Robust Multivariable Strategy and its application to a Powered Wheelchair

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***Abstract*—The paper proposes a systematic robust multivariable control strategy based on combination of systematic triangularization technique and robust control strategies. Two design stages are required. In the first design stage, multivariable control problem is reduced into a series of scalar control problems via triangularization technique. For each specific scalar system, two advanced control strategies are proposed and implemented in the second design stage. The first one is based on Model Predictive Control, which is an iterative, finite horizon optimization procedure. The second control strategy is known as Neuro-Sliding Mode Control, which integrates Sliding Mode Control (*SMC*) and Neural Network Design to achieve both chattering-free and system robustness. Real-time implementation on a powered wheelchair system confirms that robustness and desired performance of a multivariable system under model uncertainties and unknown external disturbances can indeed be achieved by the combination of triangularization technique and Neuro-Sliding Mode Control.**

I. Introduction

As mobility aid, powered wheelchairs are frequently used to provide people with impairment mobility greater independence to access school, work and community environments. Safety control of conventional powered wheelchair, however, requires a significant level of skill, attention, judgment and appropriate behavior. The survey in [1] shows that nearly half of 200 participants were found unable to control a powered wheelchair. Furthermore, around 85000 serious wheelchair accidents are occurred, and the trend is expected to increase [2].

In order to accommodate powered wheelchair users with comfort and safety, a lot of progress has been conducted in the last decades. However, majority of works focuses on navigation strategies on the supervisory control level. Since dynamics of a powered wheelchair varies considerably due to environment uncertainties and external disturbances, the robustness of the overall system depends heavily on low level controller performance. Surprisingly, little research has been specifically devoted to low control level.

In term of low level control design, a powered wheelchair can be regarded as a multivariable system with uncertainties and external disturbance [3]. There have been various multivariable control techniques, but decoupling control techniques provide the very effective solution to multivariable problem by reducing it to series of scalar problems. However, its researches on robustness under system uncertainties and external disturbances have been still spare.

This paper aims at extending decoupling technique known as triangularization technique [4] introduced by Hung, an author of this paper, to provide a systematic robust multivariable control strategy for a class of multivariable system. First, a multivariable system is reduced to series of independent scalar systems by the triangilarization technique. Then a robust controller is designed for an independent scalar system. In the control design phase, two control schemes are proposed and compared. The first control scheme is Model Predictive Control (*MPC*), which is known as advanced control methodology and has been applied successfully in different application areas. The second control scheme is Neuro-Sliding Mode Control (*NSMC*), which integrates Sliding Mode Control theory and Neural Network Design to provide system robustness while eliminating chattering phenomenon and avoiding the calculation of the plant Jacobian. The effectiveness of the proposed strategy is proven via its application to a powered wheelchair system.

The paper is organized as follows. In section II, Robust Multivariable Strategy is presented in detail. Its application to a powered wheelchair is described in Section III. Real-time experimental results and discussions are shown in Section IV. Conclusion is given in the section V.

II. Robust Multivariable Strategy

Two design stages are required in this strategy. First, nominal model of multivariable system is used to construct a pre-compensator so that the resulting system matrix is Triangular-Diagonal-Dominance (*TDD)*, implying that multivariable control problem is reduced to series scalar control problems. Then two control strategies known as *MPC* and *NSCM* are proposed in the second design stage.

2.1 Design stage 1: Triangularization with TDD property

Consider a multivariable system which is given as proper square $nxn\ G_0(s)+\Delta G(s)$. In order to use decoupling technique, nominal model of the plant ($G_o(s)$) is used in this stage. Two steps are required to construct a desired compensator.

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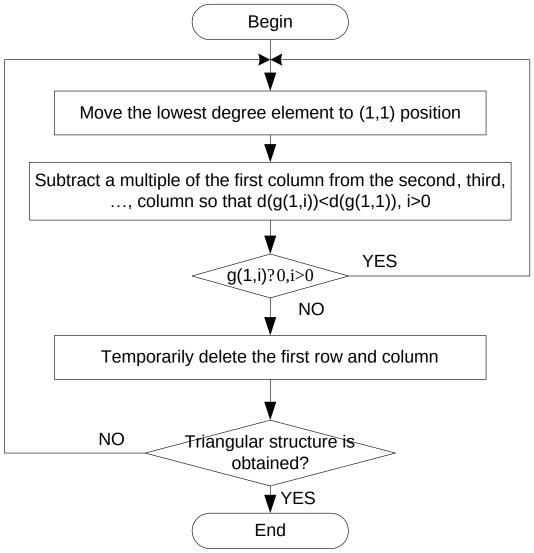


Figure 2.1: TDD compensator construction procedure

Step1: Construct a uni-modular pre-compensator matrix *D(s)* over the principal ideal domain so that resulting transfer function matrix *T(s) = G_0(s).D(s)* is triangular. As pointed out in [4], if $G_o(s)$ is stable D(is) can always be constructed. *D(s)* can be constructed in Figure 2.1.

