Bayesian Recursive Algorithm for Width Estimation of Freespace for a Power Wheelchair Using Stereoscopic Cameras

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Abstract - This paper is concerned with the estimation of freespace based on a Bayesian recursive (BR) algorithm for an autonomous wheelchair using stereoscopic cameras by severely disabled people. A stereo disparity map processed from both the left and right camera images is constructed to generate a 3D point map through a geometric projection algorithm. This is then converted to a 2D distance map for the purpose of freespace estimation. The width of freespace is estimated using a BR algorithm based on uncertainty information and control data. Given the probabilities of this width computed, a possible movement decision is then made for the mobile wheelchair. Experimental results obtained in an indoor environment show the effectiveness of this estimation algorithm.

I. INTRODUCTION

Power wheelchairs are necessary to provide mobility assistance for severely disabled people. However a conventional wheelchair in many cases is not sufficient to assist in their mobility. For this reason, certain wheelchairs are fitted with additional equipment, such as sensors, and given new, generally autonomous, capabilities such that they can be classified as smart wheelchairs, in order to enhance the independence and mobility of severely disabled people. In recent decades many significant algorithms, which utilise the additional equipment, have been developed to create these smart wheelchairs [1].

Cameras are one type of sensory equipment that can be mounted on robots and wheelchairs to obtain real-time information from the environment. This information is analysed and processed by advanced algorithms to produce necessary data and signals for the operation of the associated robots and smart wheelchairs [2, 3]. In particular, the left and right images acquired from stereoscopic cameras are matched to generate a disparity map using a Sum of Absolute Differences (SAD) algorithm [4]. This disparity map is used to produce a 3D point map, which is then applied a geometric projection algorithm to convert it to a 2D distance map [5-7]. These 2D distance maps can allow detection of obstacles and freespace distances necessary in the operation of robots and smart wheelchairs.

The Bayesian method is useful in detecting freespace from sensor information for robots. For example, in a recent application, an autonomous exploration strategy for freespace detection based on mapping uses the Bayesian update theory [8]. A Bayesian filter algorithm with statistical tool is also applied to estimate locations based on uncertainty information from sensors. This Bayesian filter algorithm uses the Markov assumption and recursive process to compute the belief probability [9].

The BR algorithm is applied to estimate the width of freespace in a 2D distance map using measurements and control data from the wheelchair system. Firstly the previous probability is computed based on the prior probability and the control data. The second step of this algorithm is that the posterior probability using Bayesian rules is carried out based on the previous probability along with the measurement update. The process is recursively iterated in the next time step to choose the high belief probability. The average of the probabilities is to make a decision for the mobile wheelchair.

This paper is organised as follows. In Section 2, the problem of stereoscopic vision is introduced. In Section 3, a BR algorithm using Bayesian rules, control data and measured information is employed to estimate the width of freespace. Section 4 shows the experimental result, in which the width of freespace in a 2D distance map is estimated to make the decision. Finally Section 5 concludes the paper.

II. STEREOVISCION

A. Stereo Disparity

The purpose of stereo vision is to perform range measurements based on the left and right images obtained from stereoscopic cameras. Basically, a correlation algorithm of the Sum of Absolution Differences (SAD) is implemented to establish the correspondence between image features in different views of the scene and then calculate the relative displacement between feature coordinates in each image to produce a disparity [5].

$$\text{SAD} = \min_{d_{\text{min}} d_{\text{max}}} \sum_{i=-M}^{M} \sum_{j=-M}^{M} |I_L(i,j,y-f) - I_R(i+d,j,y+f)| \quad (1)$$

where the centre of a window of size \((2M+1)x(2M+1)\) at the coordinates of the matching pixels \((i,j), (x,y)\), are pixel coordinates in one image. \(I_L\) and \(I_R\) are the intensity functions of the right and left images, and \(d_{\text{min}}, d_{\text{max}}\) are the minimum and maximum disparities. The disparity \(d_{\text{min}}\) of zero pixels often corresponds to an infinitely far away object and \(d_{\text{max}}\) denotes the closest position of an object.

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B. Computation of a 2D Distance Map

In this project, a Point Grey Research ‘Bumblebee 2’ stereoscopic camera system used to capture the left and right images as shown in Fig. 1 and Fig. 2 has two cameras which act like two “eyes”. This camera system has a limited field of view, with the total range denoted here as α. This vision system produces a α1 maximum angle to the left and a α2 maximum angle to the right of the wheelchair due to the angle of the cameras system relative to the wheelchair. It means that the camera system can just see freespace $A_i$, $w_2$ and obstacles $o_1$, $o_2$ and $o_3$ in the field of view (AOB) as shown in Fig. 3, in which Z is distance-axis, X is horizontal-axis and $OH=Z_{max}$ is the maximum visible distance of the cameras. The maximum range of Z-axis is $5000mm$, the 0.01 resolution of X-axis is from $-1800mm$ to $1700mm$ and the image resolution in this case is 320x640 in the 2D map.

