $R_{t+\tau^i} \equiv \ln(F_{t+\tau^i}/F_t)$. Trader $i$ updates her probabilities after observing $R_{t+\tau^i}$ according to

$$q^i_{\theta,t+\tau^i} = q^i_{\pi,t}\mathbb{P}(\theta_{t+\tau^i} = \theta^i|\pi_t = \pi_L, R_{t+\tau^i}) + (1-q^i_{\pi,t})\mathbb{P}(\theta_{t+\tau^i} = \theta^i|\pi_t = \pi_H, R_{t+\tau^i}),$$
$$q^i_{\pi,t+\tau^i} = q^i_{\theta,t}\mathbb{P}(\pi_{t+\tau^i} = \pi_L|\theta_t = \theta^i, R_{t+\tau^i}) + (1-q^i_{\theta,t})\mathbb{P}(\pi_{t+\tau^i} = \pi_L|\theta_t = -\theta^i, R_{t+\tau^i}),$$

where

$$\mathbb{P}(\theta_{t+\tau^i} = \theta^i|\pi_t, R_{t+\tau^i}) = \frac{\mathbb{P}(\theta_{t+\tau^i} = \theta^i|\pi_t)\mathbb{P}(R_{t+\tau^i}|\theta_{t+\tau^i} = \theta^i)}{\sum_{\theta_{t+\tau^i}\in\{\theta^i,-\theta^i\}}\mathbb{P}(\theta_{t+\tau^i} = \theta|\pi_t)\mathbb{P}(R_{t+\tau^i}|\theta_{t+\tau^i})},$$
$$\mathbb{P}(\pi_{t+\tau^i} = \pi_L|\theta_t, R_{t+\tau^i}) = \frac{\mathbb{P}(\pi_{t+\tau^i} = \pi_L)\mathbb{P}(R_{t+\tau^i}|\theta_t, \pi_{t+\tau^i} = \pi_L)}{\sum_{\pi_{t+\tau^i}\in\{\pi_L,\pi_H\}}\mathbb{P}(\pi_{t+\tau^i})\mathbb{P}(R_{t+\tau^i}|\theta_t, \pi_{t+\tau^i})},$$

$$\mathbb{P}(R_{t+\tau^i}|\theta_{t+\tau^i}) \propto \exp\left(-\frac{(R_{t+\tau^i}-\theta_{t+\tau^i})^2}{\sigma^2\tau^i}\right),$$
$$\mathbb{P}(R_{t+\tau^i}|\theta_t, \pi_{t+\tau^i}) \propto \pi_{t+\tau^i}\exp\left(-\frac{(R_{t+\tau^i}-\theta_t)^2}{\sigma^2\tau^i}\right) + (1-\pi_{t+\tau^i})\exp\left(-\frac{(R_{t+\tau^i}+\theta_t)^2}{\sigma^2\tau^i}\right)$$

and

$$\mathbb{P}(\theta_{t+\tau^i} = \theta^i|\pi_t) = q^i_{\theta,t}\pi_t + (1-q^i_{\theta,t})(1-\pi_t),$$
$$\mathbb{P}(\theta_{t+\tau^i} = -\theta^i|\pi_t) = q^i_{\theta,t}(1-\pi_t) + (1-q^i_{\theta,t})\pi_t,$$
$$\mathbb{P}(\pi_{t+\tau^i} = \pi_L) = q^i_{\pi,t}(1-\lambda_1) + (1-q^i_{\pi,t})\lambda_2,$$
$$\mathbb{P}(\pi_{t+\tau^i} = \pi_H) = q^i_{\pi,t}\lambda_1 + (1-q^i_{\pi,t})(1-\lambda_2)$$

for $\theta_t, \theta_{t+\tau^i} \in \{-\theta^i, \theta^i\}$ and $\pi_t, \pi_{t+\tau^i} \in \{\pi_L, \pi_H\}$.

Given her estimated probabilities $q^i_{\pi,t}$ and $q^i_{\theta,t}$, trader $i$ makes a $\tau^i$-period ahead forecast of the mean and variance of log fundamental price

$$\mathbb{E}^i_t[\ln(F_{t+\tau^i})] = \ln(F_t) + \mathbb{E}^i_t[\theta_{t+\tau^i}], \tag{2.5}$$

$$\mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i + (\theta^i)^2 - (\mathbb{E}_t^i[\theta_{t+\tau^i}])^2, \tag{2.6}$$

where

$$\begin{aligned}\mathbb{E}_t^i[\theta_{t+\tau^i}] = {} & \mathbb{P}(\pi_{t+\tau^i} = \pi_L|I_t^i)\begin{pmatrix} q_{\theta,t}^i & 1-q_{\theta,t}^i \end{pmatrix}\begin{pmatrix} \pi_L & 1-\pi_L \\ 1-\pi_L & \pi_L \end{pmatrix}\begin{pmatrix} \theta^i \\ -\theta^i \end{pmatrix} \\ & + \mathbb{P}(\pi_{t+\tau^i} = \pi_H|I_t^i)\begin{pmatrix} q_{\theta,t}^i & 1-q_{\theta,t}^i \end{pmatrix}\begin{pmatrix} \pi_H & 1-\pi_H \\ 1-\pi_H & \pi_H \end{pmatrix}\begin{pmatrix} \theta^i \\ -\theta^i \end{pmatrix}.\end{aligned}$$

Note that without sentiment ($\theta^i = 0$), the belief of BSV traders in (2.5) and (2.6) becomes

$$\mathbb{E}_t^i[\ln(F_{t+\tau^i})] = \ln(F_t), \qquad \text{and} \qquad \mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i, \tag{2.7}$$

which characterizes the true fundamental value process.