Step2: Check *TDD* property of *T(s)*by using *Lemma 6* in [4]. If *T(s)* is *TDD*, its diagonal elements suffice to determine the stability properties of the system. In another word, this multivariable control problem is reduced to a series of scalar control problem via triangularization technique.

2.2 Design stage 2: Control design

Assume that after decoupling a diagonal element of multivariable is in the controllable form as follows:

$$\begin{cases} \dot{x} = (A+\Delta A)x + (B+\Delta B)u + \Theta d(t) \\ y = Cx \end{cases} \tag{2.1}$$

where $x \in R^n$, $u \in R$, $A \in R^{nxn}$, $B \in R^{nx1}$; $\Delta A \in R^{nxn}$ and $\Delta B \in R^{1xn}$ present bounded uncertainties; $d(t)$ is external disturbance and $\Theta \in R^{nx1}$.

A. Model predictive control design

Model predictive control predicts and optimizes the future behavior of the progress based on a dynamic model of the process. At each control interval, *MPC* algorithm calculates an open loop sequence of manipulated variables in such a way to optimize the future of the plant.

Figure 2.2 presents *MPC* algorithm operating in two phases, prediction and optimization, to compute *m* moves $u_k, u_{k+1}, \ldots, u_{k+m-1}$ based on values of set points, measured disturbances and constraints specified over a finite of future sampling instants. The moves are solution of a constrained optimization problem:

$$\min_{\Delta u_k,\ldots,u_{k+m-1}} \left(\sum_{i=1}^{p} \left\| \hat{y}_{k+i} - r_{k+i} \right\|^2 W_y + \sum_{i=1}^{m} \left\| \Delta u_{k+i-1} \right\|^2 W_u \right) \tag{2.2}$$

where: $\Delta u_k = u_k - u_{k-1}$; $W_y \ge 0, W_u \ge 0$

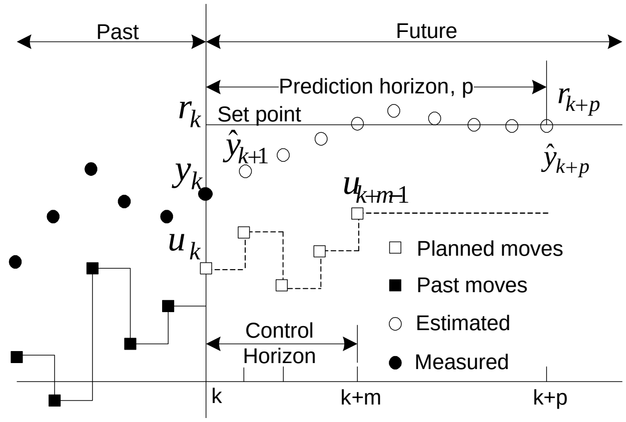


Figure 2.2: Model predictive control algorithm description

For details of formulations, see [5]

B. Neuro-sliding mode control design

The sliding surface is defined as:

$$s = h^T (x_d - x) \tag{2.3}$$

According to sliding mode theorem presented in [6], the control input is obtained as:

$$u(t) = u_{eq}(t) + u_c(t) \tag{2.4}$$

where $u_{eq}(t)$ is the equivalent control, $u_c(t)$ is the corrective control given in [6] as:

$$u_{eq}(t) = \left[h^T (B+\Delta B)\right]^{-1} \left(h^T \left[\dot{x}_d - \Theta d(t)\right] - \left[h^T (A+\Delta A)\right] x\right) \tag{2.5}$$

$$u_c(t) = \left[h^T (B+\Delta B)\right]^{-1} \cdot \delta \cdot g(s) = K \cdot g(s) \tag{2.6}$$

Since $u_c(t)$ and $u_{eq}(t)$ can not be directly calculated. Thus, these control signals are estimated by two neural networks: *CNN* and *ENN* [6].