Given the camera calibration, a stereo disparity map is processed to generate a 3D point cloud image based on sub-pixel interpolation for more accurate stereo depth extraction. From this 2D point image, four steps can be performed to convert this to a 2D distance map using geometric projection as follows:

- Step 1: $n$ 3D points $(x_i, y_i, z_i)$ are converted to $n$ 3D points $(X_i, Y_i, Z_i)$ on coordinates $X$, $Y$, $Z$.
- Step 2: Assume that a certain value $X_i$ on the horizontal X-axis corresponds to many points $(Y_i, Z_i)$. The minimum distance $Z_{min}$ from an object to the wheelchair corresponding to a point $X_i$ is chosen.
- Step 3: In order to obtain a 2D distance map $(X, Z_{min})$, all minimum distances $Z_{min}$ are computed as follows

$$Z_{min} = \min_{j \in k} Z_{ij}$$

- Step 4: The width of freespace $w$ is detected based on the maximum distance $Z_{max}$ as follows

$$w = \sum_{Z_{i=1}}^{Z_{max}}$$

III. FREESPACE ESTIMATION ALGORITHM

A. Bayesian Recursive Algorithm

The width of freespace in a 2D distance map is difficult to estimate due to noise information from the camera system. In order to solve this problem, Bayesian Recursive (BR) algorithm bases on measurements and control data of the wheelchair system to compute the conditional probability. The first step is that the algorithm requires the probability and the control $u(t)$ to compute the probability over the width of freespace $w(t)$ which is a state variable. The prior probability is expressed as follows

$$P_{po}(w(t)) = \sum_{w(t-1)} P(w(t-1) | u(t-1), P(w(t-1)))$$

where $P_{po}(w(t))$ is the posterior probability based on measurements $z_t$ and $P_{th}(w(t))$ is the previous probability and the conditional probability over the width $w(t-1)$.

The next step is that, the probability $P_{po}(w(t))$ is used along with the conditional probability based on measurements $z_t$ to make the posterior probability as follows

$$P_{po}(w(t)) = \sum_{w(t)} P(z(t) | w(t)) P_{po}(w(t))$$

B. Decision Making

This algorithm is recursively iterated many times to produce the probabilities. The average of the probabilities is computed to make a decision for the wheelchair as follows

$$P_{th}(w) = \sum_{i=1}^{n} P_{po,i}(w(t))$$

where $n$ is the amount of times.

The average probability is then compared to the threshold probability $P_{th}$ that is carried out many times in the practical operation of the wheelchair. If $P_{th}(w)$ is greater than or equal to $P_{th}$, the mobile wheelchair can be decided to move.

IV. EXPERIMENTAL RESULTS

The autonomous wheelchair has been fitted with the ‘Bumblebee 2’ camera system in the University Research Centre for Health Technologies as shown in Fig. 4.

Consider an actual width of a freespace $w_d$ of an entry through a doorway or a corridor which is compared to the safe diameter $d_s$ of a wheelchair. If $w$ measured by the camera system is greater than or equal to $d_s$, the wheelchair...
can move through this freespace. If \( w \) is less than \( d_w \), the wheelchair cannot move through. In practice, \( w \) measured by the camera system can be less than, equal to or greater than \( w_w \). This is very difficult to estimate. To solve this problem, BR algorithm is employed to estimate the width of freespace based on the conditional probabilities, control data and measurements. Two practical experiments implemented to estimate the width of freespace are shown below.

### A. Experiment 1

![Fig. 5: Left image](image1)

![Fig. 6: Right image](image2)

![Fig. 7: Stereo disparity map](image3)

![Fig. 8: 3D point map](image4)

![Fig. 9: 2D distance map](image5)

Fig. 5 and 6 show the left and right images with one freespace \( w \). From these left and right images, the disparity and 3D point maps are constructed as shown in Fig. 7 and Fig. 8. A 2D map with a freespace \( w \) is shown in Fig. 9.

