# Nonparametric Modelling of $VO_2$ Response to Exercise

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***Abstract*— This paper investigates the modelling of oxygen consumption ($VO_2$) response to jogging exercise on treadmill. Unlike most of the previous methods, which often use simple parametric models, e.g., first order linear time invariant model, this study applied a nonparametric kernel based regularised method to estimate $VO_2$ to address the ill-conditioned modelling problem and achieve accurate estimation. In particular, it is worthy to be noted that the selection of kernels will affect the results for different modelling scenarios. Therefore, in this research, both radial basis kernel and stable spline kernel were selected for testing. In order to select the favourable kernel for this system, a simulation related to $VO_2$-jogging speed was carried out. The results of simulation indicated that spline kernel can achieve higher accuracy comparing to radial basis function kernel. Experimentally, the kernel based estimation method and spline kernel were tested using six participants. From the results, an average impulse response is obtained. It showed the $VO_2$ estimation, based on the average finite impulse response, is fitted well to the six observations collected from the participants.**

## I. INTRODUCTION

Oxygen consumption on-kinetics is an important physiological parameter for the determination of functional health status and muscle energetics during physical exercise. Several experiments suggest that oxygen consumption is mainly controlled by intramuscular factor related metabolic system [1], [2]. Unlike heart rate, which is affected by mood, stress, etc., the maximum Oxygen uptake is considered as the most accurate measurement of the fitness of cardiorespiratory system. The main goal of this paper is to establish a nonparametric model describing the relation between Oxygen uptake (system output) and speed of jogging exercise on treadmill (system input).

Previous researches conducted on the Oxygen uptake modelling mainly focused on two aspects: Oxygen uptake in static status and dynamic status. For the static status, a linear static model was proposed to approximately estimate Oxygen uptake based on a given range of walking speed [3]. Furthermore, a simple nonlinear static model was discussed in [4], [5]. On the other hand, the dynamic modelling of the Oxygen uptake during exercise also attracted the attention of many researchers. For example, [6], [7] developed a first order system to approximate the process based on step response. Later, [8] developed a nonlinear dynamic model for Oxygen uptake modelling during running exercise with pseudo random binary signal (PRBS) as the input. However, broadly speaking, it is relatively difficult for the exercisers to follow the PRBS signal as the treadmill speed.

Generally, the level of noise in $VO_2$ measurement is quite large and the individual variation of the Oxygen uptake is quite different from, e.g., heart rate signal. Therefore, it can be only roughly modelled as a first order system. Although nonlinear dynamic model can characterise the system with better accuracy, but it requires a relatively complex input to stimulate the system. Mostly, for this kind of problem, a non-parametric model such as impulse response (IR) model can achieve higher accuracy, but it normally requires PRBS input [9]. Recently, a new kernel based estimation method has been developed [10], [11] for nonparametric model estimation. In addition, in order to avoid ill-conditioned solutions due to the existence of large noises, a regularised term can be incorporated into the cost function [12], which can limit the one-step variation of the estimated parameters. This new kernel based method projects the parameters of IR into a reproducing kernel Hilbert space (RKHS) which can reduce high frequency components in IR model. Furthermore, more accurate results can also be obtained by using this method enabling us to employ any simple input such as step input.

In this paper, in order to implement nonparametric modelling of $VO_2$ response to the exercises, the kernel based estimation method is adopted. For this research, we selected radial basis function (RBF) kernel and stable spline (SS) kernel. Particularly, we demonstrated that this method is suitable when the input is a step response for this specific $VO_2 - Speed$ system. Furthermore, we showed through a simulation example that SS kernel can achieve higher accuracy comparing to RBF kernel for this problem. Eventually, the proposed methods were experimentally validated by using the $VO_2$ data collected from six participants.

## II. NEW MODELLING METHOD FOR $VO_2$ DURING EXERCISE

For most of the previous studies, the Oxygen uptake during exercise (Fig.1) has been considered as an exponential function [1]:

$$VO_2(t) = VO_2^0 + \beta \times [1 - e^{(t-TD)/\tau_p}], \tag{1}$$

which can be considered as a first order system from the perspective of control theory, $TD$ is the time delay, $\tau_p$ is the time constant and $VO_2^0$ is the initial value of Oxygen uptake. However, sometimes, a first order system cannot obtain the best results due to the individual variation of Oxygen uptake. In order to obtain more acceptable results, we adopted a

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nonparametric modelling method which make use of finite impulse response to describe the system's characteristic.

