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# Vibration of Hydraulically Interconnected Suspensions due to Fluid-Structure Interaction

Jing ZHAO** and Nong ZHANG**
** Mechatronics and Intelligent Systems, Faculty of Engineering, Unibersity of Technology, Sydney,
P. O. Box 123, Broadway, NSW 2007, Australia
E-mail: Nong.Zhang@uts.edu.au

**Abstract**

The pressure changes produced inside the fluid circuits of Hydraulically Interconnected Suspensions (HIS) often lead to vibration of pipelines and associated structures and become a source of structural noise. This paper presents a theoretical investigation into the vibration of a liquid-filled hydraulic circuit that is often used in an interconnected suspension. The one-dimensional wave theory is employed to formulate the equations that govern the dynamics of the fluid-structural system. Axial and one plane of lateral vibrations as well as the effects of shear deformation on the lateral vibration of the pipe are considered. The Transfer Matrix Method (TMM) is applied to determine the steady state response of the fluid-structural system, which consists of pipe sections, damp valves, an accumulator, and supports. The overall system transfer matrix including fluid and pipe mechanics is obtained by combining with field transfer matrices representing the motion of the single pipe sections and various point transfer matrices derived in this work for describing specified junction conditions. The simulation results show the hydraulic components have apparent impact on the dynamics of combined pipe structural and fluid system.

***Key words***: Hydraulic circuit, fluid-structural system, transfer matrix method

## Notation

**Arguments** (for pipe wall):

| | | | |
|---|---|---|---|
| $f_z$ | axial force | $F_z$ | axial force amplitude |
| $f_y$ | lateral shear force (y-z) | $F_y$ | lateral shear force amplitude (y-z) |
| $m_x$ | bending moment (y-z) | $M_x$ | bending moment amplitude (y-z) |
| $u_z$ | axial velocity | $U_z$ | axial displacement amplitude |
| $u_y$ | lateral velocity (y-z) | $U_y$ | lateral displacement amplitude (y-z) |
| $\psi_x$ | rotational velocity (y-z) | $\Psi_x$ | rotation amplitude (y-z) |

**Arguments** (for fluid):

| | | | |
|---|---|---|---|
| $p$ | axial pressure | $P$ | axial pressure amplitude |
| $v$ | axial velocity | $V$ | axial displacement amplitude |

**Coefficients**:

| | |
|---|---|
| $A$ | cross-sectional area |
| $e$ | pipe wall thickness |
| $E$ | Young's modulus of pipe wall |
| G | shear modulus |
| I | area moment of inertia |
| J | polar moment of inertia |

**Independent arguments**:

| | |
|---|---|
| $t$ | time |
| $z$ | axial coordinate |

| | |
|---|---|
| $k$ | coefficient |
| $L$ | length between accumulator and main pipe |
| m | mass |
| $K$ | fluid bulk modulus |
| $K^*$ | modified fluid bulk modulus |
| $p$ | pressure |
| $r$ | inner radius of pipe wall |
| $V$ | volume |
| Z | accumulator impedance |
| $\gamma$ | specific heat ratio |
| $\kappa$ | shear coefficient |
| $\mu$ | fluid dynamic viscosity |
| $\nu$ | Poisson ratio |
| $\rho$ | mass density |
| $\omega$ | angular frequency |

**Subscripts**:

| | |
|---|---|
| $0$ | system initial status |
| $a$ | accumulator |
| $f$ | fluid |
| $iy$ | $y$ direction of pipe section $i$ |
| $iz$ | $z$ direction of pipe section $i$ |
| $orf$ | orifice |
| $p$ | pipe wall |
| $pre$ | pre-charged gas |

## 1. Introduction

Safety associated with four-wheel drive vehicles, especially rollover occurring on-road or off-road, is an important issue for vehicle manufacturers. A type of Hydraulically Interconnected Suspensions (HIS) have recently been applied to race cars and passenger cars for improving drive performance and preventing rollover. This suspension can overcome the ride/handling compromise, which is the unavoidable shortcoming for individual wheel station suspension systems, without adding any other part by applying passive manner. The interconnection between wheel stations can provide more control on suspension performance.