BR algorithm is utilised to estimate the width \( w \) of this freespace which is assigned as follows: if its measured width \( w \) is greater than or equal to the diameter \( d_w \) of the wheelchair, \( w \) is the freespace, called ‘\( w=FS \)’ and the wheelchair can go through. If \( w \) is less than \( d_w \), \( w \) is the obstacle, called ‘\( w=OB \)’, the wheelchair can’t go through.

Assume that the actual width of freespace \( w_w \) is equal to the safe diameter \( d_w=1000\text{mm} \) of the wheelchair. However the measured width \( w \) of the freespace estimated by the camera system can be noisy. For this reason, BR algorithm is used to estimate this measured width \( w \).

Initially, the equal prior probabilities are given as follows

\[
P(w(0)=FS) = 0.5 \quad \quad (7a)
\]

\[
P(w(0)=OB) = 0.5 \quad \quad (7b)
\]

Furthermore, the conditional probabilities corresponding to noise measurements and the width of freespace \( w(t) \) at time \( t \) are carried out in fact by trial times

\[
p(z(t) = FS \mid w(t) = FS) = 0.7 \quad \quad (8a)
\]

\[
p(z(t) = OB \mid w(t) = FS) = 0.3 \quad \quad (8b)
\]

\[
p(z(t) = FS \mid w(t) = OB) = 0.2 \quad \quad (9a)
\]

\[
p(z(t) = OB \mid w(t) = OB) = 0.8 \quad \quad (9b)
\]

In which \( z(t) \) is the measurement and \( w(t) \) is the measured width.

The conditional probabilities depend on the wheelchair control \( u(t) \) and its previous state as follows

\[
p(w(t) = FS \mid w(t-1) = FS, u(t) = move) = 0.8 \quad \quad (10a)
\]

\[
p(w(t) = OB \mid w(t-1) = FS, u(t) = move) = 0.2 \quad \quad (10b)
\]

\[
p(w(t) = FS \mid w(t-1) = OB, u(t) = move) = 0.9 \quad \quad (11a)
\]

\[
p(w(t) = OB \mid w(t-1) = OB, u(t) = move) = 0.1 \quad \quad (11b)
\]

For similarity, the control case is \( u(t) = stop \), the conditional probabilities are determined as follows

\[
p(w(t) = FS \mid w(t-1) = FS, u(t) = stop) = 0.1 \quad \quad (12a)
\]

\[
p(w(t) = OB \mid w(t-1) = FS, u(t) = stop) = 0.9 \quad \quad (12b)
\]

\[
p(w(t) = FS \mid w(t-1) = OB, u(t) = stop) = 0.1 \quad \quad (13a)
\]

\[
p(w(t) = OB \mid w(t-1) = OB, u(t) = stop) = 0.9 \quad \quad (13b)
\]

In practice, relatively the error probabilities can occur as shown in Eqs. (10b), (11b), (12a) and (13a).

Firstly, the wheelchair takes on control action \( u(t) = move \) and the measurement \( z(t) = FS \) at time \( t_1 \). From Eq. (4), the probabilities \( P_{po}(w(1)) \) using the prior probability are computed for the width \( w(t) \) as follows

\[
P_{po}(w(1) = FS) = \sum_{w(0)} P(w(1) \mid w(0), u(1))P(w(0)) = 0.5 \quad \quad (14a)
\]

\[
P_{po}(w(1) = OB) = \sum_{w(0)} P(w(1) \mid w(0), u(1))P(w(0)) = 0.5 \quad \quad (14b)
\]

Given the measurement \( z(t) = FS \) and \( P_{po}(w(1)) \), the posterior probabilities \( P_{po}(w(1)) \) using Eq. (5) are determined as follows

\[
P_{po}(w(1) = FS) = \sum_{w(1)} P(z(1) = FS \mid w(1))P_{po}(w(1)) = 0.778 \quad \quad (15a)
\]

\[
P_{po}(w(1) = OB) = \sum_{w(1)} P(z(1) = FS \mid w(1))P_{po}(w(1)) = 0.222 \quad \quad (15b)
\]

This BR algorithm is iterated for the next time step corresponding to \( u(t) = move \) at time \( t_2 \). We obtain the probabilities using Eq. (4) as follows

\[
P_{po}(w(2) = FS) = 0.8778 \quad \quad (16a)
\]

\[
P_{po}(w(2) = OB) = 0.1222 \quad \quad (16b)
\]

As the measurement is still \( z(t) = FS \), we have the posterior probabilities using Eq. (6) as follows

\[
P_{po}(w(2) = FS) = 0.9617 \quad \quad (17a)
\]

\[
P_{po}(w(2) = OB) = 0.0383 \quad \quad (17b)
\]

This process is recursively iterated ten times to produce the probabilities \( P_{po}(w=FS), P_{po}(w=OB) \) and the average probabilities, \( P_{av}(w=FS), P_{av}(w=OB) \) as shown in Table 1. If \( P_{po}(w) \geq d_w \), there is a freespace and if \( P_{po}(w) < d_w \), there is an obstacle. In particular, the average probability \( P_{av}(w=FS) = 0.8821 \) is greater than \( P_{av}=0.8 \), it means that the
width \( w \) is the freespace and the wheelchair is possibly decided to go through.

