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MODELING OF A GAS CONCENTRATION MEASUREMENT SYSTEM

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ABSTRACT- Energy expenditure can be calculated via measurement of oxygen consumption and carbon dioxide production. Precise measurement of expired gas concentrations and volume is required for this determination. For a given gas concentration measurement system, the establishment of a model is a good way to effectively use the equipments and achieve more accurate energy expenditure calculations. This paper proposes a simple but effective approach for the modeling of a gas concentration measurement system.

I. INTRODUCTION

Indirect calorimetry is the calculation of energy expenditure via measurement of oxygen consumption ($\dot{V}O_2$) and carbon dioxide production ($\dot{V}CO_2$) [1][2]. However, the complexity and expense of indirect calorimeters made the implementation of this idea technically challenging [3][4]. The most difficult part perhaps is the dynamic measurement of the concentrations of O_2 and CO_2 with the desired level of precision.

The majority of commercially available indirect calorimeters use mixing chambers [2] for the collection of expired gases. Mixing chambers use baffles to interrupt gas flow and thus prevent streaming of gases and uneven gas concentrations. This is a necessary step to ensure desired calculation accuracy of energy expenditure. However, gas samples collected from a mixing chamber are averaged gas fractions over time. The sensitivity to changes in metabolism is therefore reduced. Patents for improved mixing chamber have been reported. However, for the user of commercial indirect calorimeters, an acceptable strategy is to set up a model for the system for the individual user's setting and environment. This may lead to more efficient and accurate use of these systems.

This paper investigates the modeling of the gas (O_2 and CO_2) concentration measurement systems. These systems are typically used to estimate energy expenditure during exercise based on expired air analysis. For breath-by-breath analysis, dynamic characteristics, such as time delays and time constant, are very important for dynamically estimating energy expenditure.

There is no report on the systematic modeling of concentration measurement systems. This paper proposes a simple but effective approach for the measurement system modeling. Firstly, the model structure is obtained based on physical principles. Then, model parameters are determined by using least square method and step change response analysis [5][6]. Finally, the overall system is modeled by a two order linear system with time delay. It is found that both the time constant and time delay are functions of breath flow-rate and are therefore not time invariant. In the next section, a brief description of the experimental setup is given. Model identification is presented in section III, and experimental results and analysis in section IV.

II. EXPERIMENTAL DESCRIPTION

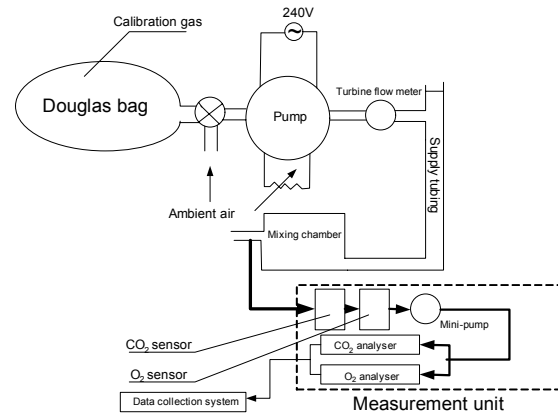


Fig. 1. Experimental settings.

The experimental settings are described in Fig.1. The concentration measurement system has the following parts: supply tubing, mixing chamber, CO_2 and O_2 sensors and CO_2 and O_2 analysers.

First, a Douglas bag is filled with calibration gas (16.3% O_2 and 4% CO_2), which is pumped to the mixing chamber through the supply tubing with a constant flow rate. Inlet flow rate is measured by a turbine flow meter. The outputs of the analysers under different inlet flow rate are all

automatically recorded by a computer-based data collection system.

III. SYSTEM MODELING

The typical outputs of CO₂ analyzer and O₂ analyzer are given in Fig 2.

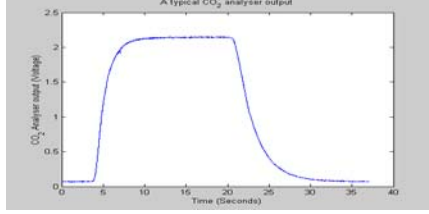


Fig 2.a. Typical outputs of CO₂ analyzer.

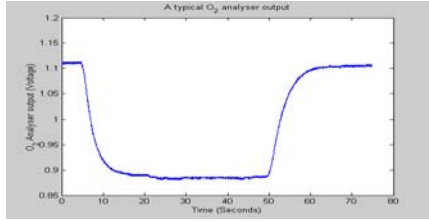


Fig. 2.b. Typical outputs of O₂ analyzer.