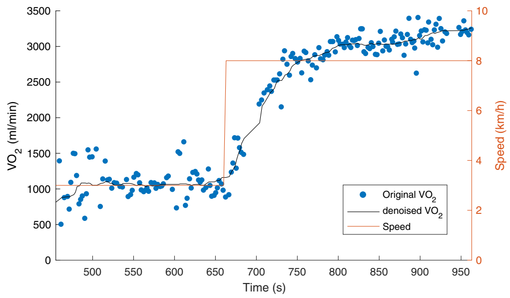


Fig. 1. Oxygen uptake during running on treadmill.

### A. Kernel Based Estimation method for Finite Impulse Response

In this section, a new kernel based nonparametric estimation method is exploited to model the Oxygen uptake during running exercise.

Let us select $t$ with sampling time $T$ as the time index, the relationship between the running speed ($u$) and Oxygen uptake ($y$) can be considered as a single input single output (SISO) dynamic system. Therefore, the discrete time output calculated by using impulse response of this system can be expressed as:

$$y(t) = \sum_{\tau=0}^{\infty} u(t-\tau)g(\tau) + \varepsilon(t), \quad t = 1, 2, \cdots N \tag{2}$$

where $g(\cdot)$ represents the parameters of IR, $\varepsilon(t)$ is the Gaussian white noise and $N$ is the total sampling number.

A widely used cost function of Eq.(2) can be expressed as:

$$\sum_{t=1}^{N}(y(t) - L_t[g])^2, \tag{3}$$

where $L_t[g]$ is:

$$L_t[g] = \sum_{\tau=0}^{\infty} u(t-\tau)g(\tau). \tag{4}$$

Although minimising the cost function (3) can be solved by least square (LS) estimation or maximum likelihood (ML) estimation directly, it is not appropriate for modelling the Oxygen uptake, as the measurements are normally extremely noisy [8]. Hence, to guarantee the validity of the obtained model and avoid any ill-conditioned solution, a regularisation term is crucial to weight the variation of the estimated parameters in the objective function. Then, the cost function can be rewritten as:

$$\sum_{t=1}^{N}(y(t) - L_t[g])^2 + \gamma ||g||^2, \tag{5}$$

where the first term implies the modelling error, $\gamma$ is a positive coefficient controlling the trade off between the error term and regulariser $||g||^2$. For a normal regulariser $||g||^2$, regularised least square estimation (ReLS) is a standard solution to solve Eq.(5). In order to obtain a better IR model of the Oxygen uptake model, we introduce a newly developed kernel method [10], [11].

Assuming that function $g \in R^m$, then function $g$ in regularisation term can be projected into a reproducing kernel Hilbert space (RKHS), i.e., $g \to g_{\mathscr{H}}$ $(R^m \times R^m \to R^{m\times m}(\mathscr{H}))$. The advantage of this transform is penalising the high frequency components in function $g$ [11]. Furthermore, unlike support vector regression (SVR) [13], the inputs and system parameters from the error term are not projected to a higher dimension which the original system parameters are hard to recover from projected parameters. Eventually, the IR model can be identified by minimising the cost function:

$$\min_{g\in\mathscr{H}} \left( \sum_{t=1}^{N}(y(t) - L_t[g])^2 + \gamma ||g||^2_{\mathscr{H}} \right). \tag{6}$$

To solve Eq.(6), an output kernel $\boldsymbol{O} \in R^{N\times N}$ is defined as:

$$O(i,j) = \sum_{x=1}^{N} u(i-x) \left( \sum_{a=1}^{N} u(j-a)K(i,a) \right), \tag{7}$$

where $K(\cdot,\cdot)$ is a selected kernel which is discussed later in this section.

From the Representer Theorem for the system identification [11], the solution of Eq.(6) is given by:

$$\hat{g}(t) = \sum_{s=1}^{N} \hat{c}_s a(t,s), \tag{8}$$

where $a(t,s)$ is defined as:

$$a(t,s) = \sum_{\tau=1}^{N} (u(s)-\tau) K(t,\tau), \tag{9}$$

and $\hat{c}_s$ is the $s-th$ element of $\hat{\boldsymbol{c}}$:

$$\hat{\boldsymbol{c}} = (\boldsymbol{O} + \gamma \boldsymbol{I}_N)^{-1} \boldsymbol{Y}, \tag{10}$$

where vector $\boldsymbol{Y} = [y(1), y(2), \cdots, y(N)]^T$ and $\boldsymbol{I}_N$ is the identity matrix with $N$ dimension.

