NONLINEAR MODELLING AND CONTROL OF HEART RATE RESPONSE TO TREADMILL WALKING EXERCISE

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Abstract: In this study, a nonlinear system was developed for the modelling of the heart rate response to treadmill walking exercise. The model is a feedback interconnected system which can represent the neural response and peripheral local response to exercise. The parameters of the model were identified from an experimental study which involved 6 healthy adult male subjects, each completed 3 sets of walking exercise at different speeds. The proposed model will be useful in explaining the cardiovascular response to exercise. Based on the model, a 2-degree-of-freedom controller was developed for the regulation of the heart rate response during exercise. The controller consists of a piecewise LQ and an H_∞ controllers. Simulation results showed that the proposed controller had the ability to regulate heart rate at a given target, indicating that the controller can play an important role in the design of exercise protocols for individuals.

1 INTRODUCTION

During dynamic exercise, the cardiovascular system increases the delivery of blood and oxygen to working muscles as the metabolic demand increases, resulting in an increase in heart rate (HR) and stroke volume. Obtaining a model that describes the HR response to exercise will improve our understanding of exercise physiology. Understanding the aetiology of HR response during, and recovery after an exercise, may also be beneficial to predicting cardiovascular disease mortality (Savonen et al., 2006) (Cole et al., 1999). This may also lead to an improvement in developing training protocols for athletics and more efficient weight loss protocols for the obese, and in facilitating assessment of physical fitness and health of individuals (Achten and Jeukendrup, 2003). Furthermore, knowing the cardiovascular system responses to the stress induced by physical exercise provides us another perspective on how this system functions. For instance, this may give us some measures for the prevention of cardiac failure from dialysis.

Studying and modelling of HR response during exercise have been carried out by a number of researchers (e.g. (Brodan et al., 1971; Hajek et al., 1980; Rowell, 1993; Coyle and Alonso, 2001; Su et al., 2007)). Broden et al. (Brodan et al., 1971) and Hajek et al. (Hajek et al., 1980) modelled the HR response from a regulation point of view. Their models are reliable for short duration exercises, but are not sufficient for explaining long duration exercises. As shown in, e.g. (Coyle and Alonso, 2001), HR will continue to increase during prolonged exercise. In reference (Su et al., 2007), exercising HR response was modelled by a Hammerstein system[1]. Besides modelling, they also studied the control of the HR response during exercise.

The ability to control the HR during exercise is of importance in the design of exercise protocols for patients with cardiovascular diseases and in developing rehabilitation exercises to aid patients recovering from cardiothoracic surgery. The control of heart rate response during exercise has been reported in the references (Kawada et al., 1999; Cooper et al., 1998; Su

[1]A system consists of a static nonlinearly cascaded at the input of a linear system.

et al., 2007). Among them, a number of different control strategies or algorithms have been successfully applied, e.g. classical PID control, H_∞ control, and model reference control. Each has its merits or disadvantages and therefore, it is interesting to investigate the usefulness of other control algorithms and techniques which have been developed by the control society.

The objective of this paper is twofold. First, a nonlinear model is proposed to describe the HR response to treadmill walking exercise during both the exercising and the recovery phases. We model the HR response from the neural and the local responses perspective. The advantage of this approach is that the model may describe the HR response over a longer exercise duration. Secondly, using the proposed model, we develop a controller-using the treadmill's speed as a control variable-that regulates the HR during exercise. The controller consists of feedforward and feedback components which provide better performance without trading off robustness.

2 THE MODEL

In this paper, we propose the following nonlinear state-space control systems to model the HR response to treadmill walking exercise:

$$\begin{aligned} \dot{x}_1(t) &= -a_1x_1(t) + a_2x_2(t) + a_2u^2(t) \\ \dot{x}_2(t) &= -a_3x_2(t) + \phi(x_1(t)) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where $\phi(x_1(t)) := \frac{a_4x_1(t)}{1+\exp\left(-15(x_1(t)-a_5)\right)}$ and $x(0) = [x_1(0) \quad x_2(0)]^T = 0$, $y(t)$ describes the change in HR from rest, and $a_1,\ldots,a_5$ are positive scalars. The control input $u(t)$ represents the speed of the treadmill.