2.2. **Optimal Demand.** We now consider the investment decision of traders. Following Chiarella et al. (2009), we assume that traders maximize a CARA utility function of their wealth. When trader $i$ enters the market at time $t$, she determines the optimal demand on the risky asset to maximize the expected utility of her wealth at the end of her trading horizon at time $t + \tau^i$ based on her belief. Let $s_t^i$ be the number of shares of the risky asset and $c_t^i$ be the amount of cash trader $i$ holds at time $t$. Trader $i$ submits an order with price $p_t^i$ and quantity $z_t^i$, thus her wealth at the end of the trading period is given by[5]

$$W_{t+\tau^i}^i = (s_t^i + z_t^i)p_{t+\tau^i} + c_t^i - z_t^i p_t^i. \tag{2.8}$$

[5]We assume the risk-free rate is zero.

We assume that trader $i$'s expected utility can be approximated by the conditional mean and variance of her terminal wealth, thus her objective can be written as

$$\max_{z_t^i} \left( \mathbb{E}_t^i[W_{t+\tau^i}] - \frac{\alpha^i}{2} \mathbb{V}_t^i[W_{t+\tau^i}] \right), \tag{2.9}$$

where $\alpha^i$ measures trader $i$'s risk aversion. Solving equation (2.9) yields

$$z_t^{i*} = \frac{\mathbb{E}_t^i[r_{t+\tau^i}]}{\alpha^i p_t^i \mathbb{V}_t^i[r_{t+\tau^i}]} - s_t^i,$$

where $r_{t+\tau^i} \equiv p_{t+\tau^i}/p_t^i - 1$ is the rate of return over the period $[t, t+\tau^i]$. For convenience, we use $r_{t+\tau^i} \approx \ln(p_{t+\tau^i}/p_t^i)$, which is a good approximation when $\tau^i$ is small. Thus, the optimal demand in equation (2.10) becomes

$$z_t^{i*} = \frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^i)}{\alpha^i p_t^i \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} - s_t^i. \tag{2.10}$$

2.3. **Order submission.** Now to determine the submission price $p_t^i$ for trader $i$, we assume that traders cannot short sell and nor can they borrow at the risk-free rate. For trader $i$, this implies that

$$z_t^{i*} \geq -s_t^i \qquad \text{and} \qquad z_t^{i*} p_t^i \leq c_t^i.$$

From which we obtain the following low and upper bounds for the submission price $p_t^i$ of trader $i$,

$$p_t^{i,m} \leq p_t^i \leq p_t^{i,M},$$

where

$$p_t^{i,M} = \exp\{\mathbb{E}_t^i[\ln(p_{t+\tau^i})]\}$$

and $p_t^{i,m}$ is determined implicitly by

$$\frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^{i,m})}{\alpha^i \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} = c_t^i + s_t^i p_t^{i,m}.$$

Define $p_t^{i*}$ as the *no trade price* for agent $i$, which solves

$$\frac{\mathbb{E}_t^i[\ln(p_{t+\tau^i})] - \ln(p_t^{i*})}{\alpha^i p_t^{i*} \mathbb{V}_t^i[\ln(p_{t+\tau^i})]} = s_t^i.$$

It can be show that

$$p_t^{i,m} \leq p_t^{i*} \leq p_t^{i,M}.$$

Furthermore, we assume that

$$\mathbb{E}_t^i[\ln(p_{t+\tau^i})] = \mathbb{E}_t^i[\ln(F_{t+\tau^i})], \qquad \mathbb{V}_t^i[\ln(p_{t+\tau^i})] = \mathbb{V}_t^i[\ln(F_{t+\tau^i})],$$

meaning that traders belief about the future price is determined by their belief about the future fundamental value.

The order submission of trader $i$ is in the following way. When entering the market, trader $i$ tries to either sell $s_t^i$ shares of the risky asset at a maximum price of $p_t^{i,M}$ or buy $c_t^i/p_t^{i,m}$ shares of the risky assets at a minimum price of $p_t^{i,m}$. If the best ask $a_t < p_t^{i,m}$ or the best bid $b_t > p_t^{i,M}$, then trader $i$ submits a market order to buy or order. Otherwise she submits a limit order at price $p_t^i$ to buy (when $z_t^{i*} > 0$) or sell (when $z_t^{i*} < 0$) and the number of shares is determined by (2.10).[6] Furthermore, we assume that the probability of submitting a buy or sell order is given by

$$P_{buy} \equiv \mathbb{P}(z_t^{i*} = c_t^i/p_t^{i,m}) = \frac{p_t^{i*} - p_t^{i,m}}{p_t^{i,M} - p_t^{i,m}},$$
$$P_{sell} \equiv \mathbb{P}(z_t^{i*} = -s_t^i) = \frac{p_t^{i,M} - p_t^{i*}}{p_t^{i,M} - p_t^{i,m}}.$$

Intuitively, the further the no-trading price is away from the minimum (maximum) price, the higher the probability to buy (sell).

[6]Note that this way of determining the submission price is different from Chiarella et al. (2009), where traders randomly pick a price $p_t^i \in [p_t^{i,m}, p_t^{i,M}]$, which may not be the optimal price for the optimal demand.

Upon entering the market, trader $i$ chooses to either place a market order or a limit order which will be stored in the limit order book. A transaction occurs when a market order hits a quote on the opposite side of the order book. As usual, limit orders are executed using both price and time priorities. At time $t$, trader $i$ submits a buy or sell order with price level $p_t^i$ and order size $z_t^{i*}$. The order leads to a market buy when $p_t^i \geq a_t$ or market sell when $p_t^i \leq b_t$, where $b_t$ and $a_t$ are the best bid and ask price respectively. If there is enough depth at the best bid or best ask, then the entire order submitted by trader $i$ is executed at $a_t$ or $b_t$; otherwise part of the order may be executed at prices further away from the best bid or ask or it may become a limit order with price $p_t^i$ as the new best bid or ask price.[7]

| | buy/sell | | Limit/Market | Volume |
|---|---|---|---|---|
| $X \leq P_{buy}$ | buy | $a_t^1 \leq p_t^{i,m}$ | Market order | $c_t^i/p_t^{i,m}$ |
| $X \leq P_{buy}$ | buy | $a_t^1 > p_t^{i,m}$ | Limit order | $c_t^i/p_t^{i,m}$ |
| $X > P_{buy}$ | sell | $b_t^1 \geq p_t^{i,M}$ | Market order | $s_t^i$ |
| $X > P_{buy}$ | sell | $b_t^1 < p_t^{i,M}$ | Limit order | $s_t^i$ |

TABLE 2.1. Summary of submission rules of trader $i$, $0 \leq X \leq 1$ is drawn from a uniform distribution.

