

Approximate broadcasting of quantum correlations

Wei Xie^{1,*}, Kun Fang^{1,†}, Xin Wang^{1,‡} and Runyao Duan^{1,2,§}

¹Centre for Quantum Software and Information, Faculty of Engineering and Information Technology,
University of Technology Sydney, NSW 2007, Australia and

²UTS-AMSS Joint Research Laboratory for Quantum Computation and Quantum Information Processing,
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
(Dated: August 10, 2017)

Broadcasting quantum and classical information is a basic task in quantum information processing, and is also a useful model in the study of quantum correlations including quantum discord. We establish a full operational characterization of two-sided quantum discord in terms of bilocal broadcasting of quantum correlations. Moreover, we show that both the optimal fidelity of unilocal broadcasting of the correlations in an arbitrary bipartite quantum state and that of broadcasting an arbitrary set of quantum states can be formulized as semidefinite programs (SDPs), which are efficiently computable. We also analyze some properties of these SDPs and evaluate the broadcasting fidelities for some cases of interest.

I. INTRODUCTION

Copying information is a rather simple task in the classical realm, but unfortunately not in the quantum realm. It is not allowed to create an identical copy of an arbitrary unknown pure quantum state due to the no-cloning theorem [1, 2]. One can clone a set of pure states if and only if they are orthogonal. The no-broadcasting theorem [3] generalizes this result to mixed states, saying that a set of quantum states can be broadcast if and only if the states commute with each other.

These no-go theorems can be further extended to the setting of local broadcast for composite quantum systems. Given a bipartite quantum state ρ_{AB} shared by Alice and Bob, their objective is to perform local operations only (without communication) to produce a state $\hat{\rho}_{A_1 A_2 B_1 B_2} = (\Lambda_{A \rightarrow A_1 A_2} \otimes \Gamma_{B \rightarrow B_1 B_2}) \rho_{AB}$ such that $\text{Tr}_{A_1 B_1} \hat{\rho}_{A_1 A_2 B_1 B_2} = \text{Tr}_{A_2 B_2} \hat{\rho}_{A_1 A_2 B_1 B_2} = \rho_{AB}$ (see Section II for notational convention). It is shown in [4] that this task can only be performed if ρ_{AB} is classically correlated. Even if the task is relaxed to obtain two bipartite states with the same correlation as ρ_{AB} (measured by the mutual information), it is feasible to do the task if and only if the given state ρ_{AB} is classically correlated. This is called the no-local-broadcasting theorem [4]. Furthermore, when the local operations are only allowed for one party (e.g., Alice), the task can be done if and only if ρ_{AB} is classical on A [5–7].

When the task of perfect broadcasting cannot be accomplished, it is natural to ask whether the broadcasting can be performed in an approximate fashion, and how to design the optimal broadcasting operation. We shall study the approximate broadcasting of states and correlations by utilizing semidefinite programs (SDPs).

In Ref. [8] the Bose-symmetric channel is considered as unilocal broadcasting operation and an SDP is derived for this problem. Semidefinite programming optimization techniques [9] have found many applications to the theory of quantum information and computation (see, e.g., [10–18]), and also to the study of quantum correlations (see, e.g., [8, 19–21]).

Quantum discord (see Section III for definition), as an indispensable measure of quantum correlation beyond entanglement, is introduced in [22] and [23] independently. It is argued [24] that quantum discord is responsible for the quantum speed-up over classical algorithms. Quantum discord is a quite useful concept in many fields of quantum information processing, such as local broadcasting of correlations [4, 25], quantum computing [26], quantum data hiding [27], quantum data locking [28], entanglement distribution [29, 30], common randomness distillation [31], quantum state merging [32–34], entanglement distillation [34, 35], superdense coding [34], quantum teleportation [34], quantum metrology [36], and quantum cryptography [37]. Quantum discord has become an active research topic over the past few years [38, 39].

The local broadcasting paradigm can provide a natural operational interpretation to quantum discord. Remarkably, the minimum average loss of mutual information resulting from local operation $\Lambda_{A \rightarrow A_1 \dots A_n}$ on A for arbitrary quantum state ρ_{AB} approaches the quantum discord $D_A(\rho_{AB})$ of ρ_{AB} as n goes to infinity. This result is established in Ref. [25] and it generalizes the work in Ref. [40] which considers pure states ρ_{AB} only. However, it remains open whether there is an analogous connection for the two-sided setting of redistributing correlations [39].

In this paper, we study the approximate broadcasting of quantum correlations in both asymptotic and non-asymptotic settings. In the asymptotic regime, we rigorously prove the conjecture in Ref. [39] and show an operational meaning of the two-sided discord in terms of bilocal broadcasting of correlations; that is, the asymptotic minimum average loss of correlation after opti-

*Electronic address: xievvvei@gmail.com

†Electronic address: kun.fang-1@student.uts.edu.au

‡Electronic address: xin.wang-8@student.uts.edu.au

§Electronic address: runyao.duan@uts.edu.au

mal bilocal broadcasting is exactly the two-sided quantum discord of the initial state. In the non-asymptotic regime, we give an alternate derivation for the SDP characterization of the optimal unilocal broadcasting fidelity and show that the universal quantum clone machine (UQCM) can also serve as the optimal universal unilocal broadcasting operation. Moreover, the optimal state-dependent unilocal broadcasting operation for pure two-qubit states is analytically solved. Similarly, we establish the SDP for the optimal broadcasting fidelity of a finite set of quantum states.