System (1) can be viewed as a feedback interconnected system, i.e. x_1 in the forward path and x_2 in the feedback path. The component $x_1(t)$ can be viewed as the change of HR due to the neural response to exercise, including both the parasympathetic and the sympathetic neural inputs (see e.g. (Rowell, 1993)). The component x_2 is utilised in describing the complex slow-acting peripheral effects from, e.g. the hormonal systems, the peripheral local metabolism, and/or the increase in body temperature, etc.. Generally, these effects cause vasodilatation and hence HR needs to be increased in order to maintain the arterial pressure (see (McArdle et al., 2007))). So, the feedback signal x_2, which can be thought of as a dynamic disturbance input to the x_1 subsystem, is a reaction to the peripheral local effects. By observing system (1), the input s drives the system nonlinearly, describing the nonlinear increase of the HR in response to the increase in walking speed. It has been observed that there is a curvilinear relationship between aerobic demand and walking speed (see, e.g. (McArdle et al., 2007)).

Table 1: Physical characteristics of the subjects: age, height, weight, and BMI (Body Mass Index).

	Age (yr)	Height (cm)	Weight (kg)	BMI (kg/m^{-2})
mean	29.3	174	68.5	22.5
std	5.8	3.4	12.6	3.4
range	23–38	169–178	53–85	18–27

2.1 Experimental Setup

The parameters in system (1) were identified from experimental data. The setup of the experiment is described in this section.

Subject. Six healthy male subjects were studied. The physical characteristics of the subjects are given in Table 1.

Procedure. Each subject completed three exercise sessions in separate occasions. In each session, a subject was requested to walk on a treadmill at a given speed (5km/h, 6km/h, and 7km/h) for 15 minutes with a recovery period of 15 minutes. After three sessions, each subject completed the treadmill walking exercise at the three different speeds.

Data Acquisition. In this study, the Powerjog fully motorised medical grade treadmill was used. The HR of the subjects was monitored by the wireless Polar system and recorded by LabVIEW. The Polar system generated pulses which were used to determine the HR. To remove noises, the HR measurements were then filtered using the moving average with a 5-second window.

Parameter Estimation. Using the measured HR data and the Levenberg-Marquardt method, the parameters in system (1) were estimated for each subject and for the average response of all subjects . Since there were three sets of input-output measurements for each subject (where the input is the speed of the treadmill and the output is the HR), we estimated the parameters as if the following multi-input multi-output system:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), a, u(t)), \quad y(t) = C\mathbf{x}(t), \quad \mathbf{x}(0) = 0 \tag{2}$$

where $C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, $\mathbf{x} = [\mathbf{x_1} \quad \mathbf{x_2} \quad \mathbf{x_3}]^T \in \mathbb{R}^6$, $a = [a_1 \quad a_2 \dots a_5]^T \in \mathbb{R}^5$, $u = [u_1 \quad u_2 \quad u_3]^T \in \mathbb{R}^3$ and $y = [y_1 \quad y_2 \quad y_3]^T \in \mathbb{R}^3$. For $i = 1,2,3$, the vector $\mathbf{x_i} := [x_{i,1} \quad x_{i,2}]^T$ and y_i are the state vector and the output from the input u_i. The unit of time t is in minute. To make the estimation process more robust, the speeds of the treadmill were normalised by 8 km/h, assuming the maximum walking speed was 8 km/h. In other words, the input vector u in (2) is in fact $u = [5/8 \;\; 6/8 \;\; 7/8]^T$. Similarly, the output $y_i(t)$ from the input $u_i(t)$ was defined as $y_i(t) = (HR_i(t) - 73.4)/60$, where $HR_i(t)$ is the absolute HR at time t, 73.4 bpm is the average resting HR for all the subjects[2], and 60 bpm is a normalising factor.