Table 2.1 summarizes the order submission rules of trader $i$ in which $X$ is drawn from a uniform distribution on $[0, 1]$. Note that trader $i$'s submission price is either $p_t^{i,m}$ (for buy orders) or $p_t^{i,M}$ (for sell orders). If the depth at the best bid (ask) is not enough to fully satisfy the order size, the remaining volume of the order is executed against limit orders in the book. The trader thus takes the next best buy (sell) order and repeats the process as many times as necessary until the order is fully executed. This mechanism applies when quotes of these orders are above (below) price $p_t^{i,M}$ ($p_t^{i,m}$). If the limit order is not matched by the time $t + \tau^i$, it is removed from the book.

[7]There can be multiple traders who arrive at the market at the same time with same order, in which case we assume those traders trade in a randomized order.

2.4. **Simulation setting.** In the simulations, the trading time horizons $\tau^i$ of trader $i$ follows a uniform distribution between $\tau(1-\Delta)$ and $\tau(1+\Delta)$ where the reference investment horizon $\tau = 60$ (equal to one hour) and the range is specified by $\Delta = 0.5$. Furthermore we restrict the investment horizons to be integers. Each trader $i$ is initially given $s_0^i = 10$ shares of the risky asset and $c_0^i = s_0^i F_0$ amount of cash, where the initial fundamental price $F_0 = 50$. At the beginning of each period $t$, each trader $i$ has a probability $1/\tau^i$ of entering the market. Traders observe the fundamental value $F_t$ after entering the market before submitting an order. Upon entering the market, trader $i$ cancels any of her unmatched limit order and submits a new order according the order submission rules in Table 2.1. The volatility of the log fundamental price per period is set to $\sigma = 4$ basis points (bp) and the risk aversion is set to $\alpha^i = 0.1$ for every trader $i$.[8] Furthermore, we follow BSV98 and assume $\pi_L = \frac{1}{3}$, $\pi_H = \frac{3}{4}$, $\lambda_1 = 0.1$ and $\lambda_2 = 0.3$. Upon entering the market, trader $i$ estimates the probabilities $q_t^{i,\pi}$ and $q_t^{i,\theta}$ based on her information $I_t^i = \{F_t, F_{t-\tau^i}, \cdots, F_{t-N^i\tau^i}\}$ and we choose $N^i = 60$ and the initial priors $q_{t-N^i\tau^i}^{i,\pi} = q_{t-N^i\tau^i}^{i,\theta} = 0.5$. The minimum tick size is given by $0.01$.

Apart from the BSV traders, we assume there are also liquidity traders. Liquidity traders' trading horizons and arrival rates follow the same uniform distribution as the BSV traders. However they choose between buy and sell orders and between market and limit orders randomly with equal probability. Therefore, liquidity traders either provide or demand liquidity with equal probability. The order size is uniformly distributed between 1 and 10. Moreover, their limit orders are always at the best bid or ask price. We assume there are 900 BSV traders and 100 liquidity traders, which makes the total number of traders equal to 1000.

[8] If each trading period is treated as one minute, then the annualized volatility is approximately $10\%$ p.a, which is the same as Chiarella and Iori (2002). Moreover, we set the risk aversion to $0.1$ which is the average risk aversion of the agents in Chiarella et al. (2009).

Moreover, as a benchmark for comparison, we also consider a situation where the BSV traders are replaced by traders who's beliefs are given by

$$\mathbb{E}_t^i[\ln(F_{t+\tau^i})] = \ln(F_t) + \tilde{\theta}_t^i \qquad \text{and} \qquad \mathbb{V}_t^i[\ln(F_{t+\tau^i})] = \sigma^2\tau^i + (\theta^i)^2 - (\tilde{\theta}_t^i)^2,$$

where $\tilde{\theta}_t^i \overset{i.i.d}{\sim}$ Uniform$[-\theta^i, \theta^i]$. This means that, in this case, trader's belief simply deviates randomly from the objective (true) belief. We thus call such traders as *noise traders*. By comparing with the benchmark model, we aim to distinguish between the effect of sentiment trading and that of noise trading. In particular, we compare the simulation results in a market populated by (900 BSV traders+100 liquidity traders) with a market populated by (900 noise traders + 100 liquidity traders). If certain stylized facts can be replicated with BSV traders but not with noise traders in the market, then it would provide support for behavioral sentiment being the driving force behind those stylized facts rather than noise trading. The results reported are the outcome of 30 simulations of 72,000 periods with the first 60,000 steps used as a burn-in period.[9]

## 3. Replication of Stylized facts

As we have discussed in the introduction, replication of most of the stylized facts in limit order book markets presents a serious challenge to the current literature (Gould et al. (2013)) . In this section, we examine the effect of investor sentiment in replicating some of these stylized facts. We also compare the results to the model with noise traders to distinguish between the effect of sentiment trading and that of noise trading.

3.1. **Fail tails in the return distribution.** Fat tails in the return distribution are well documented in empirical studies(such as Cont (2001) and Chakraborti et al. (2011a)). Table 3.1 shows that the market with BSV traders leads to much fatter tails in return distribution compared to the market with noise traders. Moreover, the BSV trading

[9]The results remain similar for different simulations.

also leads to a negative skewness whereas skewness is close to zero with the noise trading.

| | Mean | Skewness | Kurtosis |
|---|---|---|---|
| BSV | -8.90E-07 | -0.15 | 41.98 |
| Noise | -8.70E-07 | 0.03 | 3.15 |

TABLE 3.1. Mean, skewness and kurtosis of mid-price returns.

