Robust Optimization in HTS Cable Based on DEPSO and Design for Six Sigma

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***Abstract*—The non-uniform AC current distribution among the multi-layer conductors in a high-temperature superconducting (HTS) cable reduces the current capacity and increases the AC loss. In this paper, Particle swarm optimization coupled with differential evolution operator (DEPSO) has been applied in structural optimization of HTS cables. While the existence of fluctuation in design variables or operation conditions has a great influence on the cable quality, in order to eliminate the effects of parameter perturbations in design and improve the design efficiency, a robust design method based on design for six sigma (DFSS) is applied in this paper. The optimization solutions show that the proposed optimization procedure can not only achieve a uniform current distribution, but also improve significantly the reliability and robustness of the HTS cable quality.**

Keywords-current distribution; high temperature superconducting (HTS) cable; particle swarm optimization (PSO); perturbation analysis; differential evolution (DE) operator; design of six sigma.

I. Introduction

HTS cables for large current transmission in general have a multi-layer structure consisting of parallel connected tapes, twisted in each layer. Due to the difference of inductances among layers, the currents flowing in these layers are different. Therefore, the control of current distribution among these layers is an important issue for design and optimization of an HTS cable because this significantly affects the current transmission capacity and power losses.

Usually, for an HTS AC cable, the current distribution among layers is substantially determined by the inductive impedances of these layers. The distribution of inductive impedances is however dependent on the structural parameters of the cable conductor. The main method to obtain a uniform current distribution is to alternate the inductive impedances of layers by adjusting the structural parameters of the cable conductors. Many optimization methods, such as the genetic algorithm (GA) and the particle swarm optimization (PSO) algorithm, have been applied [1]. In this paper, an improved PSO with differential evolution operator (DEPSO) is presented to optimize the structural parameters of HTS cable conductors to achieve the uniform current distribution among layers.

However, the performance of an HTS cable may to a certain degree be affected by perturbation of parameters possibly caused by imperfect manufacturing or non-ideal properties of superconducting tapes, e.g. shrinkage at low-temperature [2]. Traditional optimization methods cannot take into account the perturbations, so they may lead to unreliable or non-robust solutions.

In this paper, a robust design method based on design for six sigma (DFSS) [3] is introduced for HTS cable optimization. The philosophy of DFSS in quality engineering is applied in this optimization procedure to improve the process quality and design reliability. Taking a cold dielectric type HTS cable as an example, the optimized parameters are compared with those obtained by DEPSO.

II. Improved Particle Swarm Optimization Method

A. Particle Swarm Optimization (PSO)

The particle swarm optimization (PSO) method is a population based stochastic optimization technique developed in 1995 by Kennedy and Eberhart, inspired by the social behavior of birds flocking and fish schooling [4].

Suppose that the search space has D-dimensions. The position of the *i*-th particle in the swarm can then be expressed as a vector $X_i(t)=(X_{i,1}(t), X_{i,2}(t), \ldots, X_{i,D}(t))$. The velocity of this particle can be represented by another vector $V_i(t)=(V_{i,1}(t), V_{i,2}(t), \ldots, V_{i,D}(t))$. The *i*-th particle also maintains a memory of its previous best position in the vector $pbest_i$, and in each iteration step, *gbest* is designated as the index of the best particle in the swarm. Subsequently, the swarm is manipulated according to the following two equations [5]:

$$V_{i,d}(t) = wV_{i,d}(t-1) + c_1 r_1 \times (pbest_{i,d} - X_{i,d}(t-1))/\Delta t + c_2 r_2 \times (gbest_d - X_{i,d}(t-1))/\Delta t \quad (1)$$

and

$$X_{i,d}(t) = X_{i,d}(t-1) + V_{i,d}(t) \cdot \Delta t \quad (2)$$

where $d=1,2,\ldots,D$, and $i=1,2,\ldots,N$, N is the size of the swarm, c_1 and c_2 are two positive constants, namely social and cognitive parameters, r_1 and r_2 two random numbers distributed within the range [0,1], t is the iteration number, $\Delta t=1$, and w is inertia weight.

B. DEPSO Algorithm

To overcome the premature of multi-model function search by the standard PSO, a hybrid particle swarm with differential evolution operator (DEPSO) is utilized, which also provides the bell-shaped mutations with consensus on the population diversity, while keeps the particle swarm dynamics [6].