### B. Kernel Selection

Several kernels can be applied to this kernel estimation method, such as radial basis function (RBF) kernel, stable spline (SS) kernel, diagonal/correlated (DC) kernel, etc. In this research, the RBF kernel and the SS kernel are selected. RBF kernel is widely used in kernel related methods since the development of support vector machine (SVM). SS kernel is developed in [10]. As it is known, the impulse response is treated as a function which decays exponentially with a certain rate, the SS kernel which belongs to amplitude modulated locally stationary (ALMS) kernel can achieve deserved results when modelling this impulse response model. Therefore, the RBF kernel and SS kernel are selected for the estimation of the impulse response of the Oxygen uptake:

- RBF kernel:

$$K(i,j) = e^{-\rho ||i-j||^2}, \tag{11}$$

where $\rho > 0$.

- SS kernel:

$$K(i,j) = \frac{c}{2}e^{-\beta \min(i,j)} - \frac{c}{6}e^{(-3\beta \max(i,j))}, \tag{12}$$

where $c, \beta > 0$.

More details about the kernel and kernel estimation method can be found in [14].

## III. Experimental results and analysis

### A. Simulation

Generally, the relationship between the Oxygen uptake and the jogging speed was considered as a first order system. Therefore, the performance of this method can be tested on a first order system involving a relatively large noise, the system is assumed as:

$$Y(s) = \frac{7U(s)}{35s+1}. \tag{13}$$

As seen, the time-constant and the gain are 35 and 7, respectively.

In the system (13), we let the input $U(s)$ be a step input, assumed a Gaussian white noise with the SNR of $1dB$, and set the sampling time ($Ts$) as $1s$. The input and output is shown in Fig.(2)

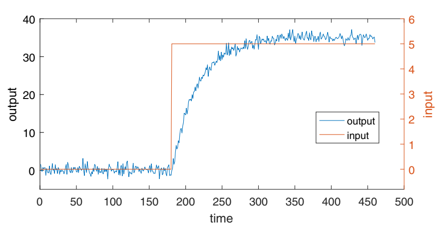


Fig. 2. Input and noisy output ($1dB$ SNR) of the system for simulation.

Then, the kernel method is applied to solve the IR of the system. The settings of kernels and regulariser are listed below:

- RBF kernel: $\rho = 1 \times 10^{-7}$
- SS kernel: $c = 0.015$, $\beta = 0.008$
- regulariser: $\gamma = 0.2$

Based on these values, the IR of the system and the estimations are shown in Fig.(3) and Fig.(4). As we can see from Fig.(3), the IR from SS kernel is closer to the true value comparing to IR from RBF kernel. Moreover, from Fig.(4), the estimation output from SS kernel is very close to the true output without over-fitting. Therefore, we choose SS kernel in our experiment section, as it can provide a better IR for this problem.

A classic ridge regression without kernel is also applied to solve the IR as a comparison. From Fig.(5), it can be seen that the IR model from ridge regression without kernel is inaccurate and noisy comparing to the kernel method. The results from the kernel method is far better than the classic ridge regression in this specific problem.

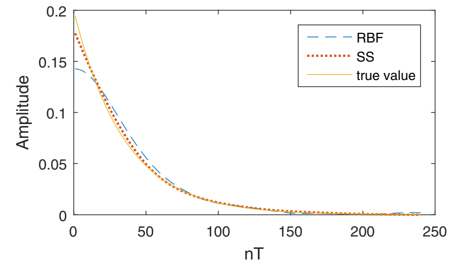


Fig. 3. Comparison among true IR and estimated IR based on RBF kernel and SS kernel.

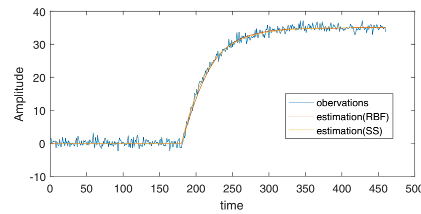


Fig. 4. Estimated output from RBF kernel and SS kernel and observations.

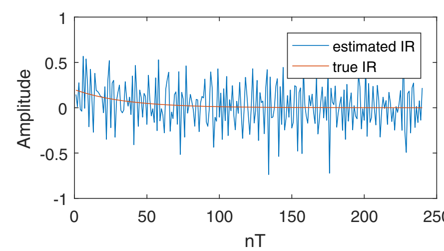


Fig. 5. IR from Ridge regression ($\gamma = 0.2$).

