Modeling of a New Active Suspension for Roll Control

Nong Zhang*, Lifu Wang* and Haiping Du**

* Mechatronics and Intelligent Systems, University of Technology, Sydney,,
P.O. Box 123, Broadway, NSW 2007, Australia
E-mail: nong.zhang@uts.edu.au,
lifu.wang@eng.uts.edu.au

** School of Electrical, Computer and Telecommunications Engineering
University of Wollongong
E-mail: hdu@uow.edu.au

Abstract

The paper presents the computer simulations and experimental validation on a new active suspension used for vehicle roll control. This study focuses primarily on the dynamics of Hydraulically Interconnected Active Suspensions and the dynamics of a vehicle fitted with the suspension. The proposed active suspension, also called as Demand Dependent Active Suspension, consists of four double direction hydraulic actuators which are hydraulically interconnected, and a pressure controlled fluid supply unit. The suspension can actively tilt the vehicle against its vibration modes (roll, pitch, bounce and articulation), by applying required restoring forces or moments. It therefore far exceeds the capability of passive or semi-active suspensions. This study evaluates the first part of the vehicle multi-mode control based on the proposed active suspension, i.e., implements vehicle roll control. This is because of the dangerous nature and fatal injuries associated with vehicle rollover incidents. Modeling, simulations and preliminary experiment of system dynamics of a vehicle fitted with the proposed suspension are carried out and the obtained simulation results agree with those obtained from experiment.

Key words: Active suspension, Hydraulically Interconnected Active Suspensions, Vehicle System Dynamics, Vehicle roll control.

1. Introduction

Hydraulically Interconnected Active Suspensions (HIAS), a novel active suspension, is of great potential value to the automotive industry due to its simplicity and reduced overall cost. Few studies have analyzed the use of independent active suspension to improve the vehicle comfort and handling performance.

The increased demand of vehicles promises another century's vigorous vitality in the automotive industry. Vehicle safety is the most important factor considered in vehicle design and will maintain its priority in the drawing board. Vehicle accidents have inherent dangerous consequences, and many reported fatal accidents are involved with vehicle rollover. The conventional means of anti-roll approach have limited ability to deal with vehicle rollover. Therefore, to meet the industry's needs, new technologies to improve vehicle anti-roll ability are developed, such as the hydraulically interconnected passive suspension system [1]. A successful commercial application of this technology is the Kinetic Dynamic Suspension System (KDSS) in some passenger cars [3]. This hydraulically

interconnected passive suspension recently has been theoretically and experimentally studied by Zhang et al, [4-5].

Active suspensions utilize more advanced technologies to yield improved suspensions with adjustable parameters [6]; they have therefore been studied and implemented in recent decades. Fully active or semi-active suspension technology emerged from top brand racing and luxury vehicle design. For example, Audi used these technologies in their sports vehicles [7]. Another example is the Bose suspension which uses linear electromagnetic motors installed at each wheel, controlled by a set of proprietary algorithms [8]. The subject of conventional active suspension design has been intensively reviewed by Hedrick and Wormley [9], Godall and Kortum [10], Sharp and Crolla [11] and Masao Nagai [12]. It is noticeable that Hrovat [13] presented an extensive survey covering the area of active suspension developments.

However, there are some drawbacks with the use of conventional active suspensions, which typically have independent structures, i.e. four independently controlled actuators. Better performance should be expected from this ideal suspension arrangement but the issues of the independent arrangement include increased cost, reduced system reliability, increased power consumption and inherent complexity. For these reasons, alternative active suspensions that enhance vehicle safety with less energy consumption, simplified structure, and reliable function and reduced cost are of better value to the automotive industry.

Two prominent trends attempting to overcome this aforementioned limitation of active suspension have been observed in the past decade. One attempt made to optimize the active suspension is to implement advanced methodology to seek the main vibration or motion mode for actuators to deal with, by eliminating the insignificant components from the vehicle vibration signal. Thus less energy will be consumed due to fewer tasks assigned to active suspensions. For instance, the slow-active suspension only acts in a limited low frequency range [14].

The second trend observed involves adopting compact interconnected fluid circuits into suspension design, instead of using the ideal independent layout of four actuators. The benefit of this type of active suspension will be low cost, reduced power requirements, less complexity and therefore increased reliability. For example, "Dynamic Ride Control (DRC) Sports suspension system" [7] uses diagonally interconnected active suspensions,. The system is mainly mechanical; using a pump to provide additional pressure in the diagonally linked shock absorbers during cornering, to counteract rolling and pitching.