Table 1: The estimated values of the width of one freespace

<table>
<thead>
<tr>
<th>Times</th>
<th>( w(t) )</th>
<th>( P_{av}(w(t)=FS) )</th>
<th>( P_{av}(w(t)=OB) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1020</td>
<td>0.7778</td>
<td>0.2222</td>
</tr>
<tr>
<td>2</td>
<td>1020</td>
<td>0.9617</td>
<td>0.0383</td>
</tr>
<tr>
<td>3</td>
<td>980</td>
<td>0.7640</td>
<td>0.2360</td>
</tr>
<tr>
<td>4</td>
<td>1020</td>
<td>0.9613</td>
<td>0.0387</td>
</tr>
<tr>
<td>5</td>
<td>1020</td>
<td>0.9679</td>
<td>0.0321</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>0.9682</td>
<td>0.0318</td>
</tr>
<tr>
<td>7</td>
<td>970</td>
<td>0.7652</td>
<td>0.2348</td>
</tr>
<tr>
<td>8</td>
<td>990</td>
<td>0.7269</td>
<td>0.2731</td>
</tr>
<tr>
<td>9</td>
<td>1010</td>
<td>0.9600</td>
<td>0.0400</td>
</tr>
<tr>
<td>10</td>
<td>1020</td>
<td>0.9617</td>
<td>0.0383</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{Average probability } P_{av} ) 0.8821</td>
<td>( \text{Average probability } P_{av} ) 0.1179</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{Variance } ) 0.0115</td>
<td>( \text{Variance } ) 0.0115</td>
</tr>
</tbody>
</table>

B. Experiment 2

Assume that the actual widths of two freespaces \( w_{1d} \) and \( w_{2d} \) are equal to the safe diameter of the wheelchair \( d_s \). This is difficult for the wheelchair to estimate and choose one of them to go through. For this reason, BR algorithm is used to find an optimal freespace and make the possible decision for the mobile wheelchair.

Consider the measured widths of two freespaces \( w_1 \) and \( w_2 \) as shown in Fig. 14. For similarity as Experiment 1, the high belief probabilities \( P_{av}(w_1) \), \( P_{av}(w_2) \) and their average probability are shown in Table 2.

In this Table 2, the average probability \( P_{av}(w_j) \), which is greater than not only the average probability \( P_{av}(w_j) \), but also the threshold probability \( P_{th} \) results in this freespace \( w_j \) being selected for the wheelchair to move through.

Table 2: The estimated values of the widths of two freespaces

<table>
<thead>
<tr>
<th>Times</th>
<th>( w_1(t) )</th>
<th>( P_{av}(w_1(t)=FS) )</th>
<th>( w_2(t) )</th>
<th>( P_{av}(w_2(t)=FS) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1010</td>
<td>0.7778</td>
<td>1020</td>
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</tr>
<tr>
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<td>0.9617</td>
<td>1010</td>
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</tr>
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<td>4</td>
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<td>0.7652</td>
<td>1000</td>
<td>0.9682</td>
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<tr>
<td>5</td>
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<td>0.9613</td>
<td>1010</td>
<td>0.9682</td>
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<td>1010</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( \text{Average probability } P_{av} ) 0.9065</td>
<td>( \text{Average probability } P_{av} ) 0.9032</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>( \text{Variance } ) 0.0090</td>
<td>( \text{Variance } ) 0.0104</td>
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</table>

V. CONCLUSION

In this paper, stereoscopic vision, freespace detection in a 2D distance map and a BR algorithm are presented. The stereo disparity map is based on a combination of the left and right stereoscopic camera system images. From this disparity map, a 3D point map is computed based on a geometric projection approach. In addition, a 2D distance map is generated from this 3D map. Computation is performed on the 2D distance map to identify the presence of freespaces. For the freespace detection implementation for an autonomous wheelchair, a BR algorithm is applied to estimate the width of freespace. Experiment results on the wheelchair in a practical environment have illustrated the effectiveness of this estimation method.

REFERENCES