The objective function was chosen as

$$S(a) = \sum_{i=1}^{N} (y(t_i) - \hat{y}(t_i,a))^T Q (y(t_i) - \hat{y}(t_i,a)) \quad (3)$$

where, for $i = 1,2,\dots,N$, $y(t_i)$ is the measurement of the output vector at time t_i, $\hat{y}(t_i,a)$ is the output of system (2) with the parameter vector a, and Q is a given diagonal weighting matrix. In this study, $Q := diag([2.5 \;\; 1.5 \;\; 1])$ was used. With the objective function (3), the Levenberg-Marquardt method was used to determine an estimate of a which was denoted as $\hat{a} := [\hat{a}_1 \;\; \hat{a}_2 \dots \hat{a}_5]^T$ (see, e.g. (Stortelder, 1996)). Based on a linear approximate method (see e.g. (Stortelder, 1996)), an approximate $100(1-\alpha)\%$ independent confidence interval for each estimate was given by $(\hat{a}_i - \delta a_i, \hat{a}_i + \delta a_i)$, for $i = 1,2,\dots,5$. An α level of 0.05 was used for obtaining the confidence intervals of parameter estimates. Table 2 summaries the estimated parameters for each subject and it also shows the estimated parameters for the average response from all the subjects. The simulated HR responses with the proposed model based on the average response are shown in Figure 1.

3 CONTROLLER DESIGN

In the second part of this paper, a controller design is proposed for the regulation of HR. The controller essentially controls the speed of the treadmill and in turns controls the HR. It is desirable to design a controller that is suitable for all the subjects, rather than designing a controller for each individual subject. To design such a controller, the model for the average response was utilised (see Table 2). Substituting the parameters estimated from the average response, system (1) is written in the state-space form as follows:

[2]Resting HR was estimated from the 3-minute resting period before exercise.

Table 2: Estimated parameter values for 6 different subjects and the average response of all subjects.

	Parameter estimates (Confidence intervals, δa)				
Subject	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$
1	2.374 (0.180)	2.319 (0.161)	0.024 (0.014)	0.018 (0.003)	0.000 (0.308)
2	3.351 (0.334)	3.591 (0.340)	0.126 (0.008)	0.071 (0.004)	0.683 (0.009)
3	1.940 (0.180)	1.597 (0.138)	0.038 (0.007)	0.054 (0.004)	0.507 (0.010)
4	1.041 (0.078)	0.787 (0.052)	0.072 (0.011)	0.069 (0.007)	0.491 (0.015)
5	3.665 (0.489)	2.394 (0.304)	0.169 (0.023)	0.107 (0.013)	0.476 (0.029)
6	1.782 (0.166)	1.442 (0.123)	0.110 (0.009)	0.105 (0.007)	0.562 (0.013)
average response	1.858 (0.119)	1.655 (0.099)	0.057 (0.007)	0.046 (0.003)	0.550 (0.009)

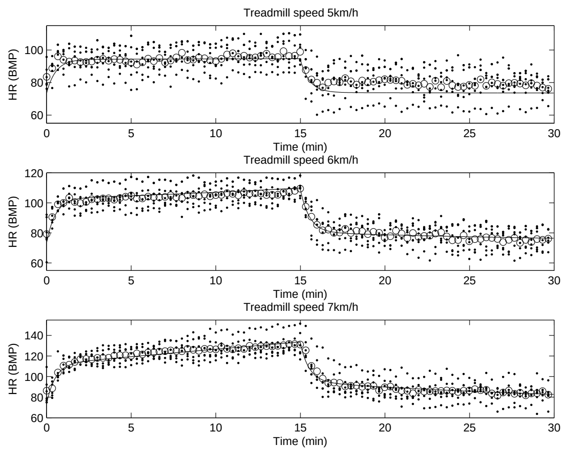

Figure 1: HR responses: actual responses from all subjects (dots), average response (circles) and simulated response (solid line).

$$\dot{x} = Ax + B_1\phi(x_1) + B_2 g(u), \quad y = Cx \qquad (4)$$

where

$A = \begin{bmatrix} -1.858 & 1.655 \\ 0 & -0.057 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1.655 \\ 0 \end{bmatrix}$, $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $g(u) := u^2$, $\phi(x_1) := \frac{0.046 x_1}{1+\exp\left(-15(x_1-0.55)\right)}$.