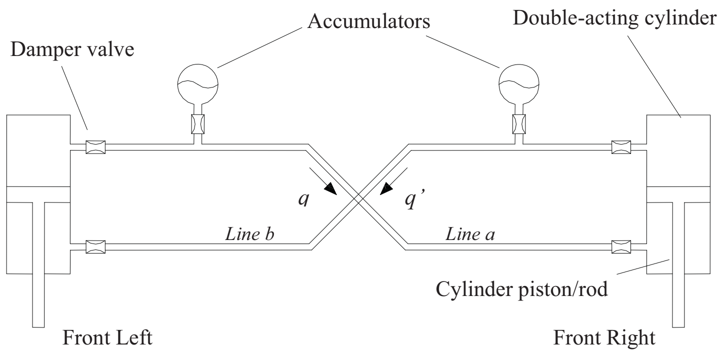


Fig. 1 Schematic Diagram for Hydraulic Circuit of Half-Car HIS System[1]

Fig. 1 illustrates the basic structure of the fluid circuit system of the suspension on the base of front half car. The simplified diagram represents a sealed liquid-filled system that consists of double-acting cylinders, diaphragm-type hydraulic accumulators, damper valves, and two interconnected fluid circuits, which is the principal part of the HIS. When installed on a vehicle, the pistons of the double-acting cylinders are fixed on each of the road wheels while the cylinders and the hydraulic circuit mount on the vehicle's chassis.

This HIS can separately increase roll stiffness and have almost no effect on bounce stiffness, which means that the vehicle handling effectiveness is improved without deducting the vehicle capability of reducing shock. It has been shown that the HIS can significantly improve vehicle handling and increase rollover stability compared with conventional suspension systems[2]. Therefore, the performance and safety of passenger cars can be improved through using this HIS. However, for the HIS, the pressure changes produced inside the fluid circuits often lead to vibration of pipelines and associated structures and become a source of structural noise.

When a vehicle fitted with a HIS runs, the displacement between wheels and the chassis produces relative movement of the piston to the cylinder. The pressure ripple induced by the movement is propagated within the hydraulic circuit and generates objectionable vibrations. The vibrations can be transferred to the vehicle structure and become an excitation force to the vehicle. The low- and mid-frequency vibrations influence vehicle handling and ride whilst the high-frequency vibrations are associated to the vehicle noise. Since the HIS mounts on the chassis of a vehicle, the vibration induced by fluid flow dynamically interacts with some of the vehicle assemblies and/or parts. The natural frequencies of some structures are high, so the resonance of high-frequency vibrations is obvious and becomes noise that cannot be ignored.

Due to the competition of commercial environment of the modern automotive market, the level of noise and vibration harshness (NVH) is one of the important quality indicators of a passenger and has to be eliminated or minimized cost-effectively. It influences not only the direct commercial profit but also the future success of a company. Therefore, a theoretical model of the HIS should be developed to understand the system and provide theories to optimise the design and assembly manner.

## 2. Current Research on HIS

The low-frequency handling effectiveness of the HIS was investigated in a previous research and the experimental results showed that for a motor vehicle, the suspension can provide greater rollover resistance than conventional suspension systems without significantly sacrificing ride quality [(2)]. The system performance of this HIS in low-to-mid-frequency ride performance is well understood by establishing and investigating an experimentally validated model of the system [(3)]. For low-to-mid-frequency investigation, pipes of the hydraulic system can be assumed as rigid body. However, it is inappropriate to use this assumption for high-frequency vibration investigation, thus an important phenomenon named Fluid-Structure Interaction (FSI) occurred within the hydraulic system must be considered when the model is developed. The FSI in piping systems has been investigated by many researchers. There are three coupling mechanisms which contribute to dynamic FSI: Poisson coupling, junction coupling, and friction coupling. In most practical systems, friction coupling is insignificant when compared to the other two coupling mechanisms[(4),(5)]. Both Poisson and friction coupling are distributed along the axis of a pipe element, whereas junction coupling acts at discrete locations in piping systems such as bends, supports, concentrated masses, orifices, etc.[(4),(6)]