Although some attempts have been made to apply the active suspension technology into engineering practice, designing the cost-effective active suspension, in terms of both energy consumption and manufacturing cost is still a challenge in the foreseeable future. Zhang proposed a novel design of hydraulic interconnected active suspension and a Demand Dependent Active Suspension (DDAS) [15, 16]. These active suspensions consist of four double direction hydraulic actuators, hydraulically interconnected and controlled by a compact manifold, and a pressure controlled fluid supply unit. DDAS can actively tilt the vehicle against its motions, e.g. rollover, by supplying different required restoring forces. This ability far exceeds the capabilities of passive or semi-active suspension systems.

2. Design

The details of the DDAS design and experiment set up can be found in [15, 16]. Here a brief description of the DDAS structure and function is provided. This active suspension design consists of four double direction hydraulic actuators, hydraulically interconnected, powered by a mechanical steering pump and controlled by a compact manifold. The oil from the mechanical power steering pump is regulated by a proportional valve in the manifold, and the hydraulic circuits can be reconfigured in real time by switching a group

of solenoid valves. Using signal collected from the vehicle, the controller controls the proportional pressure relief valve to change the pressure and directs the solenoid valves to deal with the vibration modes by setting up the required hydraulic circuits.

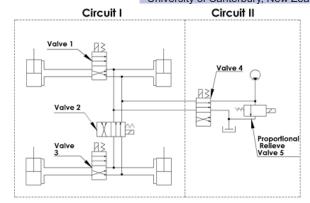


Fig. 5. DDAS hydraulic layout [15]

3. Modeling

The dilemma likely to be encountered in this modeling is the need to develop a system dynamic model that is as simple as possible and yet to a certain degree of complexity that can predict reasonably accurately the system response. In this project, the dynamic behavior of a multi-body system is characterized by a composition of rigid bodies, interconnected by joints, springs, dampers, and actuators. Furthermore, the active suspension system couples with vehicle through the boundary condition inside the actuators. Joints constrain the motion of the bodies in the vehicle system and the mechanical-hydraulic boundary conveys the hydraulic energy that alters the vehicle motion.

3.1 A bicycle model of kinematic steering characteristics

The roll moment resulted from lateral acceleration during cornering is used as the input disturbance to the vehicle in consideration. The moment input is derived from the rigid car model and vehicle steering bicycle model, as shown in Fig. 1 and Fig. 2. From the bicycle model, the lateral acceleration can be determined by the steering angle and the vehicle speed. From the rigid car model, the centripetal force supplied by the friction between the tyre and ground, is equal to a couple (or called a moment) and a *Y* direction force to the center of the vehicle mass, which is roughly the mass center of the vehicle body.

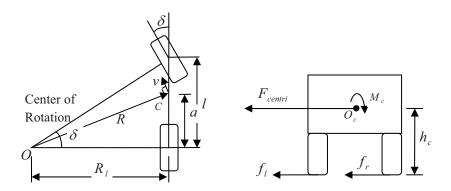


Fig.1 A bicycle vehicle steering model

Fig.2 A rigid model for vehicle lateral analysis

From Fig.1, the radius R of the vehicle turning is then determined by the Eq. (1) and further modified by the compensating method detailed in [17].

$$R = \sqrt{a^2 + l^2 \cot^2 \delta} \tag{1}$$

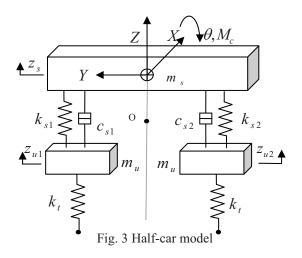
Where the δ is the turning angle of the steering wheel, C is the center of the vehicle, V is the vehicle velocity in Fig. 1, and f_{l} f_{r} are the friction forces from the tyre in the

lateral direction.

From Fig. 2, the roll moment is determined by:

$$f_l + f_r = F_{centri} = m_v (\frac{v^2}{R})$$
 ; $M_c = (f_l + f_r)h_c$ (2)

3.2 A 4-DOF half-car modeling with the input of a couple



The differential equations for this half-car model are given in the Appendix.