The mutations are provided by DE operator on the $pbest_i$, with a trail point $tbest_i = pbest_i$, which for the dth dimension:

If (*rand*()<CR OR *d*==*k*)

$$
\begin{aligned}
tbest_{i,d} &= pbest_{i,d} + \lambda \cdot \left(gbest_d - pbest_{i,d} \right) + \beta \cdot \delta_{2,d} \\
\delta_{2,d} &= \left(\sum_{1}^{2} \Delta_d \right) \Big/ 2 \\
\Delta_d &= pbest_{A,d} - pbest_{B,d}
\end{aligned}
\tag{3}
$$

where k is a random integer value within [1, D], which ensures the mutation at least one dimension, CR is a crossover constant, λ and β are two weighted factors respectively, δ_2 is the general difference vector, Δ means the difference between two elements that are randomly chosen from a common point set, which includes all the $pbest$ in the current case, $pbest_A$ and $pbest_B$ are chosen from the $pbest$ set at random.

III. Robust Optimization Using Design for Six Sigma

Because of errors and uncertainties in design process, manufacturing process, and operating condition in real-world engineering designs, the idea of robust optimization considering both the optimality and the robustness of objective function and constraints has been paid attention for real-world design problems in recent years.

Design for six sigma (DFSS) is one of conventional robust optimization approaches. The term “sigma” refers to standard deviation σ, which is a measure of dispersion, and “six sigma” is one of the management reform techniques aiming at the establishment of business process with very small dispersion.

In a traditional optimization (minimization) problem, the objective function f of design variable $\boldsymbol{X}$ should be minimized as follows.

$$
Minimize: \quad f(\boldsymbol{X})
$$

Combining the probabilistic elements of reliability, robust design, and six sigma, variability is incorporated into a robust optimization formulation through the definition of uncertain random variables, formulation of reliable input constraints and output constraints, and objective robustness (minimize variation in addition to mean performance on target). The formulations of robust optimization based on DFSS can be established as follows [3]:

$$
\begin{aligned}
Minimize: \quad & F(\mu_f(X), \sigma_f(X)) \\
& = \mu_f(X) + \omega_1 \sigma_f(X) \\
subject\ to: \quad & \mu_{g_i(X)} - n\sigma_{g_i(X)} \geq 0 \\
& X_L + n\sigma_X \leq \mu_X \leq X_U - n\sigma_X \\
& \mu_f - n\sigma_f \geq Lower\ specification\ limit \\
& \mu_f + n\sigma_f \leq Upper\ specification\ limit
\end{aligned}
\tag{4}
$$

where the set of design variables $\boldsymbol{X}$ includes the input parameters that may be design variables, random variables, or both. The objective function F and constraints are described by the mean value $\mu_f(\boldsymbol{X})$ and standard deviation $\sigma_f(\boldsymbol{X})$.

The main steps to achieve DFSS are described as Fig. 1 [7].

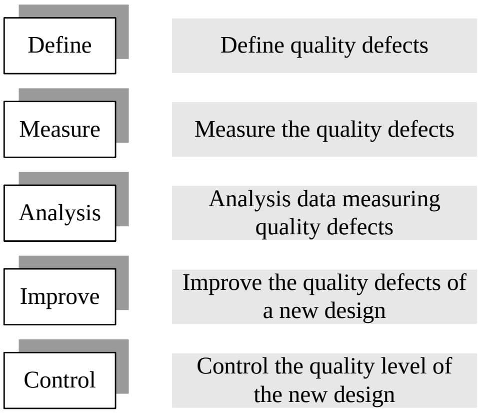


Figure 1. DFSS procedure

IV. Model of HTS Cable

The structure of single-phase cold dielectric type HTS power cable, consisting of four layers of conductors and two layers of shield of Ag/Bi-2223 tapes, is shown in Fig. 2 [8].

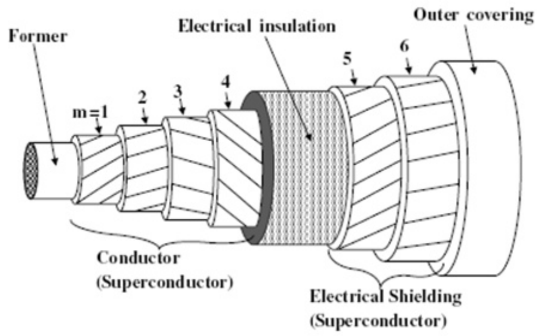


Figure 2. Schematic diagram of the cold dielectric type HTS cable.