System (4) is a nonlinear system with nonlinearity $\phi(x_1)$ and nonlinear control input $g(u)$. To overcome the control input nonlinearity, a transformed input $v = g(u)$ is defined. The function $\phi(x_1)$ can be approximated by a piecewise linear function

$$\gamma(x_1) = \begin{cases} 0 & \text{if } x_1 \leq 0.418 \\ 0.090x_1 - 0.038 & \text{if } x_1 > 0.418. \end{cases}$$

In fact, $\gamma(x_1)$ is obtained by linearising the function $\phi(x_1)$ at $x_1 = 0$ and 0.5. As a result, system (4) can be approximated by a piecewise affine system (see e.g. (Rantzer and Johansson, 2000)).

In this paper, we propose a two-degree-of-freedom (2-DOF) controller consisting of a piecewise linear quadratic (LQ) feedforward and a H_∞ feedback controllers, as shown in Figure 2, for the control and regulation of HR response.

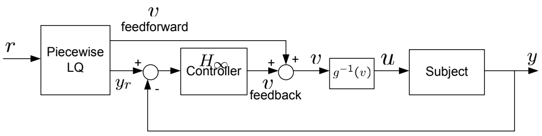

Figure 2: Control configuration.

3.1 LQ Feedforward Controller Design

First, we design the piecewise LQ feedforward controller using the piecewise LQ optimal control technique (Rantzer and Johansson, 2000). We also incorporate an integral action in the controller.

Define two partitions of the state space:

$$X_1 := \{[x_1 \;\; x_2]^T \in \mathbb{R}^2 \,|\, x_1 < 0.418\}$$
$$X_2 := \{[x_1 \;\; x_2]^T \in \mathbb{R}^2 \,|\, x_1 \geq 0.418\}$$

Next, define

$$\bar{A}_i = \begin{bmatrix} A_i & 0 & a_i \\ -C & 0 & 0 \\ 0_{n\times 1} & 0 & 0 \end{bmatrix}, \; \bar{B} = \begin{bmatrix} B_2 \\ 0 \\ 0 \end{bmatrix}, \; \bar{x} = \begin{bmatrix} x \\ e \\ 1 \end{bmatrix}$$

for $x \in X_i$ and $i = 1, 2$, where

$$A_1 = \begin{bmatrix} -1.858 & 1.655 \\ 0 & -0.057 \end{bmatrix}, A_2 = \begin{bmatrix} -1.858 & 1.655 \\ 0.090 & -0.057 \end{bmatrix},$$

$a_1 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $a_2 = \begin{bmatrix} 0 & -0.038 \end{bmatrix}^T$, $e(t) = \int_0^t (r - Cx(t))dt$ and r is the constant reference input. Therefore, we have

$$\dot{\bar{x}} = \bar{A}_i\bar{x} + \bar{B}v, \quad y = \bar{C}\bar{x}, \quad \text{for } x \in X_i. \qquad (5)$$

where $\bar{C} = [C \; 0 \; 0]$. Then, the control problem is to find a control law v that minimises the following cost function: $J = \int_0^\infty (\bar{x}^T \bar{Q}\bar{x} + v^T Rv)dt$, for any given $\bar{Q} \geq 0$ and $R > 0$. In the control design, the matrix $\bar{Q}$ and the value of R were chosen as follows: $\bar{Q} = diag([0 \;\; 0 \;\; 10 \;\; 0])$, $R = 0.5$. By using the technique in (Rantzer and Johansson, 2000), the minimising control law was $v(t) = L_i\bar{x}, x \in X_i$, $i = 1, 2$, where $L_1 = \begin{bmatrix} -1.457 & -0.989 & 4.471 & 0 \end{bmatrix}$, $L_2 = \begin{bmatrix} -1.48 & -1.001 & 4.471 & 0.009 \end{bmatrix}$. In turn, the LQ feedforward controller is in the form:

$$\dot{\bar{x}} = \bar{A}_i\bar{x} + \bar{B}v + B_r r, \quad y_r = \bar{C}\bar{x}, \quad v(t) = L_i\bar{x}, \qquad (6)$$

for $x \in X_i$ where $\bar{x}(0) = [0\;0\;0\;1]^T$, $B_r = [0\;0\;1\;0]^T$ and r is the reference input. In other words, the input to this feedforward controller is the reference r and the output are the feedforward control v and the "filtered" reference y_r.

3.2 H_∞ Controller Design

To cope with the uncertainty in the model, we design a feedback controller based on the H_∞ control technique (see e.g. (Petersen et al., 2000)). We first linearise the system (4) and then formulate the control problem as a mixed sensitivity problem (see e.g. (Skogestad and Postlethwaite, 1996) for details). In a mixed sensitivity problem, the idea is to choose some weighing functions, namely $W_1(s)$, $W_2(s)$ and $W_3(s)$ to reflect the control objectives. Generally, $W_1(s)$ is chosen to meet a performance specification and $W_3(s)$ is chosen to characterise the modelling errors. Whereas the weighing function $W_2(s)$ may be used to reflect some restrictions on the actuator signal.

In order to apply the mixed sensitivity technique, the system (4) was linearised at $x_0 = [0.5 \;\; 0.13]^T$, $v_0 = 0.43$, and the transfer function of the linearised model is given by

$$G(s) = \frac{1.655s + 0.094}{s^2 + 1.915s - 0.043} \qquad (7)$$

The weighting functions were then chosen as: $W_1(s) = \frac{0.02(s+5)}{(s+0.0001)}$, $W_2(s) = \frac{700(s+0.3)}{(s+2100)}$, $W_3(s) = \frac{100(s+7.13)}{(s+800)}$. By using MATLAB Robust Control Toolbox, we obtained a controller $K(s)$ that is fifth order, resulting in a complicated control strategy. In fact, by observing the Hankel singular values of the controller $K(s)$, a second order controller K_{reduced} was in fact adequate to approximate $K(s)$ and it is in the form

$$K_{\text{reduced}}(s) = \frac{0.927s + 0.009}{s^2 + 0.060s + 6.008 \times 10^{-6}} \qquad (8)$$

4 SIMULATION RESULTS

As shown in Figure 2, a 2-DOF controller were constructed by combining the LQ feedforward controllers (6) and the H_∞ feedback controller (8). Since both the feedforward and feedback controllers were

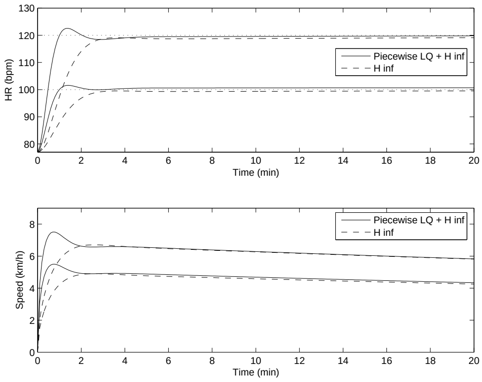

Figure 3: Subject 1–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

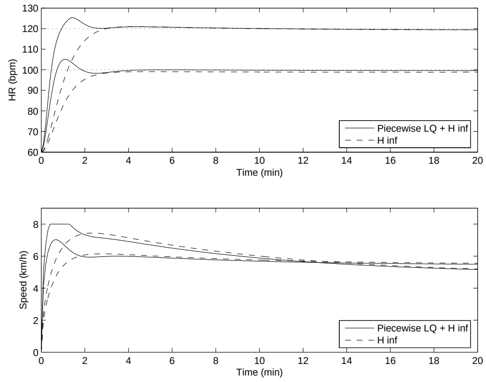

Figure 4: Subject 2–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

obtained by considering the model of the average response from the six subjects, it would be realistic to validate the 2-DOF controller by applying it to each of the subject without re-tuning the control parameters for each subject. From the previous section, we have a model for each of the subject and the estimated parameters of the model are shown in Table 2.