Previous researches are mostly concern with fluid transients and how to smooth the peak pressure in pipeline. Wiggert and Tijsseling reviewed the majority of the studies in this field[(4),(6),(7)]. Li and other researchers performed frequency-domain analyses for liquid-filled single pipe to obtain the natural frequencies and mode shapes[(8)]. The characteristics of individual hydraulic element are investigated by Edge and Johnston. The focus of their studies is on the resist coefficients of hydraulic elements such as valves and accumulators[(9),(10)]. There are some hydraulic system models that are developed for particular applications[(3),(11),(12)]. However, the consideration of FSI is avoided to reduce the complication of the big system models.

Piping systems are conventionally demonstrated as one-dimension models due to the significant disparity between different dimensions. Li et al. use the straight pipe as their research object[(8)], whereas Hu and Phillips investigate the curved pipe[(13)]. Wiggert, Lavooij and Tijsselling et al. modelled the large piping system by treating each straight pipe section as a pipe reach[(4),(5),(6),(7)] and combining several pipe reach with elbows, which are considered as internal boundary conditions[(5)]. The one-dimension model for straight pipe section is applied in this paper because the curve-pipe model is not appropriate to be combined with the hydraulic elements' models. The transfer matrix method (TMM) is used to analyse the hydraulic system in frequency domain(14) since the natural frequencies and mode shapes of the system are the key characteristics to investigate the vehicle noise that is produced by the HIS.

## 3. Mathematical Modelling

### 3.1 System Assumption

Like the majority of the studies reviewed, the pipe element analysed in this study is liquid-filled, slender, straight, prismatic, and with circular cross-section(6). It is assumed that the inside liquid and the pipe-wall material are linearly elastic and friction free. The phenomenon of cavitations will not occur because of high initial pressure in the hydraulic circuit. The theory developed is valid for the low-frequency acoustic behaviour of the pipe section. This means that the ratios of fluid velocity to wave speed, pipe wall thickness to inner radius of pipe section, inner diameter of pipe section to wavelength, and liquid pressure to fluid bulk modulus are small with respect to unity(4). Axial, lateral and torsional vibrations are assumed not to influence each other along the straight pipe section.

### 3.2 Governing Equations

The pipe element transmits torsional waves, transverse shear and bending waves in the pipe wall, and axial compression waves in both the pipe wall and the liquid. Such motion can be described by 14 equations. In the initial research, the axial and one lateral direction (y-z) of the piping is studied first, thus the eight-equation model will be applied and the pipe element is assumed thin-walled pipe to simplify the model(4),(15),(16).

$$\frac{\partial p}{\partial z} + \rho_f \frac{\partial v}{\partial t} = 0 \qquad \frac{\partial m_x}{\partial z} - \left(\rho_p I_p + \rho_f I_f\right)\frac{\partial \psi_x}{\partial t} - f_y = 0$$

$$\frac{\partial p}{\partial t} + K^* \frac{\partial v}{\partial z} - 2\nu K^* \frac{\partial u_z}{\partial z} = 0 \qquad \frac{\partial m_x}{\partial t} - EI_p \frac{\partial \psi_x}{\partial z} = 0$$

$$\frac{\partial f_z}{\partial z} - \rho_p A_p \frac{\partial u_z}{\partial t} = 0 \qquad \frac{\partial f_y}{\partial z} - \left(\rho_p A_p + \rho_f A_f\right)\frac{\partial u_y}{\partial t} = 0$$

$$\frac{\partial f_z}{\partial t} - EA_p \frac{\partial u_z}{\partial z} - \frac{r\nu A_p}{e}\frac{\partial p}{\partial t} = 0 \qquad \frac{1}{\kappa G A_p}\frac{\partial f_y}{\partial t} - \frac{\partial u_y}{\partial z} - \psi_x = 0$$