### B. Experiments

To develop the impulse response model of $VO_2$ response to treadmill exercise, six healthy males participated in this experiments; their physical characteristics are shown in Table. I. All data were acquired by a portable gas analyzer $K4b^2$ (COSMED), which is the first portable system for pulmonary gas exchange measurement with true breath-by-breath analysis. The UTS Human Research Ethics Committee (UTS HREC 2009000227) approved this study and an informed consent was obtained from every participant before commencement of data collection.

TABLE I

Age and BMI of Participant

| Participant | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Age | 45 | 37 | 52 | 52 | 43 | 50 |
| BMI($kg/m^2$) | 30.3 | 24.6 | 29.6 | 29.1 | 31.3 | 26.0 |

Prior the experiments, all participants were seated for $5mins$, then stand next to treadmill for $2mins$. During the experiments, the participants were walking at $3km/h$ for $3mins$, then they started to running for $7mins$ at $8km/h$, following by another walking at $3km/h$ for $3mins$ before stop. The protocol of this experiment is shown in Fig.(6).

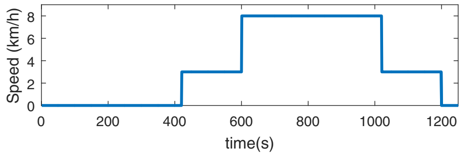


Fig. 6. Protocol of exercise.

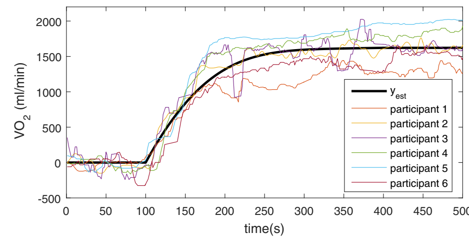


Fig. 8. Comparison between estimated $VO_2$ and measurements from six participants.

For the research, in this stage, we only focus on the onset (from walking to running) $VO_2$ response of treadmill exercise. Therefore, we only took the data from $t_1 = 500s$ to $t_2 = 1000s$ as shown in Fig.(6) for impulse response modelling. Since the gas response recorded by $K4b^2$ are breath by breath based, the sampling is irregular, and the quality of the data is often influenced by the breath frequency of the subject. Thus, prior the modelling, we applied a median filter for the data and interpolate the data with $1s$ sampling time by using Matlab. For the developed impulse response model, the sampling time is selected as $1s$, and the order of the model is selected as 300. The IR model can therefore be expressed as:

$$\begin{aligned} y[n] &= g[0]u[n] + g[1]u[n-1] + \cdots + g[299]u[n-299] \\ &= \sum_{i=0}^{299} g[i]u[n-i]. \end{aligned} \tag{14}$$

With the selected 500 observations, firstly, we removed the offset which is the average value of the initial 100 data. Then, we applied the introduced kernel estimation approach to estimate the IR model by using stable spline kernel ($c = 0.05$, $\beta = 0.01$, $\gamma = 0.4$). The results are shown in Fig.(7) and Fig.(8). Fig.(8) shows the estimated impulse responses for all 6 participants. Although the values of the impulse responses are slight different, the pattern of the responses among participant is similar. We calculated the averaged impulse response and highlighted it in Fig.(7). Based on the estimated average impulse response model, we also calculated the predicted $VO_2$ output, and then compared it with the experimental data, which is shown in Fig. (8). From Fig. (8), we can observe that the estimation fits *properly* with the experimental data without overfitting.

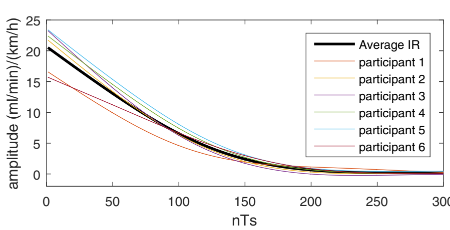


Fig. 7. Average IR and individual IR from six participants.

## IV. CONCLUSIONS

This paper established the first nonparametric model of $VO_2$ response to treadmill exercise by using the recently proposed kernel based modelling method. Both stable spline kernel and radial basis function kernel have been studied and tested by using numerical simulation, and it is observed for impulse response modelling, the stable spline kernel outperforms the radial basis function kernel. The averaged impulse response model has been finally established based on the experimental data of six treadmill exercisers. The results of both simulation and experiment indicate that the kernel based nonparametric modelling method is an effective method for the estimation of impulse response of oxygen consumption, and can provide accurate prediction of $VO_2$ response during treadmill exercise.

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