From Fig 2, it can be seen that the outputs of the system look like the output of a low order stable linear system with time delay. The time delay is summarized in Table I.

Table I

Test No	Flow rate(Liter/s)	Time delay(s)	
		CO2	O2
5	1.54	4.53	5.53
3	3.40	3.745	4.885
4	7.24	3.5	4.4
6	11.8	2.97	3.89

Basis physical principles together with the above experiment results leads to the development of a suitable model structure.

As the calibration gas takes time to pass through the supply tubing, a time delay caused by this is approximately = *Volume of supply tubing (2.7 liters) / inlet flow rate*.

Based on the data in Table I, a linear relationship has been found between time delay (τ) and reciprocal of inlet flow rate. Parameters were calculated as follows (least square):

$$O_2: \quad \tau_{O_2} = 2.6 / f_{in} + 3.9 \quad (1)$$

$$CO_2: \quad \tau_{CO_2} = 2.5 / f_{in} + 3.0 \quad (2)$$

The estimation results about time delay are summarized in Fig. 3 and Fig. 4.

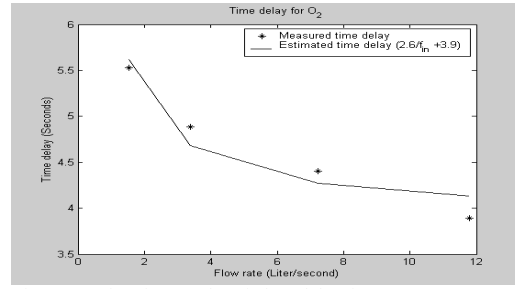


Fig. 3.a. Estimation results of time delay for O₂.

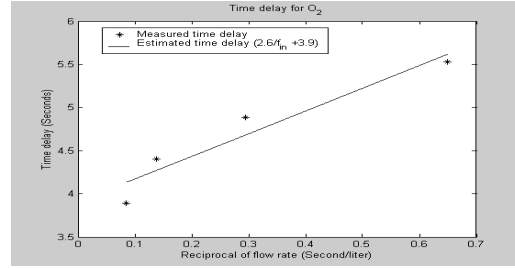


Fig. 3.b. Estimation results of time delay for O₂.

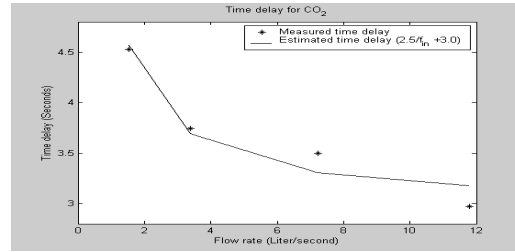


Fig. 4.a. Estimation results about time delay for CO₂.

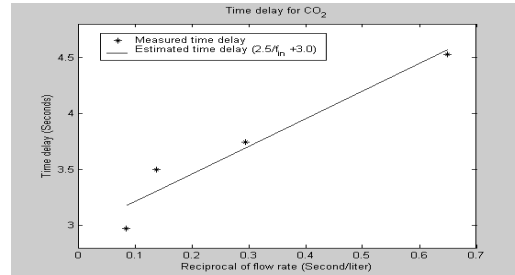


Fig. 4.b. Estimation results for time delay for CO₂.

Now, we consider the model structure of the mixing chamber (see Fig. 5.)

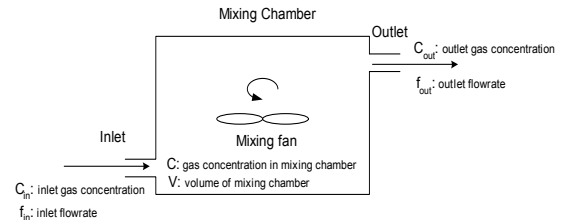


Fig. 5. A simple physical model of the mixing chamber.

Based on physical principles, it can be seen that $\dot{C} = (C_{in}f_{in} - C_{out}f_{out})/V$. Because the gas in the mixing chamber is well mixed, it is reasonable to assume that $C = C_{out}$. Thus:

$$\dot{C} = (C_{in}f_{in} - Cf_{out})/V \quad (3)$$

Generally, equation (1) is a nonlinear differential equation. However, if we assume that inlet gas concentration (C_{in}) and outlet flow rate (f_{out}) is constant or with a very slow variant rate¹, equation (3) can be simplified as a first order linear differential equation:

$$C(s) = \frac{k_m}{T_m s + 1} f_{in}(s) \quad (4)$$

Where $k_m = \frac{C_{in}}{f_{out}}$ and $T_m = \frac{V}{f_{out}}$

with the volume of mixing chamber V approximately 4.2L.