For thin-wall pipe, the modified fluid bulk modulus is defined as(4):

$$\frac{1}{K^*} = \frac{1}{K} + \frac{2r}{eE}\left(1-\nu^2\right)$$

In Timoshenko's beam theory, the shear coefficient for hollow circle cross section of thin-wall pipe is(17):

$$\kappa = \frac{2(1+\nu)}{4+3\nu}$$

### 3.3 Transfer Matrix Method

By adopting TMM, The establishment of this model requires the integration of existing pipe element model and new-developed models of the hydraulic components such as orifice and accumulator. The state vector of the TMM is:

$$\mathbf{S} = \left[U_z(z), P(z), V(z), F_z(z), U_y(z), \Psi_x(z), M_x(z), F_y(z)\right]^T$$

$$u_z(z,t) = j\omega U_z(z)e^{j\omega t} \qquad p(z,t) = P(z)e^{j\omega t}$$

$$v(z,t) = j\omega V(z)e^{j\omega t} \qquad f_z(z,t) = F_z(z)e^{j\omega t}$$

$$u_y(z,t) = j\omega U_y(z)e^{j\omega t} \qquad \psi_x(z,t) = j\omega \Psi_x(z)e^{j\omega t}$$

$$m_x(z,t) = M_x(z)e^{j\omega t} \qquad f_y(z,t) = F_y(z)e^{j\omega t}$$

The field matrix expresses the forces and displacements at one section of a chain-type structure in terms of the corresponding forces and displacements at an adjacent section(15),(16). The point matrices express the relation of the two sides of the discrete locations.

A damper valve is simulated as a sharp-edged constant-area orifice. The point transfer matrix that describes the orifice is:

$$\begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Right}^{i} = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 & 0 \\ j\omega\dfrac{A_f}{k_{orf}} & 1 & -j\omega\dfrac{A_f}{k_{orf}} & 0 \\ 0 & 0 & 1 & 0 \\ -j\omega\dfrac{A_f^2}{k_{orf}} & 0 & j\omega\dfrac{A_f^2}{k_{orf}} & 1 \end{matrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Left}^{i}$$

where $k_{orf} = \dfrac{0.7^2}{3}\dfrac{\pi\, r_{orf}^3}{\mu}$

A diaphragm-type hydraulic accumulator consists of two chambers: a pre-charged gas chamber and a fluid chamber connected to a hydraulic system, which are separated by a kind of elastic diaphragm(18). The point transfer matrix that describes the accumulator is:

$$\begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Right}^{i} = \begin{bmatrix} 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & & & \mathbf{0} & \\ 0 & -\dfrac{1}{A_f Z_a} & 1 & 0 & & & & \\ -\omega^2 m_a & 0 & 0 & 1 & & & & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\omega^2 m_a L_a & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\omega^2 m_a & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Left}^{i}$$

where $Z_a = \dfrac{\gamma\, p_0^2}{p_{pre} V_{pre}} + \dfrac{j\omega}{k_{orf}}$

A support constrains the pipe movement on both axial and lateral directions. The point transfer matrix that describes the accumulator is:

$$\begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Right}^{i} = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -k_{iz} & 0 & 0 & 1 \end{matrix} & \mathbf{0} \\ \mathbf{0} & \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -k_{iy} & 0 & 0 & 1 \end{matrix} \end{bmatrix} \begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_{Left}^{i}$$

The entire transfer matrix for a piping system that includes pipe sections, supports, orifice, and accumulator can be derived by combining the field matrices and point matrices together. The relation is represented in Eq. 1. After applying boundary conditions, Eq. 2 can be derived and employed to determine the natural frequencies of the system, in which the order of matrix **TT** is smaller than the order of the entire transfer matrix **T**.