As described in [1], the winding angle, direction and radius of each layer are selected as the design variables. For a cable of n layers, the optimized variables can be expressed as a vector:

$$
\boldsymbol{X} = \left[\beta_1, a_1, R_1, \beta_2, a_2, R_2, \cdots, \beta_n, a_n, R_n \right]
$$

Without quenching, the objective function for optimization of the cold dielectric type HTS cable is derived to achieve the uniform current distribution among the phase conductors and the shields respectively, which is subject to the mechanical properties and critical current of the tape. The critical current is derived from its relationship of the magnetic field and temperature of superconducting tape.

$$\min\ f(\boldsymbol{X})=\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left|I_{ix}(\boldsymbol{X})-I_{jx}(\boldsymbol{X})\right|+\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left|I_{iy}(\boldsymbol{X})-I_{jy}(\boldsymbol{X})\right| + \sum_{i=m+1}^{n-1}\sum_{j=i+1}^{n}\left|I_{ix}(\boldsymbol{X})-I_{jx}(\boldsymbol{X})\right|+\sum_{i=m+1}^{n-1}\sum_{j=i+1}^{n}\left|I_{iy}(\boldsymbol{X})-I_{jy}(\boldsymbol{X})\right| \quad (5)$$

where $I_{ix}(\boldsymbol{X})$ and $I_{iy}(\boldsymbol{X})$ are the real and imaginary components of current $\dot{I}_i(\boldsymbol{X})$ in the i-th layer. $\dot{I}_i(\boldsymbol{X})$ as a function of the parameter vector $\boldsymbol{X}$ can be derived from [1]. The current distribution among layers should become more uniform when $f(\boldsymbol{X})$ is closer to a minimal value.

The associated constraints are:

1) Constraints of mechanical properties

Considering the mechanical properties of a tape, such as the tensile strain characteristic and the bending strain characteristic, the constraints of mechanical properties can be expressed as

$$\begin{cases}\beta_i-\arcsin\left(\dfrac{2R_i\varepsilon_{cb}}{t}\right)^{1/2}\le 0\\[2mm] \beta_i-\arcsin\left(\dfrac{\varepsilon_{ct}+\varepsilon_p-\varepsilon_{fc}}{\varepsilon_p-\varepsilon_r}\right)^{1/2}\ge 0\end{cases}\quad i=1,2,\cdots,n \quad (6)$$

where ε_{cb} and ε_{ct} are the critical bending and tensile strain of the tape at 77 K, ε_p and ε_{fc} the thermal shrinkages of the winding pitch and the tape, respectively, ε_r is the radial thermal shrinkage of the former, and t the thickness of the tape.

2) Constraints of radii

The constraints of radii can be expressed as

$$\begin{cases}R_1-\dfrac{D_{\min}}{2}+(t_f+\dfrac{t}{2})\ge 0\\[2mm] R_{i+1}-R_i-(t_f+t)\ge 0 \qquad i=1,2,\cdots,n-1\\[2mm] \dfrac{D_{\max}}{2}-R_i-(n-i)(t_f+t)-\dfrac{t}{2}\ge 0 \quad i=1,2,\cdots n\end{cases} \quad (7)$$

where D_{min} and D_{max} are used to limit the inner and outer diameters of the cable conductors, and t_f is the thickness of the dielectric between layers.

3) Constraints of critical current

These constraints are used to restrict the currents in layers below their critical currents, and can be expressed as

$$I_i<N_iI_ck_1k_2k_3k_4 \qquad i=1,2,\cdots,n \quad (8)$$

where N_i is the number of tapes wound on the i-th layer, I_c the mean of critical currents of HTS tapes in the cable, k_1, k_2 and k_3 are the deteriorations of the critical current considering the magnetic field and the temperature, manufacture, and the thermal cycles, respectively, and k_4 is the design safety margin.

V. Structural Parameter Optimization

Taking a cold dielectric type HTS cable with 4-layer conductors and 2-layer shields as an example, Table I tabulates the structural parameters of the cold dielectric type HTS cable before the structural optimization.

TABLE I. STRUCTURAL PARAMETERS OF COLD DIELECTRIC TYPE

Layer Index	1	2	3	4	5	6
α_i	+1	+1	-1	-1	+1	+1
β_i(°)	27.0	27.0	27.0	27.0	27.0	27.0
R_i (*mm*)	10.0	10.45	10.90	11.35	18.50	18.95

Note: α is the winding direction, β the winding angle, R the radius; Layers 1-4 are conductors and Layers 5 and 6 the shields.

The length of the cable for calculation is chosen to be 100 m. The AC voltage source u is 10 kV (rms), and the load R_L is 10 Ω. Fig. 3 plots the current distributions before the structural optimization. It is found that the currents in different layers before optimization differ greatly in both the amplitude and phase angle.