Thus, the mixing chamber can be described by a first order system. However, its DC gain k_m and time constant are the functions of outlet flow rate f_{out} and concentration C_{in} .

Now, let us consider model structure for the measurement unit (includes sensors and analysers). Normally, these can be modeled by a first order system with time delay. However, in order to suppress high frequency noise, we have added RC filters to the output of the analysers. It would seem reasonable to treat the measurement units as second order systems, however the time constant for the RC filters is less than 0.015 seconds, which is much less than the time constant of the measurement unit. From the following results obtained from singular perturbation analysis [7], it can be seen that the first order model is a rational selection:

When two or more first order stable system are serially connected, if one of the system has a very big time constant T_{max} , then the overall system can be simplified as a first order system with time constant T_{max} .

Therefore, two serially connected first order systems with time delay can be selected as the model structure of the overall measurement system.

Now, let us determine the parameters of the second order system for the overall measurement system by using the “limit theorems of the Laplace transform” method (see [8]). Specifically, by applying equation (36) of [8], we approximately calculate the coefficients of the second order systems based on the data of the third experiment (flow rate 3.4). The second order model for O_2 measurement unit is then given by:

$$\frac{1}{(\frac{3.7}{3.4}s + 1)(0.53s + 1)}$$

and for the CO_2 measurement unit:

$$\frac{1}{(\frac{3.9}{3.4}s + 1)(0.48s + 1)}$$

Finally, the following models are achieved for the overall measurement systems under different flow rates:

$$O_2: e^{-(2.6/f_{in}+3.9)s} \frac{1}{0.53s + 1} G_{mixing}(s) \quad (5)$$

$$CO_2: e^{-(2.5/f_{in}+3.0)s} \frac{1}{0.48s + 1} G_{mixing}(s) \quad (6)$$

Where $G_{mixing}(s) = \frac{k_m}{\frac{3.8}{f_{in}}s + 1}$ as given in equation (4).

IV. ESTIMATION RESULTS

The following figures (Figures 6-13) show the estimation errors of models (5) and (6)². It can be observed that the estimation error is relatively small when inlet flow rate is low (1.5 L/S, 3.4 L/S), and the estimation error is relatively high when inlet flow rate is high (7.24 L/S, 11.8 L/S). The reason for the difference may be that the pump speed may vary when the flow rate is high. However, because the breath flow rate for moderate exercising levels is low, models (5) and (6) are good models for use in energy expenditure measurements.

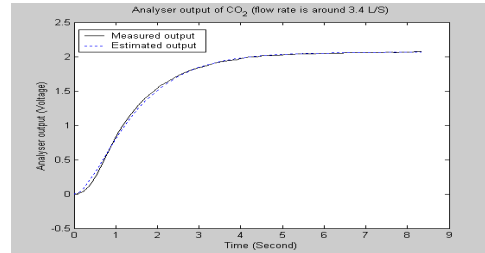


Fig. 6.

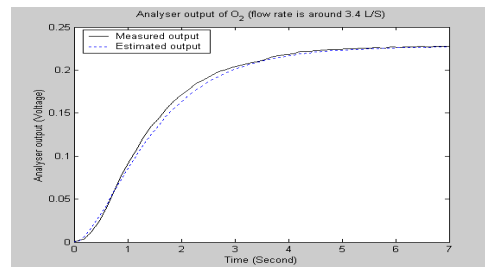


Fig. 7.

¹ Normally, this condition can be satisfied when doing mild strength exercises.

² Only the dynamic part of the step response is compared in the following figures. The difference of time delay is shown in figure 4 and 5.

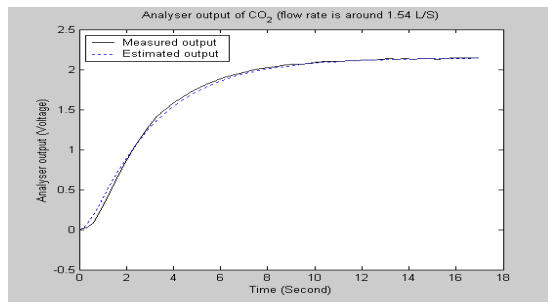


Fig. 8.