Based on the differential equations, we obtain the following state equation for the half-car system:

$$\dot{x}_{car} = A_{car} x_{car} + B_{car} u_{car}(t) \tag{3}$$

Where the vectors of state is:

$$\boldsymbol{x}_{car} = \begin{bmatrix} \boldsymbol{\theta} & \boldsymbol{z}_s & \boldsymbol{z}_{u1} & \boldsymbol{z}_{u2} & \dot{\boldsymbol{\theta}} & \dot{\boldsymbol{z}}_s & \dot{\boldsymbol{z}}_{u1} & \dot{\boldsymbol{z}}_{u2} \end{bmatrix}^T$$

Where θ and z_s stand for the roll angle and the vertical displacement of sprung mass; while z_{u1} and z_{u2} are the vertical displacements of the unsprung masses.

3.3 8-state half-DDAS modeling

The full DDAS has four working modes, such as anti-bounce, anti-roll, anti-pitch and anti-articulation; however, because the half-car model was used for this study, the DDAS system only has two working modes, anti-bounce and anti-roll. Hence, it is appropriately titled half-DDAS modeling. In Fig. 6, the half-DDAS Model used in this paper is an 8-state, first order linear system with two distance inputs and two force outputs. The compact manifold for pressure control and hydraulic circuit reconfiguration is realized by handily controlling the pressure ports, P_{rs} and P_{r2} .

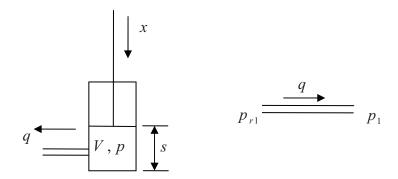


Fig. 4 Hydraulic cylinder chambers

Fig. 5 Fluid line model

In Fig. 5, the oil inside the chambers is treated as compressible fluid and modeled by using the bulk modulus of the fluid, to capture the hydraulic dynamics of the system;

Bulk modulus is $E = \frac{-dp}{dV}V$, then

$$\dot{p} = \frac{E}{sA_a} (A_a \dot{x} - q) \tag{4}$$

Where s is the height of the analyzed oil volume, A_a is the piston area, and $A_a\dot{x}$ is the volumetric flow due to piston motion, and the pressure differential \dot{p} is related to the flow rate change $(A_a\dot{x}-q)$. It is noted that here the oil volume change inside the chamber is assumed constant in order to linearize the system, and a time varying equation will be considered in the further study.

In Fig. 6, the oil inside the pipe is treated as an incompressible fluid due to its insignificant affect to the hydraulic force output from the piston. This part of the fluid is consequently satisfying Newton's second law, $m\ddot{x}=F$, and we can therefore obtain:

$$\rho L_p \dot{q} = p_{r1} A_p - p_1 A_p \tag{5}$$

Where ρ is the fluid density, L_p and A_p is the length and the pipe area, the flow rate differential is related to the pressure applied at both sides, p_{r1} and p_1 .

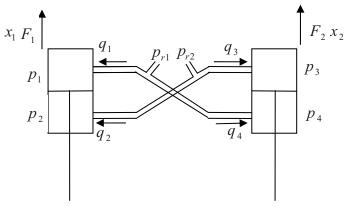


Fig. 6 Half-DDAS model

Now, combining the two hydraulic modeling components from Fig. 4 and Fig. 5 to model the half-DDAS model in Fig. 6, we get:

$$\dot{p}_{1} = \frac{E}{sA_{a}} (\dot{x}_{1}A_{a} + q_{1})$$

$$\dot{p}_{2} = \frac{E}{sA_{a}} (-\dot{x}_{1}A_{a} + q_{2})$$

$$\dot{p}_{3} = \frac{E}{sA_{a}} (\dot{x}_{2}A_{a} + q_{3})$$

$$\dot{p}_{4} = \frac{E}{sA_{a}} (-\dot{x}_{2}A_{a} + q_{4})$$
(6)

Where from the half-car model we can get:

$$\dot{x}_{1} = l_{1}\dot{\theta} + \dot{z}_{s} - \dot{z}_{u1} \quad ; \quad \dot{x}_{2} = -l_{2}\dot{\theta} + \dot{z}_{s} - \dot{z}_{u2}$$

$$p_{r1} - p_{1} = \dot{q}_{1} \frac{\rho L_{p}}{A_{p}}$$

$$p_{r2} - p_{2} = \dot{q}_{2} \frac{\rho L_{p}}{A_{p}}$$

$$p_{r2} - p_{3} = \dot{q}_{3} \frac{\rho L_{p}}{A_{p}}$$

$$p_{r1} - p_{4} = \dot{q}_{4} \frac{\rho L_{p}}{A_{p}}$$
(7)