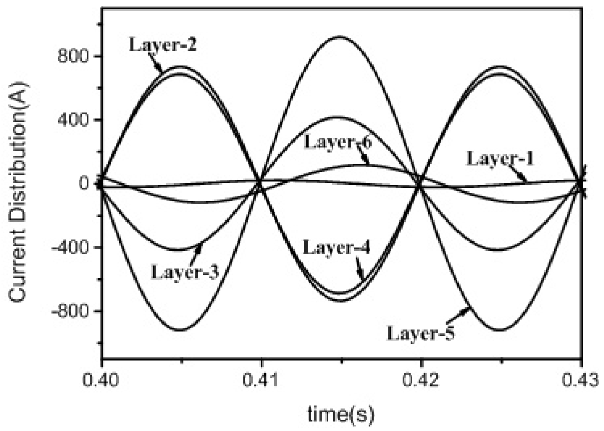


Figure 3. Current distribution in the cold dielectric type HTS cable before optimization

1) Comparision between PSO and DEPSO

PSO and DEPSO are performed under the same condition. A uniform current distribution, shown in Fig. 4, could be achieved with both two algorithms, and the comparison between PSO and DEPSO is listed in Table II, and also illustrated in Fig. 5. In Table II, F_{best} is the average of the best fitness function value of 50 evolutions, and T_{iter} is the average of the iterative times of 50 evolutions. It can be concluded that DEPSO could have more opportunities to find out the optimal result and could converge easily. In Fig. 5, F_{norm} is defined as the average of the best fitness function value of 50 evolutions in the same generation. The result shows that DEPSO provides a higher performance than PSO when the number of generations is greater than 1200.

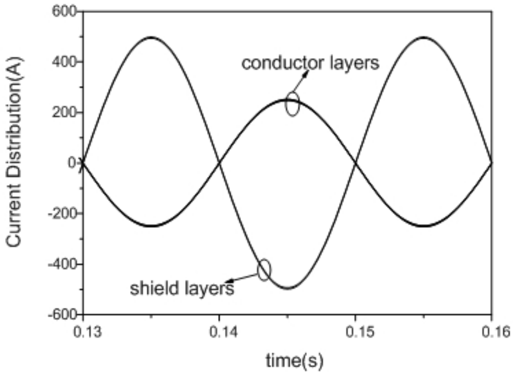


Figure 4. Current distribution in the cold dielectric type HTS cable after optimization

TABLE II. PERFORMANCE COMPARISION BETWEEN PSO AND DEPSO

Algorithm	F_{best}	T_{iter}
PSO	1039.07	4948.48
DEPSO	244.786	3189.14

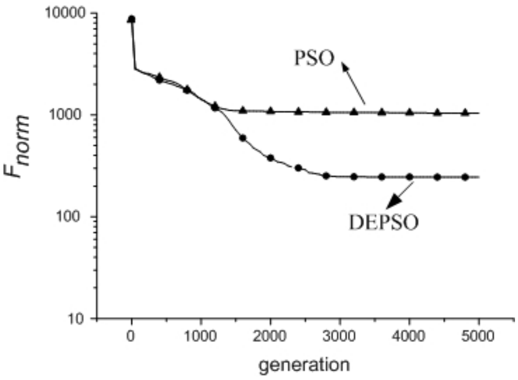


Figure 5. Performance of PSO and DEPSO

2) Comparision between DEPSO and DFSS

When the perturbation range of design variables is ±0.1%, the DEPSO algorithm and robust optimization with DFSS are performed. And the comparisons of the optimized structural parameters are shown in Table III.

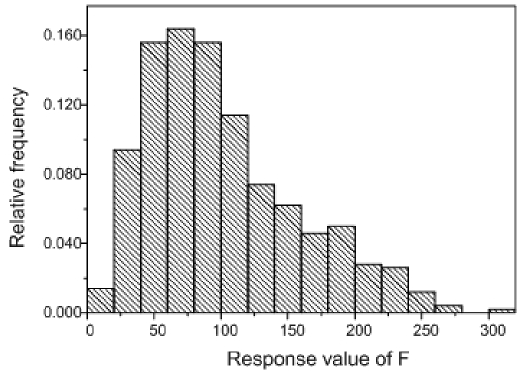
From Table III, it can be seen that the current distributions optimized by DEPSO and DFSS are almost the same. Fig. 6 demonstrates the probability distribution of fitness function *F* in different optimization results. It can be seen that in DEPSO the distribution covers a wider range, while in DFSS the probability distribution shrinks much thinner. By this way, the DFSS optimization increases the robustness of the design.

Table IV shows the quality improvement by using DFSS. With DEPSO algorithm, the mean value of the objective function μ_F=101.729, the standard deviation is σ_F=56.677 and the reliability is 63.8723%. By using the six sigma robust optimization, the mean value and the standard deviation of the objective function decreased to 54.52 and 24.67, respectively. The constraints have almost 0% probability in exceeding their limits.