3.2. **Absence of autocorrelation in returns.** Empirical studies have shown that except weakly negative autocorrelations (ACs) on very short time scales, return series do not display any significant ACs in many markets (Chakraborti et al. (2011a) and Gould et al. (2013)). In particular, Gould et al. (2013) report that several empirical studies find some negative ACs of the return of mid-price over short lags in some hybrid markets but they disappear very quickly for long lags. Figure 3.1 shows the ACs of returns for market with the BSV traders and the market with the noise traders. Both markets exhibit the empirically observed pattern in the ACs of mid-price returns. Apart from a significant and negative AC in the first lag, the ACs for all other lags are insignificant, though there are some small and positive ACs in the first few lags for the market with the BSV traders.

3.3. **Volatility clustering and long memory in volatility.** Volatility clustering is a common stylized fact in stock markets and an important justification for agent-based models. For zero-intelligence models, such as LiCalzi and Pellizzari (2003), is able to generate fat tails but not the volatility clustering; and Raberto and Cincotti (2005) find that the volatility clustering only occurs when the zero-intelligence agents take into account of the volatility of the previous period when submitting orders. For heterogenous agent models, such as Chiarella and Iori (2002), point out that fundamentalist, chartist and noise are necessary in some form to generate volatility clustering.

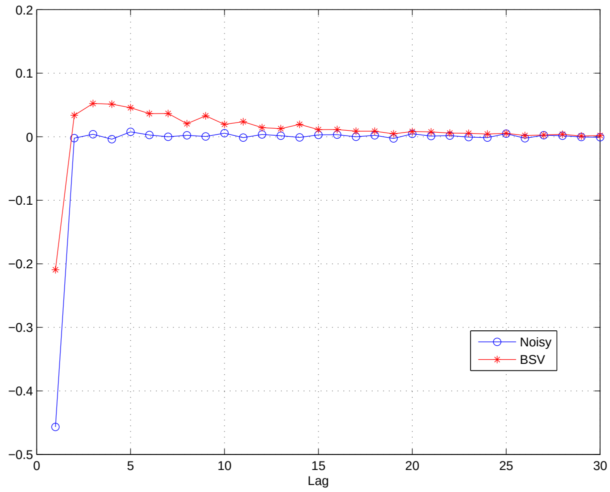



FIGURE 3.1. Autocorrelation function for mid-price returns.

Figure 3.2 shows that both noise and BSV trading can lead to long memory in the volatility of mid-price returns, which is indicated by the significant and slow decaying ACs in the absolute returns. Also, according to Gould et al. (2013), the Hurst exponent $H$ is an important measure for volatility clustering and long memory.[10] Table 3.2 reports that both BSV and noise trading leads to $H = 0.68$ and $H = 0.66$ respectively for the absolute returns of mid-price, which are higher than $H \approx 0.58$ in the Shenzhen Stock market (Gu and Zhou (2009)) and lower than $H \approx 0.8$ in the Paris Bourse (Chakraborti et al. (2011a)). Note that the Hurst exponent for the absolute return of the fundamental value is 0.49, which is very close to the true value of 0.5. The results show that both BSV and noise trading can generate volatility clustering.

3.4. **Long memory in the bid-ask spread and trading volume.** Apart from long memory in volatility, empirical studies also find long memory in the bid-ask spread (Groß-Klußmann and Hautsch (2013)) and trading volume (Covrig and Ng (2004),

[10]The series have long memory with positive long-range ACs when $0.5 < H < 1$, have long memory with negative long-range ACs when $0 < H < 0.5$, and follow a random walk when $H = 0.5$. Following Di Matteo (2007), we use the generalized Hurst exponent method to estimate $H$, the Matlab code of generalized hurst exponent can be download from http://www.mathworks.com.au/matlabcentral/fileexchange/30076-generalized-hurst-exponent.

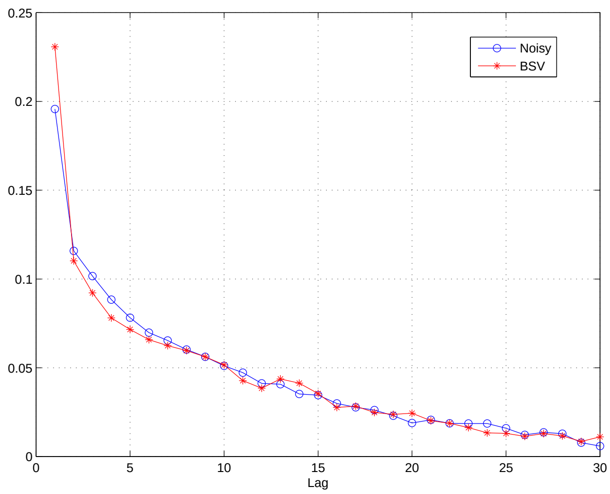


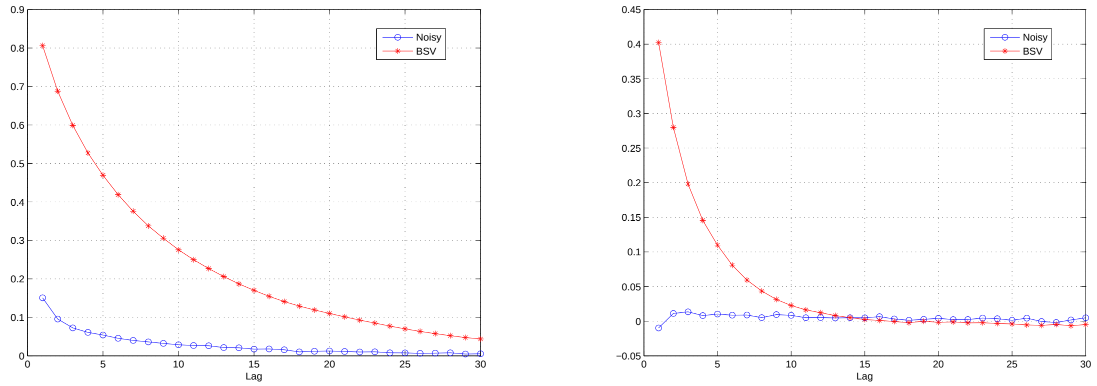

FIGURE 3.2. Autocorrelation functions for absolute mid-price returns.