$$\mathbf{S}_N^R = \mathbf{P}_N \left( \prod_{1=N-1}^{1} \mathbf{F}_i\, \mathbf{P}_i \right) \mathbf{S}_1^L = \mathbf{T}\, \mathbf{S}_1^L \tag{1}$$

$$\mathbf{0}_N^R = \mathbf{TT}\ \mathbf{SS}_1^L \qquad (2)$$

## 4. Results and Discussion

The TMM is used for modelling non-rigid liquid-filled straight pipe to get its axial and lateral modes separately. The results are respectively compared with those come from Li et al.[8] and Zhang and Hayama[19], which result in the difference is very small (under 0.1%). Therefore, this method is appropriate to be used to model the hydraulic piping system. The TMM is also used for modelling a 90° bend non-rigid liquid-filled piping, through which the field matrix and point matrix of elbow are improved and the errors are corrected.

### 4.1 Natural Frequency Comparison for 8-Equation Model Simulation

Fig. 2 shows a pipe guided fluid circuit system, which includes pipe sections, supports, an orifice, and an accumulator. The axial and in plane (y-z) vibration of this system is analysed to obtain the natural frequencies of the system.

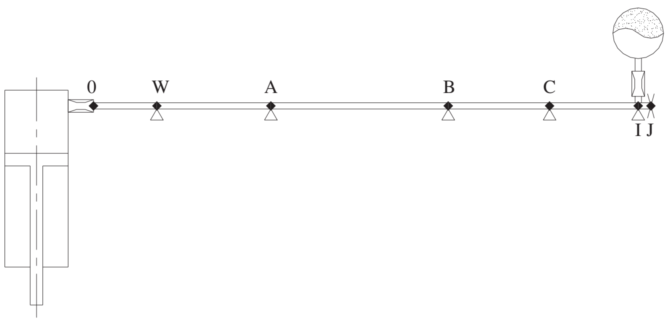


Fig. 2  Pipe guided fluid circuit system 1

Fig. 3 is the obtained results, which show that the hydraulic elements apparently influence the system characteristics. The blue line represents the natural frequencies of the system that only includes constraints of pipes, whereas the red line represents the natural frequencies of the system that consists of constraints, an orifice, and an accumulator.

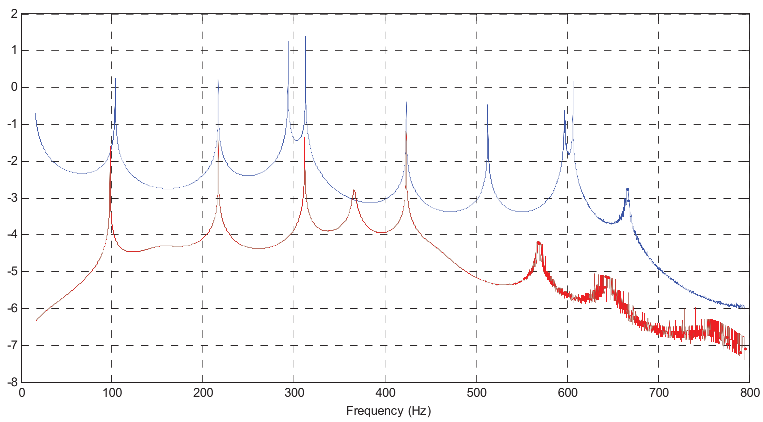


Fig. 3  Obtained natural frequencies of the pipe guided fluid circuit system 1 with and without hydraulic components

### 4.2 Accumulator's Influence in Axial Direction

The hydraulic piping system shown in Fig. 4 is almost the same system as Fig. 2 except having fewer supports. The axial direction of this system is analysed and the accumulator location is changed to show its influence.