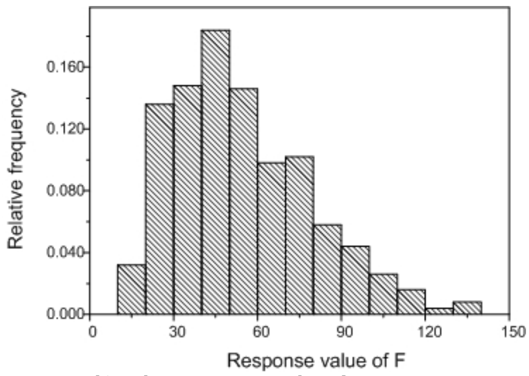
TABLE III. OPTIMIZED STRUCTURAL PARAMETERS OF HTS CABLE

Layer Index	DEPSO Algorithm			DFSS Optimization		
	a_i^{Δ}	β_i^{Δ}	R_i^{Δ} (mm)	a_i^{*}	β_i^{*}	R_i^{*} (mm)
1	-1	16.23	9.69	-1	24.15	9.00
2	-1	8.03	10.11	-1	8.00	9.91
3	1	8.92	10.64	1	10.37	10.69
4	1	30.64	10.99	1	30.00	11.55
5	-1	21.94	17.94	-1	22.81	18.05
6	-1	10.59	18.51	-1	14.41	19.21

Note: Δ represents PS0 algorithm, and * DFSS optimization; the definitions of *α*, *β* or *R* is similar to those in Table I.



a) DEPSO algorithm



b) Robust optimization based on DFSS

Figure 6. Histogram of DEPSO algorithm and robust design for six sigma

TABLE IV. QUALITY IMPROVEMENT FOR HTS CABLE

Optimization	μ_F	σ_F	Reliability
DEPSO	101.729	56.677	63.8723%
DFSS	54.5188	24.6709	~100%

The perturbation analysis is applied to evaluate the influence of the distorted structural parameters. The current relative error of the *i*-th layer is introduced to investigate the current distribution with perturbed parameters as follows

$$E_{cv,i} = \frac{\left| I_{ave} - I_{err,i} \right|}{I_{ave}} \times 100\% \quad (9)$$

where I_{ave} is the average value of the currents of the layers obtained through the optimized parameters, and $I_{err,i}$ the current of the *i*-th layer with the perturbed structural parameters.

The perturbations are performed on the winding angle and the radius in a certain range. The maximum value of E_{cv} in all cases is defined as E_{max}. The curves in Fig. 7 reveal that the robust stabilization of DFSS is higher than that of PSO.

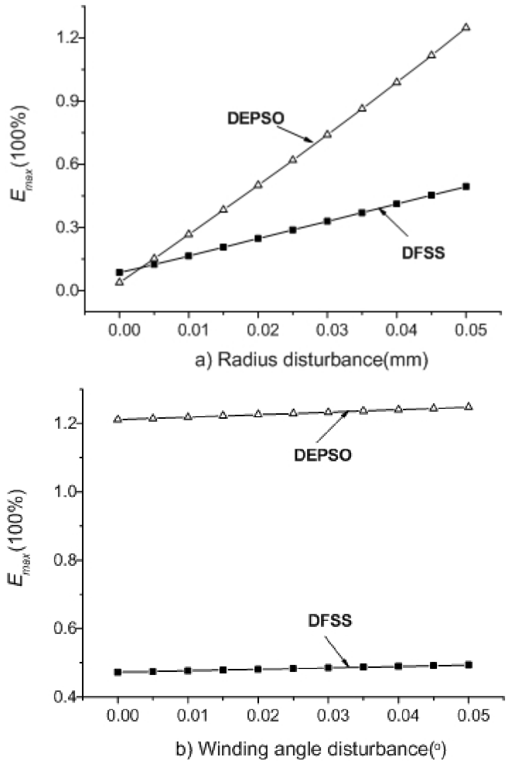


Figure 7. Influence of parameters perturbation on current distribution

VI. CONCLUSION

In this paper, an improved particle swarm optimization, which is hybrid traditional PSO with differential evolution operator, is utilized to overcome the premature of multi-model function search by the standard PSO, and it has proved that DEPSO provides a better performance than PSO. Considering the uncertainties in HTS cable structural design, a optimization algorithm based on design of six sigma is applied to perform a robust design. The comparison between DEPSO and robust optimization shows that the robust optimization using design of six sigma is superior to the DEPSO algorithm to achieve higher reliability and quality.

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