| Case | $\lvert r_t\rvert$ | $\lvert r_{mt}\rvert$ | $\lvert r_{vt}\rvert$ | Volume | Spread |
|---|---|---|---|---|---|
| BSV | 0.64 | 0.68 | 0.49 | 0.74 | 0.86 |
| Noise | 0.64 | 0.66 | 0.49 | 0.51 | 0.61 |

TABLE 3.2. The Hurst exponents. Here $r_t$ is the return of market price, $r_{mt}$ is the return of mid-price, and $r_{vt}$ is the return of fundamental value.

Fleming and Kirby (2011) and Rossi and Santucci de Magistris (2013)). In Figure 3.3, panel A shows the ACs of the bid-ask spread and panel B shows the ACs of the trading volume. Both panels show that BSV trading leads to significant and decaying ACs in the bid-ask spread and trading volume comparing to the noise trading. This is further demonstrated by the Hurst exponent in Table 3.2, which is 0.74 and 0.86 for the trading volume and bid-ask spread, comparing to 0.51 and 0.61 for nose trading, respectively. The results show that the behavioral sentiment, instead of noise trading, is driving force in generating long memory in the bid-ask spread and trading volume observed in limit order markets.

3.5. **Hump shape in mean depth profile of the order book.** In the limit order book, the depth of the bid-side or ask-side available at a given submission price $p$ corresponds

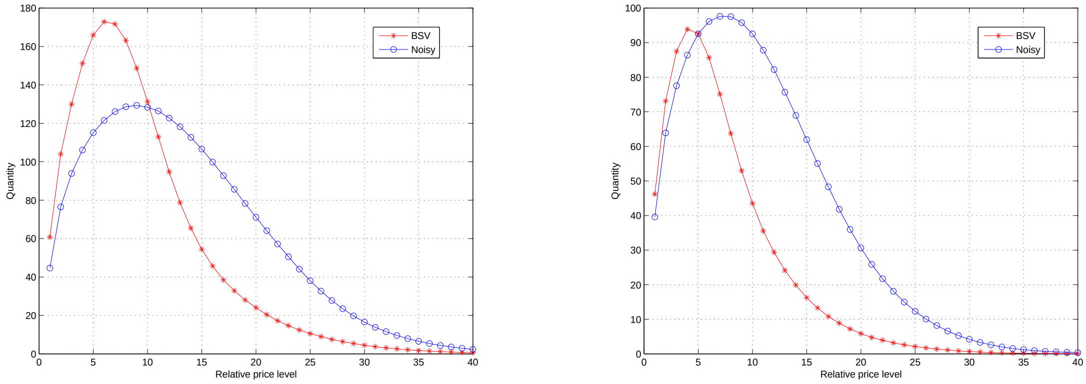



(A) ACs of bid-ask spread (B) ACs of trading volume

FIGURE 3.3. Autocorrelation functions for the bid-ask spread in panel (A) and trading volume in panel (B) for both BSV trading and noise trading.

to the total number of limit orders at price $p$. The *mean depth profile* characterizes the relationship between submission price $p$ (relative to the best quotes) and the average depth available at that price. Bouchaud, Mèzard and Potters (2002) and Chakraborti et al. (2011a) report that the peak of mean depth profile is located away from the best quotes and increases with the 5 best quote levels. Gould et al. (2013)) further show in various markets that the mean depth profile for the bid-side and the ask-side exhibits a *hump* shape, which means that the mean depth available increases over the first few relative prices (measured by the number of tick sizes away from the best bid or best ask price), and then decreases subsequently.

Figure 3.4 reports the mean depth profile for the bid (panel A) and ask (panel B) side. Both panels show that both BSV and noise trading can generate hump-shaped mean depth profiles. However, the BSV trading leads to more depth closer to the best quotes (especially for the buy side) and less depth further away from the best quote, which indicates that the market is more resilient (it is hard for a market order to move prices). With a zero-intelligence model, Ladley and Schenk-Hoppé (2009) report that the mean depth profile of the 5 best quote is nearly equal. In comparison the results




(A) Bid side (B) Ask Side

FIGURE 3.4. Mean depth profile for the bid-side in panel (A) and the ask-side in panel (B).

show that BSV trading provides a better explanation for the hump shape in the mean depth profile of limit order book.

3.6. **A non-linear relationship of trade imbalance and average mid-price return.** Various studies including Kempf and Korn (1999) and Gabaix, Gopikrishnan, Plerou and Stanley (2006) have found that the mid-price return is an increasing and non-linear function of the trade imbalances (see also Gould et al. (2013)), which is different from a linear relation in the classical models of market microstructure (see Kyle (1985)). Following Gould et al. (2013), we use the *trade imbalance size*, which is the difference between the total absolute size (quantity) of all incoming buy market orders and the total size of all incoming sell market orders that arrive during a time interval, to measure the trade imbalance. We plot in Figure 3.5 both the trade imbalance size against the average mid-price return for every 240 periods in our simulation.