The cost functions for two neural networks are as follows

$$E = \frac{1}{2}\left(u_{eq} - \hat{u}_{eq}\right)^2; J = \frac{1}{2} s^2 \qquad (2.6)$$

The weight adaptation laws for the *ENN* and *CNN* aimed at minimizing *E* and *J* are in following equations:

$$\Delta W_j = -\eta \frac{\partial E}{\partial W_j} = \eta \cdot s \cdot K_u \frac{1}{2}\left(1 - g(Unet)^2\right) Yout_j \tag{2.7}$$

$$\Delta \overline{W}_{i,j} = -\eta \frac{\partial E}{\partial \overline{W}_{i,j}} = \frac{1}{4} \eta \cdot s \cdot K_u \left(1 - g(Unet)^2\right) W_j \left(1 - g(Ynet_j)^2\right) Z_i \tag{2.8}$$

$$\Delta h_i = -\mu \frac{\partial J}{\partial h_i} = -\mu \cdot s \cdot e_i \tag{2.9}$$

Since h is adapted online, ΔB is assumed to be bounded so that K in (2.4) can be approximated as:

$$K = \delta \left[h^T B\right]^{-1} \tag{2.10}$$

Two training schemes are required. First offline training scheme aims at finding the nominal weights of two neural controllers so that a desired system performance is attained. Two trained neural networks are then used in online training scheme to reject uncertainties and external disturbances.

For details of *NSMC* algorithms, see [6].

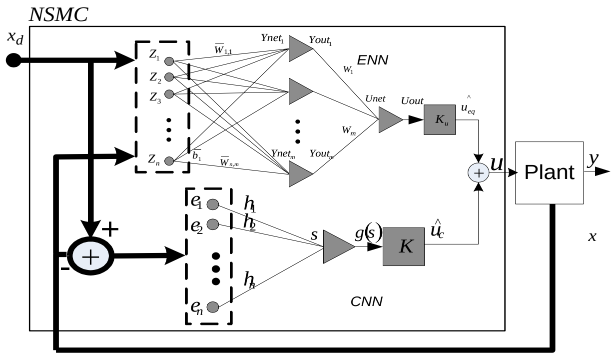


Figure 2.3: The NSCM control structure

III. APPLICATION TO A POWERED WHEELCHAIR SYSEM

In [3], the powered wheelchair model is obtained by experimental data method. This dynamics varies from lower-bounded transfer function matrix $G_1(s)$ to upper-bounded transfer function matrix $G_2(s)$. The nominal model of wheelchair $G_o(s)$ is in simplified form as:

$$G_o(s)=\begin{bmatrix}\frac{1.4}{(1+0.8s)(1+0.225s)} & \frac{0.125}{(1+0.4s)(1+0.15s)}\\ \frac{0.1}{(1+0.35s)(1+0.2s)} & \frac{1.8}{(1+0.5s)(1+0.2s)}\end{bmatrix} \quad (3.1)$$

$$G_1(s)=\begin{bmatrix}\frac{1.8}{(1+0.55s)(1+0.1s)} & \frac{0.22}{(1+0.35s)(1+0.1s)}\\ \frac{0.25}{(1+0.3s)(1+0.1s)} & \frac{2.6}{(1+0.54s)(1+0.1s)}\end{bmatrix} \quad (3.2)$$

$$G_2(s)=\begin{bmatrix}\frac{1}{(1+1.05s)(1+0.35s)} & \frac{0.1}{(1+0.45s)(1+0.2s)}\\ \frac{0.1}{(1+0.4s)(1+0.3s)} & \frac{1}{(1+0.55s)(1+0.3s)}\end{bmatrix} \quad (3.3)$$

Design stage 1: Decoupling design

The triangularization technique is used to construct the desired decoupler $D(s)$. Detail procedure can be seen in [3]. Obtained $D(s)$ is in following form:

$$D(s)=\begin{bmatrix}1 & -\frac{0.125}{1.4}\frac{(0.8s+1)(0.225s+1)}{(0.4s+1)(0.15s+1)}\\ 0 & 1\end{bmatrix} \quad (3.4)$$

Thus, the decoupled transfer function matrix

$$P_0(s)=G_0(s)D(s)=\begin{bmatrix}\frac{280}{(5+4s)(40+9s)} & 0\\ \frac{10}{(20+7s)(5+s)} & \frac{123.32(s+2.89)}{(20+7s)(5+s)(2+s)}\end{bmatrix} \quad (3.5)$$

Since $P_o(s)$ is stable and proper, it has *TDD* property

Design stage 2: Control design

After being decoupled, the wheelchair is decomposed into two scalar systems, linear velocity loop and angular velocity loop.