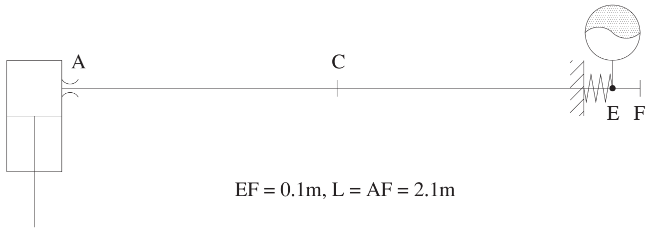


Fig. 4 Pipe guided fluid circuit system 2

Fig. 5 shows the first and second vibration modes of fluid pressure for the system without the accumulator. It represents the pressure distribution along the pipe.

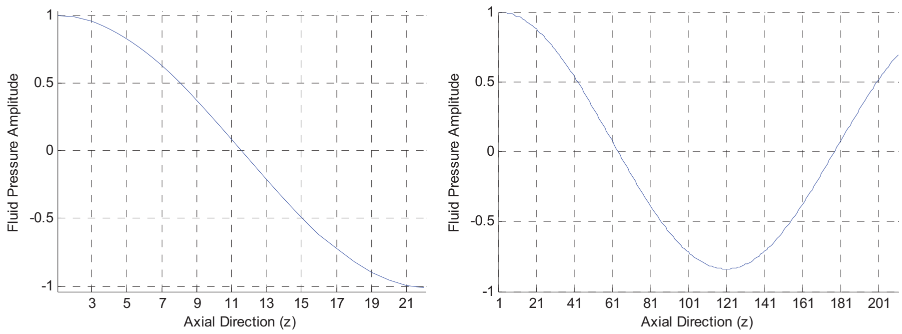


Fig. 5 1st and 2nd vibration modes of fluid pressure

It is shown in Table 1 that the accumulator has no impact on the first system natural frequency when it is fitted on the middle of the piping because the pressure amplitude almost equals zero at this point. The function of the accumulator is to reduce water hammer influence, i.e. the fluid pressure peak value, in piping systems. Therefore the accumulator cannot have its influence if it is located at the point that the pressure is always equal to zero. The same phenomenon is shown on about one quarter or three quarter position for the second system natural frequency.

Table 1. Influence of the accumulator's location on system natural frequencies

| Length (m) | support on E | CE (m) | $f_1$ (Hz) | $f_2$ (Hz) |
|---|---|---|---|---|
| AE = 2.0, EF = 0.1 | no accumulator | | 293.6 | 524.2 |
| | | CE = 0.2 | 408.4 | 522.0 |
| | accumulator on C | CE = 1.0 | 293.6 | 495.6 |
| | | CE = 1.5 | 206.1 | 501.7 |
| | accumulator on E | | 402.0 | 457.6 |
| AE = 1.0, EF = 0.1 | no accumulator | | 560.9 | |
| | accumulator on C | CE = 0.5 | 558.6 | |
| | accumulator on E | | 656.7 | |
| AE = 4.0, EF = 0.1 | no accumulator | | 146.6 | |
| | accumulator on C | CE = 2.0 | 146.7 | |
| | accumulator on E | | 217.7 | |

**4.3 Orifice's Influence in Axial Direction**

The pipe guided fluid circuit system shown in Fig. 6 is almost the same system as Fig. 4. The accumulator is taken out of the system to clearly show the influence of the orifice. The axial direction of this system is analysed and the orifice position is changed to obtain results of different situations.

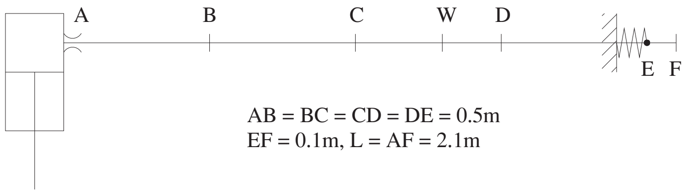


Fig. 6 Pipe guided fluid circuit system 3

Fig. 7 shows the first and second vibration modes of fluid displacement for the system without the orifice. It represents the fluid velocity distribution along the pipe.