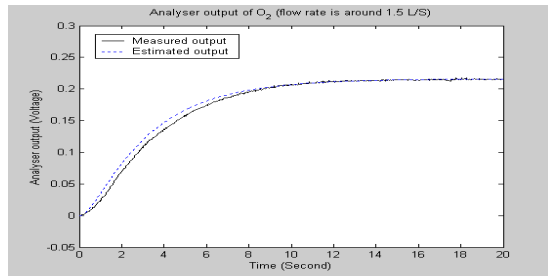


Fig. 9.

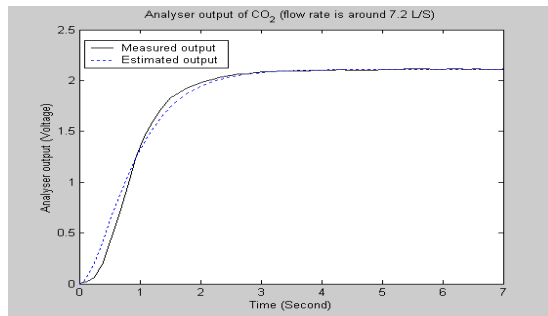


Fig. 10

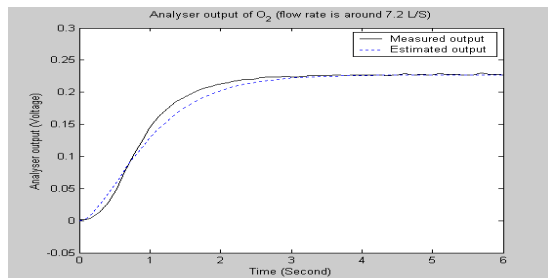


Fig. 11

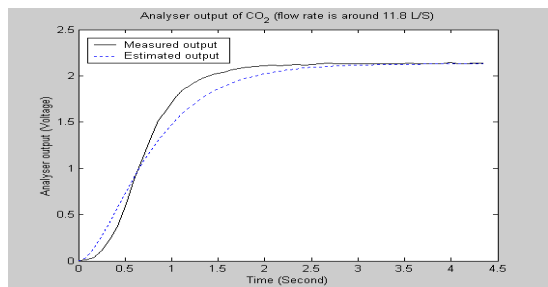


Fig. 12

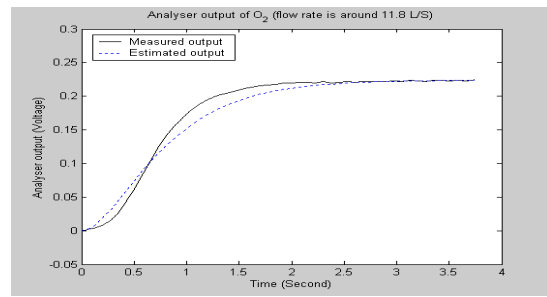


Fig. 13

V. CONCLUSION

We have presented in this paper simple but practical models for gas concentration measurement systems. Based on the models derived, some useful results can be achieved:

- The model gives us some hints to decrease the delay time:
 - a. Shorten the length of supply tubing as possible as we can.
 - b. Increase flow rate of the mini-pump and shorten the length of the thin pipe.
- The model implies that the mixing chamber is a low pass filter. Some high frequency signal cannot be measured. Thus, the volume of the mixing chamber should be carefully selected in order to ensure interested frequency band signal being captured.

VI. ACKNOWLEDGMENT

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REFERENCES

- [1] Elia M, Livesey G. Theory and validity of indirect calorimetry during net lipid synthesis. *Am J Clin Nutr* 1988;47(4):591-607.
- [2] Ferrannini E. The theoretical bases of indirect calorimetry: a review. *Metabolism* 1988;37(3):287-301.
- [3] Branson RD, Johannigham JA. The measurement of energy expenditure. *Nutr Clin Pract*. 2004;19:622–636.
- [4] Kemper MA. Indirect calorimetry equipment and practical considerations of measurement. In: Weissman C, editor. *Problems in respiratory care: nutrition and respiratory disease*. Philadelphia: JB Lippincott; 1989;2:479- 490.
- [5] Goodwin, G. C., and R. L. Payne. *Dynamic System Identification, Experimental Design and Data Analysis*. Academic Press, New York, 1977.
- [6] Ljung L. *System identification: theory for the user*. Prentice-Hall, Inc., New Jersey, 1987.
- [7] Kokotovic, P. V., Khalil, H. K. & O'Reilly, J. *Singular Perturbation Methods in Control: Analysis and Design.*, Academic Press, London etc., 1986.
- [8] Rake. V. Step response and frequency response methods *Automatica*, Volume 16, Issue 5, September 1980, Pages: 519-526