A. Model Predictive Control Design

Two model predictive controllers, *vMPC and wMPC*, are required for two sub-systems. The control structure is presented in the Figure 3.1. Both *MPCs* are turned so that the cost function defined in *(2.2)* is minimized. The inputs' constraint of these optimizations is due to the saturation of motor input voltages within *[-1;1]*.

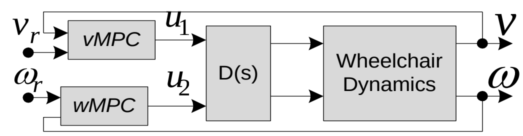


Figure 3.1: Multivariable Model Predictive Control Structure

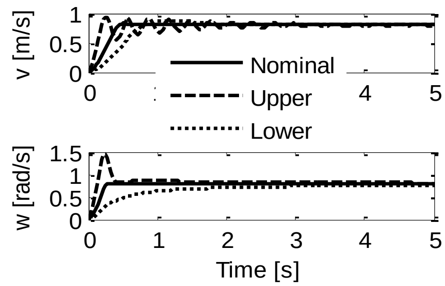


Figure 3.2: Step responses of two velocity loops obtained by MMPC

The outputs constraint of *v* is within *[-1.4;1.4]*, while that of ω is within *[-2.6;2.6]*. The chosen weighting matrix is $W_y=1$; $W_u=0.1$. After extensive simulation and experiment, predictive horizon and control horizon is chosen as *p*=7; *m*=2 for both *vMPC* and *wMPC*. The Figure 3.2 shows the step responses of two subsystems with variations.

B. Neuro-Sliding Mode Control Design

Figure 3.3 shows the control structure which requires two *NSMCs* named as *NSMC1* and *NSMC2* respectively. By trial and error, optimal structure of the *ENN* and *CNN* of the *NSMC1* are *(4,3,1)* and *(2,1,1)* while that of the NSMC2 are *(6,3,1)* and *(3,1,1)*. Off-line training algorithm is first introduced to find the nominal weights, which can provide optimal performance of whole system.

*NSMC1'*s parameters: $\eta=0.8; \mu=0.05; \delta=0.03; K_u=1$

*NSMC2'*s parameters: $\eta=0.65; \mu=0.03; \delta=0.02; K_u=1$

The integral gain used in online training scheme is chosen as K_{I1}= 0.015 for *NSMC1* and K_{I2} =0.02 for *NSMC2*. The Figure 3.4 shows the system outputs of two subsystems.

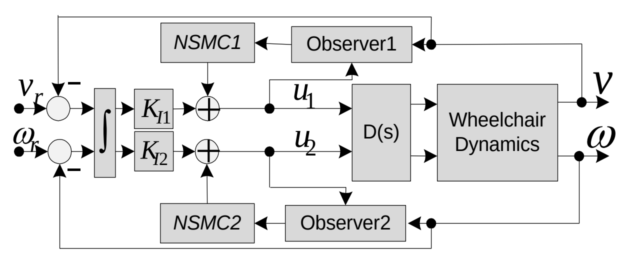


Figure 3.3: Multivariable Neuro-Sliding Mode Control Structure

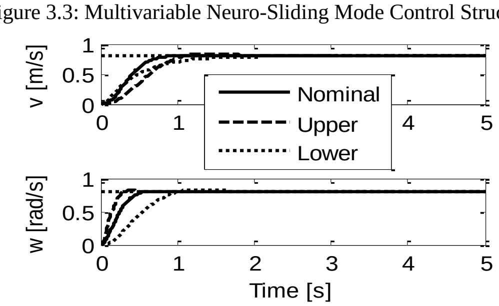


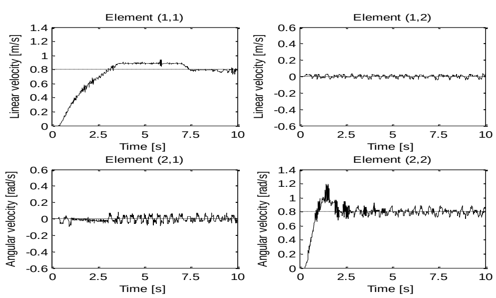
Figure 3.4: Step responses of two velocity loops obtained by MNSMC