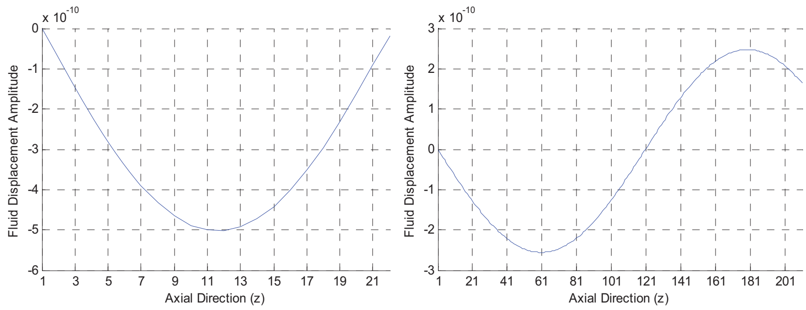


Fig. 7 1st and 2nd vibration modes of fluid velocity

If the radius of the cross hole of the orifice is too big, the influence cannot be shown in the simulation results. When the radius of the hole is equal to 1mm, the orifice shows no impact on the system wherever it is fitted in. However, when the radius is reduced to 0.1mm, the effect of the orifice is displayed in Table 2.

Table 2. Influence of the orifice's location on system natural frequencies

| Length (m) | support on E | $f_1$ (Hz) | $f_2$ (Hz) |
|---|---|---|---|
| AE = 2.0, EF = 0.1 | no orifice | 293.6 | 524.3 |
| | orifice on A | 293.6 | 524.3 |
| | orifice on B | 383.1 | 540.5 |
| | orifice on C | 518.0 | 608.8 |
| | orifice near C (WE = 0.79) | 524.7 | 700.6 |
| | orifice on D | 428.6 | 529.2 |
| | orifice near E (WE = 0.001) | 305.4 | 542.9 |

The orifice has no impact at Location A. For the first system natural frequency, it has quite little effect when it is very near Location E. When the orifice is fitted between Location C and D, the first natural frequency is much larger than that of other locations, but close to the second natural frequency of other locations. The two values are almost same if the orifice is located at C and the distance of CE is equal to 0.79m. It can be concluded that the orifice has significant influence on the first vibration mode but not so on the second vibration mode at the same location.

These phenomena can be explained by analysing the first two mode shapes shown in Fig. 7. For example, the orifice has no influence at Location A because it is the close-end and the fluid velocity at this point is always equal to zero. This is because that the function of the orifice is to limit fluid velocity in piping systems. The orifice cannot have its influence if it is located at the point that the fluid velocity is always equal to or very close to zero. According to conservation of energy, the total amount of the potential energy and the kinetic energy of the system is a constant at any point. Large pressure implies large potential energy and large velocity means large kinetic energy. At one location on which the potential energy is dominant, the kinetic energy must be small, vice versa.

## 5. Conclusion

This paper presented an extended transfer matric method for free vibration analysis of pipe guided fluid-structure circuit system, icluding the point matrices for hydraulic components like orifices, accumulators and supports. The system dynamic characteristics have been investigated for various combinations of system configurations. The obtained results show that the influence of orifice and accumulator is obvious on the dynamics of pipe guided fluid circuit systems and can be explained by their physical implication. For a certain vibration mode, the accumulator should be located at the point on which the pressure is large to reduce the pressure ripple inside the fluid circuit. On the other hand, if the velocity of one location is large, this means the orifice can be located at this point to effectively limit the fluid speed.

The presented model can describe the dynamic characteristics of a pipe guided fluid circuit system with various hydraulic components and boundaries. It can be employed to analyse the system steady state dynamics but for transient dynamics, further work is needed.

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