Figure 3.5 shows that BSV trading leads to an increasing and non-linear relation between the trade imbalance and mid-price return. Moreover, the relationship is concave (convex) when trade imbalance is negative (positive), showing that the mid-price return is more sensitive to trade imbalance when trade imbalance is large. In comparison,

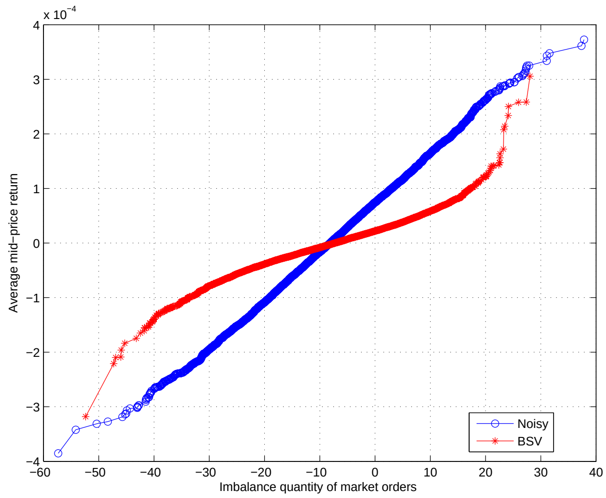



FIGURE 3.5. Relationships between trade imbalance size and the mid-price return.

noise trading leads to an almost linear relationship between trade imbalance and mid-price return. This result shows that the sentiment trading, instead of the noise trading, can lead to the increasing and nonlinear relation observed in limit order markets.

3.7. **Diagonal effect in order submission types.** Using data from 40 stocks on the Paris Bourse, Biais, Hillion and Spatt (1995) examine the probability of different types of orders and trades conditional on the last order or trade. They find that the same order type are most likely to follow each other. When listing all the conditional probabilities of each order type as a matrix, the diagonal elements of the matrix are the highest, which is also larger than the corresponding unconditional probability. This phenomena is called *diagonal effect* or *event clustering* (see Gould et al. (2013)).

We consider eight types of the submitted orders according to buy-sell direction and order aggressiveness: Market Buy (MB), Limit Buy above the best bid (aggressive Limit Buy, aLB), Limit Buy at the best bid (LB), Limit Buy below the best bid (passive Limit Buy, pLB); Market Sell (MS), Limit Sell below the best ask (aggressive Limit Sell, aLS), Limit Sell at the best ask (LS), and Limit Sell above the best ask (passive Limit Sell, pLS). We calculate the unconditional probability and conditional probability of each order type and collect them as a matrix in Tables 3.3 and 3.4 for the market with BSV traders and the market with noise traders, respectively. As in Biais et al. (1995), we highlight the two highest conditional probabilities in each column in bold. We also report the unconditional probability in the second last row and the difference between the diagonal conditional probability and the unconditional probability in the last row. Table 3.3 clearly shows that the BSV trading generates significant diagonal effect for all the order types, while Table 3.4 show that such diagonal effect cannot be generated by the noise trading.

| | Current | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Previous | MB | aLB | LB | pLB | MS | aLS | LS | pLS |
| MB | **13.46** | **3.76** | 6.16 | 25.53 | 4.52 | 0.33 | 3.53 | 42.70 |
| aLB | **12.72** | **2.27** | **8.50** | 31.02 | 2.42 | 0.02 | 2.38 | **40.66** |
| LB | 8.06 | 1.58 | **7.57** | 37.81 | 5.04 | 0.37 | 3.89 | 35.67 |
| pLB | 3.59 | 0.40 | 4.03 | **58.69** | 8.38 | 0.71 | 4.88 | 19.32 |
| MS | 3.37 | 0.26 | 2.92 | **47.08** | **22.04** | **2.36** | 6.84 | 15.13 |
| aLS | 2.28 | 0.02 | 2.43 | 40.94 | **14.64** | **2.34** | **14.07** | 23.29 |
| LS | 4.00 | 0.36 | 3.35 | 38.97 | 10.41 | 1.78 | **11.58** | 29.56 |
| pLS | 6.44 | 0.84 | 4.22 | 21.46 | 3.15 | 0.25 | 4.13 | **59.51** |
| Unconditional | 6.51 | 0.72 | 4.23 | 39.46 | 7.25 | 0.69 | 5.01 | 36.13 |
| Difference | *6.95* | *1.55* | *3.34* | *19.23* | *14.79* | *1.65* | *6.57* | *23.38* |

TABLE 3.3. Unconditional and conditional probability (in %) of the submitted order types with the BSV trading. The two largest numbers in each column are in bold. The difference in the last row in italic is the difference between the diagonal number of the conditional probability minus the corresponding unconditional probability.

Biais et al. (1995) put forward that diagonal effect and event clustering are generated by order splitting, imitation behavior and reactions to the information, Goettler, Parlour and Rajan (2005) argue that the conditional correlation is due to corrections of mis-pricing. However, Ladley and Schenk-Hoppé (2009) find that zero-intelligence model can also generate diagonal effect, and they point out that the diagonal effect is

| | Current | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| previous | MB | aLB | LB | pLB | MS | aLS | LS | pLS |
| MB | 14.50 | **10.20** | 3.08 | 24.32 | 13.57 | **7.04** | 3.05 | 24.25 |
| aLB | 13.17 | 4.41 | 3.18 | **31.27** | **18.77** | 4.00 | 3.09 | 22.13 |
| LB | 14.87 | 5.98 | **3.34** | 27.37 | 15.54 | 5.80 | **3.20** | 23.89 |
| pLB | 13.91 | 5.69 | 3.17 | 29.61 | **16.57** | 5.28 | 3.10 | 22.68 |
| MS | 13.07 | **7.31** | 3.12 | **30.07** | 14.91 | **6.85** | 3.05 | 21.62 |
| aLS | **18.08** | 4.26 | 3.14 | 25.06 | 13.76 | 4.30 | 3.11 | **28.28** |
| LS | 14.82 | 5.96 | **3.26** | 27.19 | 15.36 | 5.72 | **3.30** | 24.39 |
| pLS | **16.22** | 5.48 | 3.12 | 25.33 | 14.34 | 5.56 | 3.13 | **26.81** |
| Unconditional | 15.82 | 5.94 | 3.13 | 27.09 | 15.24 | 5.72 | 3.09 | 23.96 |
| Difference | -1.32 | -1.53 | *0.21* | *2.52* | -0.33 | -1.42 | *0.21* | *2.85* |

TABLE 3.4. Unconditional and conditional probability (in %) of the submitted order types with the noise trading. The two largest numbers in each column are in bold. The difference in the last row in italic is the difference between the diagonal number of the conditional probability minus the corresponding unconditional probability.

not dependent on individual strategic behavior, but it can emerge from the interplay of the order book and demand/supply functions. More recently, following a similar model as the one in Chiarella et al. (2009), Kovaleva and Iori (2014) find that when chartist uses technical rules based on the order book depth, their model can generate significant diagonal effect for some order types, but not for all order types. Our results compliment the existing literature and show that the sentiment trading can generate event clustering for order submission types, namely behavioral sentiment.