Eq. (6-7) can then be written in matrix State-Space form:

$$\dot{x}_{DDAS} = A_{DDAS} x_{DDAS} + B_{DDAS} u_{DDAS}(t) \tag{8}$$

$$A_{DDAS} = \begin{bmatrix} 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a \\ -b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b & 0 & 0 & 0 & 0 \end{bmatrix}$$
 (9)

Where $a = \frac{E}{sA_a}, b = \frac{A_p}{\rho L_p}$

$$B_{DDAS} = \begin{bmatrix} B_{D1} & 0 \\ 0 & B_{D2} \end{bmatrix} \quad , \quad$$

 B_{D2} controls the working modes of DDAS, i.e., anti-bounce or anti-roll.

3.2 Integrated Vehicle half-car model with DDAS system

Now we can assemble the DDAS into the half-car model. To couple the two systems, the displacements from the half-car are used as the half-DDAS input; while the hydraulic forces from the half-DDAS system are the control input of the half-car model.

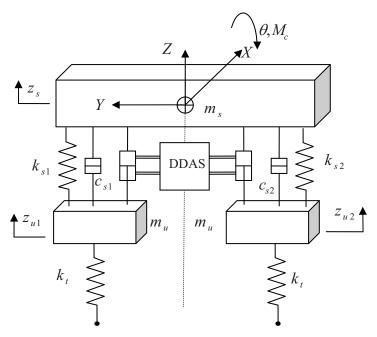


Fig. 7 Half-car fitted with a half-DDAS system

The state vectors: $\mathbf{x}_{DDAS} = [\theta \ z_s \ z_{ul} \ z_{u2} \ \dot{\theta} \ \dot{z}_s \ \dot{z}_{ul} \ \dot{z}_{u2} \ p_1 \ p_2 \ p_3 \ p_4 \ q_1 \ q_2 \ q_3 \ q_4]^T$

Where, the matrices A_{2_3} and A_{3_2} represent the coupling between the half-car and half-DDAS systems; A_{2_3} converts the pressure from the hydraulic system to the active force to vehicle system and the A_{3_2} transfers the vehicle body displacement to the pressure change of the hydraulic system.

$$A_{2_{-3}} = \begin{bmatrix} -\frac{l_{1}}{I} & \frac{l_{1}}{I} & \frac{l_{2}}{I} & -\frac{l_{2}}{I} \\ -\frac{l_{1}}{I} & \frac{l_{1}}{I} & \frac{l_{2}}{I} & -\frac{l_{2}}{I} \\ -\frac{l_{1}}{I} & \frac{l_{1}}{I} & \frac{l_{2}}{I} & -\frac{l_{2}}{I} \\ \frac{l_{1}}{I} & -\frac{l_{1}}{I} & 0 & 0 \\ 0 & 0 & \frac{l_{1}}{I} & 0 & 0 \\ 0 & 0 & \frac{l_{1}}{I} & -\frac{l_{2}}{I} & -\frac{l_{2}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & 0 \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} & -\frac{a_{1}}{I} \\ -\frac{a_{1}}{I} & -\frac{a_{1}}{I}$$

4. Simulation

In this section, the DDAS car response with different working modes in an "S" maneuver test is presented. The DDAS is set into anti-bounce and anti-roll modes, and the obtained simulation results are presented. Moreover, the experiment results of the anti-roll mode are also provided to verify the simulation results. At last, the change of the vehicle roll stiffness by DDAS is evidenced in simulation.

First, the response of a half-car's response is compared with a half-car fitted with an

inactive DDAS in anti-bounce mode and anti-roll mode setting respectively. From the Fig. 8, it can be seen that, when the DDAS is inactive, it gives no affect to the vehicle system, regardless of the anti-bounce or anti-roll mode setting.