We also assumed the treadmill speed was only allowed to be operated between 0 and 8 km/h, since speeds greater than 8 km/h may exceed the maximum walking speed of some subjects. For each subject, we tested the proposed controller by regulating the HRs at 2 levels, namely 100 and 120 bpm. In the simulations, the resting HR of each subject was assumed to be the average of the three resting HRs, since each subject performed 3 sets of walking exercise.

Figures 3–8 show the simulation results. Each figure shows the controlled HR responses and the speeds of the treadmill. It also shows the responses from the proposed 2-DOF controller and 1-DOF controller which consists of H_∞ feedback controller only. For each of the subject, the controlled HR was able to track the reference HR signals. By comparing the responses from the 2-DOF controller and the H_∞ controller, the proposed 2-DOF controller provides faster responses. It indicates that the proposed 2-DOF controller should give better performance than that of only H_∞ controller.

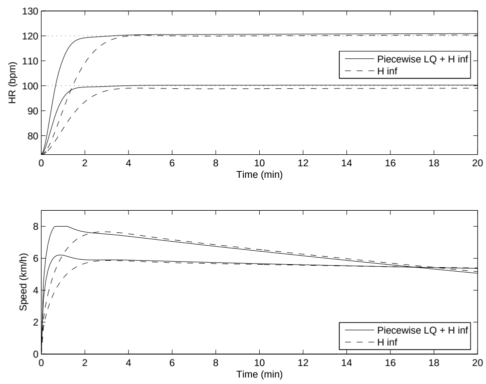

Figure 5: Subject 3–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

5 CONCLUSIONS

In this study, a nonlinear model describing the HR response to the treadmill walking exercise is proposed. The proposed model is a feedback interconnected system, consisting of a subsystem in the forward path that can be used to describe the neural response, and a feedback subsystem can be utilised to describe the peripheral local response. Utilising this model, a 2-DOF controller was developed for the regulation of HR for treadmill walking exercise. The controller consists of a piecewise LQ feedforward and a H_∞ feedback controller. One of the benefits of introducing the feedforward control is to improve the performance, since robust control such as H_∞ controller is sometimes overly conservative that impedes performance. The controller was derived from the model of average response of the six participated subjects. Simulation results showed that the proposed controller had the ability to regulate HR for all the six subjects, without the need to re-tune the controller's parameters.

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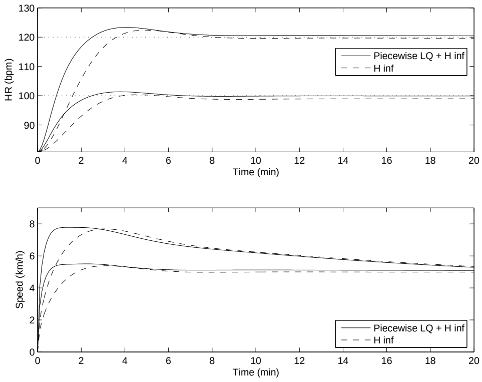