## 4. CONCLUSION

The aim of this paper was to explore the effect of behavioral sentiment in limit order markets, especially its role in replicating some the well-documented stylized facts of the time series of returns and of the limit order book.

We find that both the noise and BSV trading can generate the absence of autocorrelation in returns, long memory in the absolute returns and bid-ask spread, and the hump shaped mean depth profile of the order book. The difference is that compared to noise trading, BSV trading leads to much more significant autocorrelations in the bid-ask spread and more peaked hump shape in the mean depth profile closer to the best quotes. More importantly, BSV trading plays a unique role in explaining the fat tails in the return distribution, long memory in the trading volume, an increasing and non-linear relationship between trade imbalance and mid-price returns, and also the diagonal effect or event clustering in order submission types. The results demonstrate that behavioral sentiment is not only useful in explaining under-reation/over-reaction to news events, but also very useful in explaining some of the well-documented stylized facts in the limit order markets, which cannot be explained by noise trading.

## 5. RT FIGURES AND TABLES

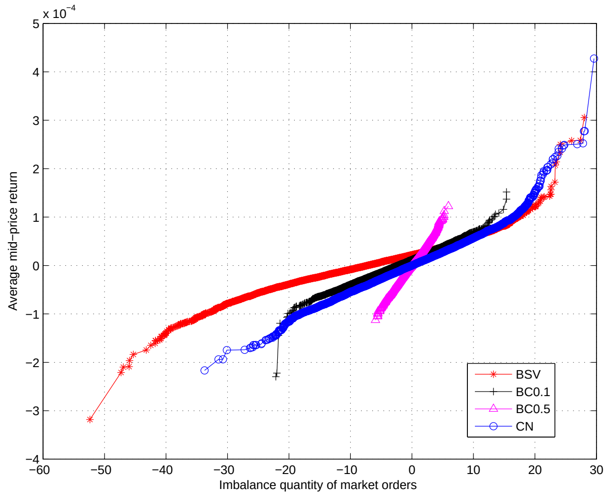



FIGURE 5.1. Relationships between trade imbalance size and the mid-price return in Robustness test.

TABLE 5.1. Add caption

| case | Forecasting correlation | $p^M - p^m$ | MO | ALO | LOA | PLO |
|---|---|---|---|---|---|---|
| BSV | 0.12 | 7.67 | 13.76 | 1.41 | 9.24 | 75.59 |
| Noise | 0 | 18.44 | 31.06 | 11.66 | 6.22 | 51.06 |

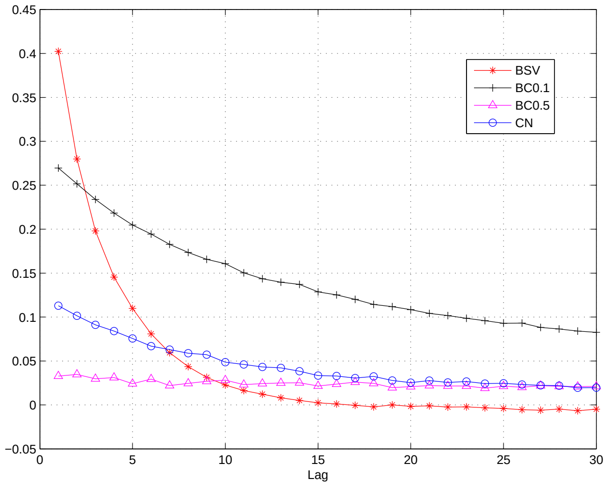



FIGURE 5.2. Autocorrelation of trading volume in Robustness test.

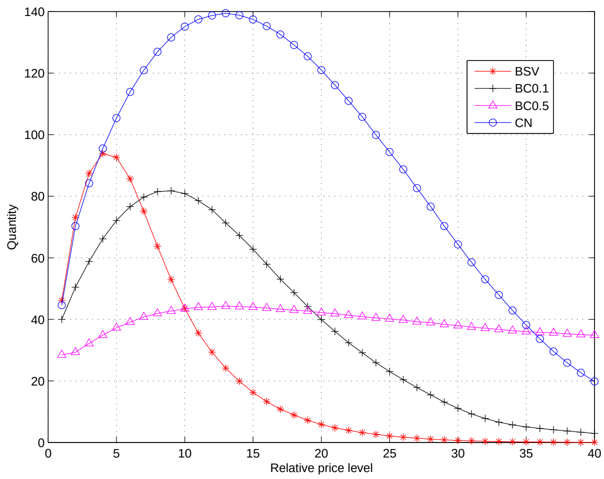



FIGURE 5.3. The sell side order book shape in Robustness test.

TABLE 5.2. Add caption

| Time horizon | $\lvert p^l - p_t^l \rvert$ | MO | ALO | LOA | PLO |
|---|---|---|---|---|---|
| 30 | 5.81 | 15.11 | 1.98 | 7.38 | 75.66 |
| 90 | 9.05 | 7.1 | 1.03 | 2.84 | 90.41 |

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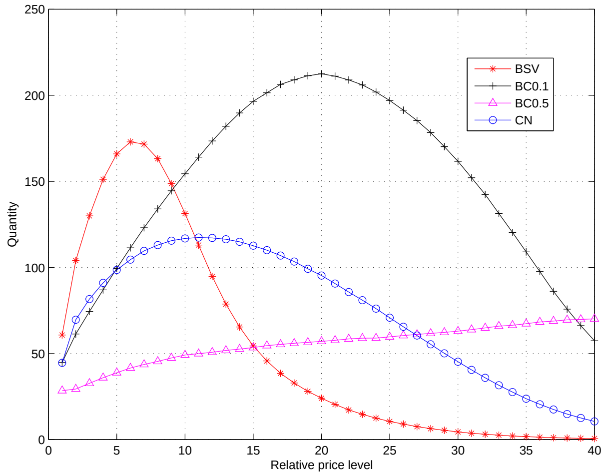


FIGURE 5.4. The buy side order book shape in Robustness test.