IV. REALTIME EXPERIMENTAL RESULTS AND DISCUSSIONS

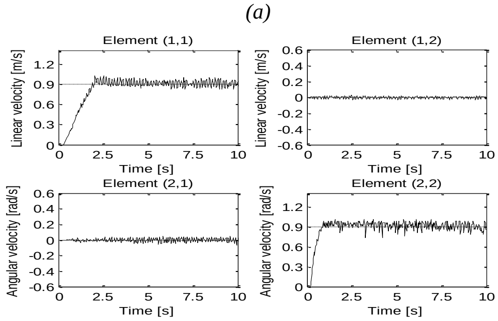
The algorithms described in the previous sections are implemented in *ANSI C LabWindow CVI 8.5* with 20 (*ms*) sampling time. Two real-time experiments are carried out to verify the design. The results obtained by Multivariable Neuro-Sliding Mode Control (*MNSMC*) are compared to that obtained by Multivariable Model Predictive Control (*MMPC*).

Experiment 1: This experiment tests the effectiveness of the proposed strategy. By exciting input signal of one subsystem while keeping other subsystem input at zero, Figure 4.1 shows the system outputs obtained by *MMPC* and that obtained by *MNSMC* method. Clearly, compared to *MMPC* method elaborate performance is obtained by *MNSMC*. Moreover, interactions between two subsystems are eliminated by pre-compensator *D(s).*

Experiment 2: This real-time experiment tests the robustness of the controlled system under system uncertainties and external disturbances in two sub-tasks: square tracking and line tracing. Each subtask experiment is conducted in different conditions.

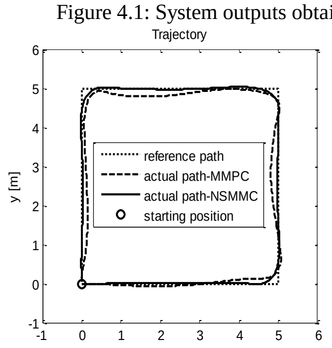


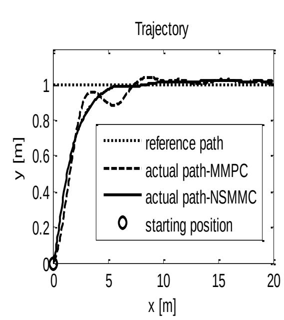
(a)



(b)

Figure 4.1: System outputs obtained: *MMPC* (a), *MNSMC (b)*





(a) *(b)*

Figure 4.2: Path-following control: square tracking (a), line tracking (b)

In line tracing task a person weighted 46 kg seats on the wheelchair and it runs on wooden surface while in square tracking task 70 kg person seats on the wheelchair and it runs on cement surface. During experiment, a person seated on the wheelchair tries to move oneself in order to change the centre of gravity of the system. The results in this experiment in Figure 4.2 confirm that system performance is still guaranteed regardless different subtasks are conducted in different conditions, and better results are obtained by MNSMC compared to *MMPC* method. This is because two *NSMCs* are trained in the offline training scheme to attain optimal performance and are trained online to adapt to the change of external conditions so that they can reject any unwanted uncertainties and external disturbances.

V. CONCLUSION

In this paper, we have proposed a robust systematic multivariable strategy for a class of multivariable systems. Two design stages are required in this approach. First, the multivariable system is decoupled into series of scalar subsystems by using a pre-compensator. Then two advanced control strategies are presented and compared in the control design stage for each scalar system. One of these control strategies is *MPC*, which is successfully applied in linear systems. Other strategy is *NSMC*, which can provide system robustness by its online adaptation ability. Real-time experiments on a powered wheelchair system are conducted to compare system performance attained by two control strategies. The results show that it is possible to combine decoupling technique and advanced control strategy to provide robust strategy for a multivariable control problem. It also confirms that optimal performance and robustness of a class of multivariable systems under system uncertainties and external disturbances can indeed be achieved by the combination of triangularization technique and *NSMC* strategy, known as *MNSMC*.

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