Figure 6: Subject 4–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

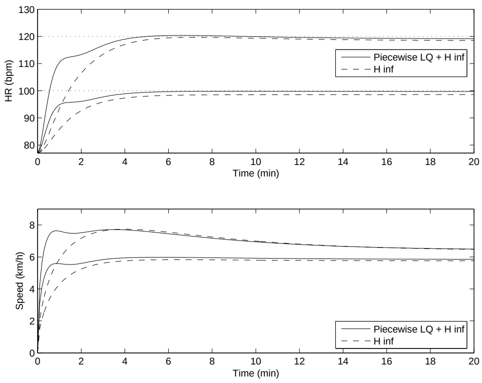

Figure 7: Subject 5–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

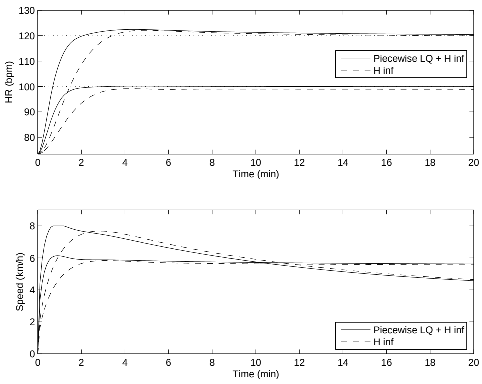

Figure 8: Subject 6–Simulation of HR regulation at 100 bpm and 120 bpm with H_∞ controller (dashed) and Piecewise LQ + H_∞ controller (solid).

REFERENCES

Achten, J. and Jeukendrup, A. E. (2003). Heart rate monitoring: Applications and limitations. *Sports Med.*, 33(7):517–538.

Brodan, V., Hajek, M., and Kuhn, E. (1971). An analog model of pulse rate during physical load and recovery. *Physiologia Bohemoslovaca*, 20:189–198.

Cole, C. R., Blackstone, E. H., F. J. Pashkow, C. E. S., and Lauer, M. S. (1999). Heart rate recovery immediately after exercise as a predictor of mortality. *The New England Journal of Medicine*, 341(18):1351–1357.

Cooper, R. A., Fletcher, T. L., and Robertson, R. N. (1998). Model reference adaptive control of heart rate during wheelchair ergometry. *IEEE Transactions on Control Systems Technology*, 6(4):507–514.

Coyle, E. F. and Alonso, G. (2001). Cardiovascular drift during prolonged exercise: New perspectives. *Exercise and Sports Science Review*, 29:88–92.

Hajek, M., Potucek, J., and Brodan, V. (1980). Mathematical model of heart rate regulation during exercise. *Automatica*, 16:191–195.

Kawada, T., Sunagawa, G., Takaki, H., Shishido, T., Miyano, H., Miyashita, H., Sato, T., Sugimachi, M., and Sunagawa, K. (1999). Development of a servo-controller of heart rate using a treadmill. *Japanese Circulation Journal*, 63:945–950.

McArdle, W. D., Katch, F. I., and Katch, V. L. (2007). *Exercise Physiology: Energy, Nutrition & Human Performance*. Lippincott Williams & Wilkins, Philadelphia, PA, 6 edition.

Petersen, I. R., Ugrinovskii, V. A., and Savkin, A. V. (2000). *Robust Control Design Using H^∞ Methods*. Springer-Verlag, London.

Rantzer, A. and Johansson, M. (2000). Piecewise linear quadratic optimal control. *IEEE Transactions on Automatic Control*, 45(4):629–637.

Rowell, L. B. (1993). *Human Cardiovascular Control*. Oxford University Press, New York.

Savonen, K. P., Laka, T. A., Laukkanen, J. A., Halonen, P. M., Rauramaa, T. H., Salonen, J. T., and Rauramaa, R. (2006). Heart rate response during exercise test and cardiovascular mortality in middle-aged men. *European Heart Journal*, 27:582–588.

Skogestad, S. and Postlethwaite, I. (1996). *Multivariable Feedback Control*. Wiley, England.

Stortelder, W. J. H. (1996). Parameter estimation in dynamic systems. *Mathematics and Computers in Simulation*, 42:135–142.

Su, S. W., Wang, L., Celler, B. G., Savkin, A. V., and Guo, Y. (2007). Identification and control for heart rate regulation during treadmill exercise. *IEEE Transactions on Biomedical Engineering*, 54(7):1238–1246.