TABLE 5.3. Add caption

| Case | Mean | Skewness | Kurtosis |
|---|---|---|---|
| BSV | -8.90E-07 | -0.15 | 41.98 |
| BC0.1 | -8.40E-07 | 0.02 | 8.92 |
| Bc0.5 | -8.90E-07 | 0 | 6.36 |
| CN | -9.00E-07 | 0.03 | 10.31 |

TABLE 5.4. Hurst expoent

| Case | $\|r_t\|$ | $\|r_{mt}\|$ | Volume | Spread |
|---|---|---|---|---|
| BSV | 0.64 | 0.68 | 0.74 | 0.86 |
| BC0.1 | 0.62 | 0.63 | 0.74 | 0.8 |
| Bc0.5 | 0.61 | 0.65 | 0.56 | 0.91 |
| CN | 0.65 | 0.66 | 0.62 | 0.75 |

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TABLE 5.5. Conditional frequencies in BC0.1

| | Current | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Previous | MB | aLB | LB | pLB | MS | aLS | LS | pLS |
| MB | **5.41** | **2.07** | 3.22 | 58.37 | 3.90 | 0.47 | 2.84 | 23.72 |
| aLB | **5.98** | **1.76** | **3.6** | 59.02 | 3.17 | 0.20 | 2.54 | **23.74** |
| LB | 4.37 | 0.72 | **3.42** | 61.63 | 4.35 | 0.52 | 2.95 | 22.03 |
| pLB | 3.64 | 0.54 | 2.81 | **65.45** | 4.54 | 0.60 | 2.90 | 19.53 |
| MS | 3.32 | 0.44 | 2.78 | 62.88 | **6.78** | **0.97** | 3.23 | 19.59 |
| aLS | 2.95 | 0.18 | 2.59 | **63.4** | **6.70** | **1.70** | **3.72** | 18.76 |
| LS | 3.78 | 0.50 | 2.93 | 62.55 | 5.08 | 0.66 | **3.51** | 20.99 |
| pLS | 4.17 | 0.64 | 2.92 | 55.78 | 4.12 | 0.46 | 2.75 | **29.17** |
| Unconditional | 4.38 | 0.59 | 2.86 | 61.22 | 4.57 | 0.59 | 2.9 | 22.89 |
| Difference | *1.03* | *1.17* | 0.56 | *4.23* | *2.21* | *1.11* | 0.61 | *6.28* |

TABLE 5.6. Conditional frequencies of BC0.5

| | Current | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Previous | MB | aLB | LB | pLB | MS | aLS | LS | pLS |
| MB | 2.67 | **1.19** | 2.67 | 41.13 | 2.66 | 0.25 | 2.54 | 46.89 |
| aLB | **2.91** | **0.98** | **3.04** | 41.69 | 2.48 | 0.11 | 2.37 | 46.4 |
| LB | **2.71** | 0.29 | **2.75** | 41.56 | 2.62 | 0.25 | 2.68 | **47.14** |
| pLB | 2.49 | 0.25 | 2.45 | **46.26** | 2.54 | 0.27 | 2.47 | 43.28 |
| MS | 2.64 | 0.26 | 2.60 | 42.28 | **2.76** | **0.31** | 2.62 | 46.53 |
| aLS | 2.35 | 0.09 | 2.57 | **42.67** | **2.79** | **0.98** | **3.16** | 45.39 |
| LS | 2.70 | 0.26 | 2.55 | 41.78 | 2.71 | 0.28 | **2.76** | 46.96 |
| pLS | 2.49 | 0.25 | 2.43 | 37.94 | 2.51 | 0.25 | 2.46 | **51.66** |
| Conditional | 3.46 | 0.25 | 2.44 | 41.24 | 2.52 | 0.26 | 2.47 | 47.36 |
| Difference | -0.79 | 0.73 | 0.31 | *5.02* | 0.24 | 0.72 | 0.29 | *4.3* |

TABLE 5.7. Conditional frequencies of BC0.5

| | Current | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Previous | MB | aLB | LB | pLB | MS | aLS | LS | pLS |
| MB | **10.87** | **3.51** | 3.63 | 25.58 | 5.72 | 1.69 | 3.34 | **45.66** |
| aLB | **9.44** | 1.37 | **3.79** | 37.27 | 8.64 | 1.19 | 3.64 | 34.66 |
| LB | 8.23 | **1.67** | **3.68** | 31.82 | 7.59 | 1.63 | 3.57 | 41.8 |
| pLB | 6.74 | 1.62 | 3.58 | **49.97** | **9.93** | 1.67 | 3.45 | 23.03 |
| MS | 5.77 | 1.67 | 3.34 | **40.42** | **11.75** | **2.37** | 3.72 | 30.95 |
| aLS | 8.05 | 1.11 | 3.55 | 32.74 | 9.70 | 1.47 | **3.98** | 39.40 |
| LS | 7.47 | 1.49 | 3.49 | 29.54 | 8.32 | **1.85** | **3.74** | 44.11 |
| pLS | 7.63 | 1.08 | 3.03 | 14.73 | 5.22 | 1.19 | 3.32 | **63.80** |
| Unconditional | 8.90 | 1.41 | 3.29 | 28.87 | 7.46 | 1.50 | 3.40 | 45.16 |
| Difference | *1.97* | -0.04 | 0.39 | *21.10* | *4.29* | -0.03 | 0.34 | *18.64* |

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