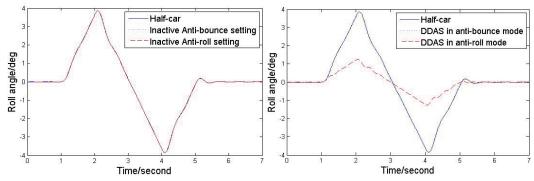


Fig. 8 Vehicle with inactive DDAS

Fig. 9 Vehicle with an active DDAS

In Fig. 9, the performance of DDAS in two modes dealing with an "S" maneuver is provided. It can be seen, the DDAS anti-roll mode works well in reducing the vehicle body roll angle, while the DDAS anti-bounce mode does not affect the vehicle roll motion.

The simulation results of DDAS anti-roll mode are also compared with those obtained from the experiment, the detail of which presented in [16], and the test results of a DDAS-vehicle in a stimulated S maneuver are provided in Fig. 10.

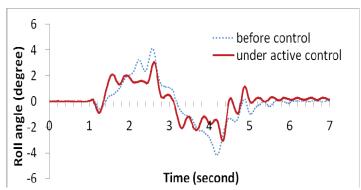


Fig. 10 DDAS Anti-roll Experiment results [15]

For this test, the DDAS was in anti-roll mode, and the results show that it can reduce the vehicle maximum roll angle, which is very useful for the vehicle anti-rollover control. From the graph, the oscillation was caused by the limitation of the experimental condition, aside from that, it is clear that the DDAS anti-roll mode has the same function in the experiment.

5. Discussion

5.1 Hydraulic-mechanical Coupled dynamics

The hydraulic system (DDAS) is coupled kinetically to the mechanical system (vehicle) through double acting hydraulic cylinders. The pressure response of the hydraulic system is a dynamic process and is affected by the change of its boundary condition, which is initiated by the relative motion between wheels and vehicle body. On the other hand, the vehicle natural roll frequency increases because the hydraulic system provides the additional roll stiffness, which results from by the working pressure. The increased vehicle roll natural frequency from 2.6Hz to 3 Hz has been observed in the experiment. This can be explained as the hydraulic system increased the vehicle system roll stiffness, which is the nature of the

hydraulic-mechanical dynamic coupling. The proposed model has successfully captured this coupling, as evidenced by the simulation results shown in Fig. 11.

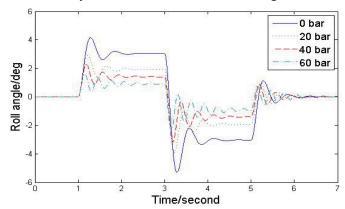


Fig. 11 The increased vehicle roll natural frequency caused by DDAS in simulation

5.2 Control strategy

A basic proportional controller is used in both simulation and experiment to prominent the system dynamic character. In the future, more sophisticated control strategies e.g., fuzzy control and H^{∞} control, may be applied in the control of the active suspension system.

5.3 Time delay issue

The consideration of actuator time delay and the valve switching time in the controller design is beyond the scope of this paper. But time delay is an important issue in active suspension controller design and will be carried on in next step modeling.

5.4 Time varying model

One limitation of this model is that the volume change of the cylinder chamber is treated as negligible to linearize the system.

6. Conclusion

This paper has presented the modeling of a novel active suspension with multi-mode control functionality. The suspension uses controllable hydraulically interconnected fluid circuit and becomes compact, energy efficient and cost effective.

This paper presents a simplified vehicle model for the control purpose and capturing the dynamics of vehicle in cornering. A simplified half-DDAS model has two working modes, i.e., to control bounce and roll respectively. The working mechanism of this half-DDAS has been demonstrated through simulation and experiment. The obtained results show the DDAS is effective in providing significant additional roll stiffness for preventing rollover from happening. Discussions on the limitations of DDAS modeling and several possible directions for future investigation are also provided.

References

- (1) W. Smith and N. Zhang, "Experimental and Theoretical Investigation into the Dynamics of a Half-Car with an Interconnected Passive Suspension," SAE Technical Paper Series, SAE 2009-01-0579, 2009.
- (2) Lexus, a Division of Toyota Motor Sales, U.S.A., Inc, "Company Website," August 26. [Online]. Available: http://www.lexus.com. [Accessed: 26 August, 2009]
- (3) Toyota, "Company Website," August 26. [Online]. Available: http://www.toyota.com. [Accessed: 26 August, 2009]

- (4) W. Smith, N. Zhang, and J. Jeyakumaran, "Ride Simulations of a Half-car with a Hydraulically Interconnected Passive Suspension," presented at FISITA-2006 World Automotive Congress, Yokohama, Japan, 22-27 October, 2006.
- (5) W. Smith, N. Zhang, and J. Jeyakumaran, "Free Vibration of Vehicles with Interconnected Suspensions," presented at XXII International Congress of Theoretical and Applied Mechanics, Adelaide, Australia, 25-29 August, 2008.
- (6) T. Carrett, K. Newton, and W. Steeds, *The motor vehicle*, 13th edition, pp. 1110-1111, 2001.
- (7) Audi, "Company Website," August 26. [Online]. Available: http://www.audi.com. [Accessed: 26 August, 2009]
- (8) Bose, "Company Website," August 26. [Online]. Available: http://www.bose.com. [Accessed: 26 August, 2009]
- (9) J. K. Hedrick and D. N. Wormley, "Active Suspension for Ground Transportation Vehicle A state of-the-art review", Mechanics of Transportation Systems, AMD, Vol. 15, pp. 21-40, (ASME),1975.
- (10) R. M. Godall and W. Kortum, "Active Control in Ground Transportation A review of the state-of the art and future potential", Vehicle System Dynamics, Vol. 12, pp. 205-227, 1983.
- (11) R. S. Sharp and D. A. Crolla, "Road Vehicle Suspension System Design a review", Vehicle System Dynamics, Vol. 16, pp. 167-192, 1987.
- (12) M. Nagai, "Recent Researches on Active Suspensions for Ground Vehicles", JSME Int. J. Ser. C, Vol. 36(2), pp. 161-170, 1993.
- (13) D. Hrovat, "Survey of Advanced Suspension Developments and Related Optimal Control Applications", Automatica, Vol. 33(10), pp. 1781-1817, 1997.
- (14) B. A. AlBassam, A. A. Fayed and M. M. ElMadany, "Optimal Linear Preview Control of Slow-active Suspension Systems", Proceedings of the 7th Saudi Engineering Conference (SEC7), 2007.
- (15) N., Zhang, Active Suspension, Australian Patent Application, PCT/Au2008-000720, filed on 21 May 2008.
- (16) L. Wang, N. Zhang and H. Du, "Design and Experimental Investigation of Demand Dependent Active Suspension for Vehicle Rollover Control", accepted for presentation in 48th IEEE Conference on Decision and Control, Shanghai, China, 2009.
- (17) R. N. Jazar, Vehicle Dynamics: Theory and Applications, Springer, pp. 382, 2008.

Acknowledgements

Financial support for this research was provided jointly by the Australian Research Council (DP0773415) and the University of Technology, Sydney, Australia.

Appendix: Differential equations for half-car and half-DDAS model

The equations for the heave and roll movements of the vehicle body, and the heave movements of the two wheels are respectively given as follows.

Vehicle body (sprung mass) movements:

$$m_{s}\ddot{z}_{s} = -k_{s1}(z_{s} + \theta l_{1} - z_{u1}) - k_{s2}(z_{s} - \theta l_{2} - z_{u2}) - c_{s1}(\dot{z}_{s} + \dot{\theta} l_{1} - \dot{z}_{u1}) - c_{s2}(\dot{z}_{s} - \dot{\theta} l_{2} - \dot{z}_{u2}) - F_{1} - F_{2}$$
(A1)

$$I\ddot{\theta} = M_c + m_s gh \theta - l_1 k_{s1} (z_s + \theta l_1 - z_{u1}) - l_1 c_{s1} (\dot{z}_s + \dot{\theta} l_1 - \dot{z}_{u1}) + l_2 k_{s2} (z_s - \theta l_2 - z_{u2}) + l_2 c_{s2} (\dot{z}_s - \dot{\theta} l_2 - \dot{z}_{u2}) - F_1 l_1 + F_2 l_2$$
(A2)

Wheel (unsprung mass) movements:

$$m_{\nu}\ddot{z}_{\nu 1} = -k_{\tau}z_{\nu 1} + k_{s1}(z_{s} + \theta l_{1} - z_{\nu 1}) + c_{s1}(\dot{z}_{s} + \dot{\theta} l_{1} - \dot{z}_{\nu 1}) + F_{1}$$
(A3)

$$m_{u}\ddot{z}_{u2} = -k_{t}z_{u2} + k_{s2}(z_{s} - \theta l_{2} - z_{u2}) + c_{s2}(\dot{z}_{s} - \dot{\theta} l_{2} - \dot{z}_{u2}) + F_{2}$$
(A4)