II. PRELIMINARIES

A quantum system A is associated to a Hilbert space \mathcal{H}_A of dimension $|A|$ with some fixed orthonormal basis $\{|j\rangle_A\}_j$. In this work, we only deal with finite-dimensional spaces, and the spaces of systems with the same letter are always assumed to be isomorphic, for example, $\mathcal{H}_A \cong \mathcal{H}_{\tilde{A}} \cong \mathcal{H}_{A_1} \cong \mathcal{H}_{A_2}$. The linear operators from \mathcal{H}_A to \mathcal{H}_B are always written with subscripts identifying the systems involved, for example, $X_{A \rightarrow B}$. We denote $\mathcal{S}(A)$ as the set of density operators [41] on system A .

A quantum operation (or channel) $\mathcal{E}_{A \rightarrow B}$ with input system A and output system B is a completely positive (CP), trace preserving (TP) linear map from the linear operators on \mathcal{H}_A to the linear operators on B . A quantum-to-classical channel \mathcal{F} is a cptp map such that $\mathcal{F}(\cdot) = \sum_j \text{Tr}(M_j \cdot) |j\rangle\langle j|$, where $\{M_j\}_j$ is a POVM. The set of all quantum-to-classical channels is denoted by QC. Since the subscript of an operator or operation specifies its input and output systems, we can write a product of operators or operations without the \otimes symbol, and omit the identity operator or operation $\mathbb{1}$, which would make no confusion, for example, $X_{AB}Y_{BC} \equiv (X_{AB} \otimes \mathbb{1}_C)(\mathbb{1}_A \otimes Y_{BC})$ and $\mathcal{E}_{B \rightarrow C}(X_{AB}) \equiv (\mathbb{1}_A \otimes \mathcal{E}_{B \rightarrow C})X_{AB}$.

The Choi-Jamiołkowski matrix [42, 43] of a quantum operation $\mathcal{E}_{A \rightarrow B}$ is $J_{\mathcal{E}} = (\mathbb{1}_{\tilde{A} \rightarrow \tilde{A}} \otimes \mathcal{E}_{A \rightarrow B})\phi_{\tilde{A}A}$, where $\phi_{\tilde{A}A} = \sum_{ij} |ii\rangle\langle jj|$ is the unnormalized maximally entangled state. The output of the channel $\mathcal{E}_{A \rightarrow B}$ with input ρ_A can be recovered from $J_{\mathcal{E}}$ by $\mathcal{E}_{A \rightarrow B}(\rho_A) = \text{Tr}_A(J_{\mathcal{E}}^T \rho_A)$, where T_A denotes the partial transpose on A .

We use $H(\cdot)$ to denote the von Neumann entropy of quantum states, $H(A|B) := H(AB) - H(B)$ the conditional quantum entropy, $I(A : B) := H(A) + H(B) - H(AB)$ the quantum mutual information. The fidelity $F(\rho, \sigma) = \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}$, as a measure of similarity between quantum states, can be viewed as the optimal solution to an SDP [44, 45]. The diamond norm can be used to give the distance of two quantum operations \mathcal{E}, \mathcal{F} , that is, $\|\mathcal{E} - \mathcal{F}\|_{\diamond} = \sup\{\|((\mathcal{E} - \mathcal{F}) \otimes \mathbb{1})X\|_1 : \|X\|_1 = 1\}$, where $\|\cdot\|_1$ is the trace norm. In addition, we denote $[n] = \{1, \dots, n\}$, and denote by $|\cdot|$ the cardinality of a set or the dimension of a linear space.

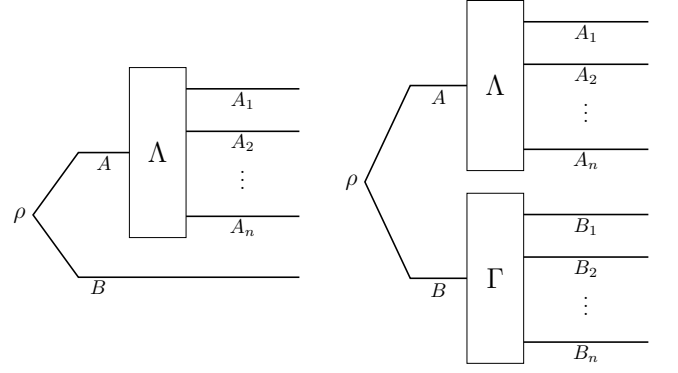


FIG. 1: Unilocal (left) and bilocal (right) broadcasting of quantum correlations in initial state ρ_{AB} . The objective is for the quantum channels Λ, Γ to make the states on $A_i B$ or $A_i B_i$ as close to ρ_{AB} as possible measured in some way.

III. ASYMPTOTIC BILOCAL BROADCASTING AND TWO-SIDED QUANTUM DISCORD

A. Previous results

The concept of quantum discord was introduced by [22, 23]. Here the one-sided and two-sided quantum discord of a bipartite state ρ_{AB} are defined by

$$D_A(\rho_{AB}) := \min_{\mathcal{E}_A \in \text{QC}} (I(A : B)_{\rho_{AB}} - I(A : B)_{(\mathcal{E}_A \otimes \mathbb{1}_B)\rho_{AB}}), \quad (1)$$

$$D_{AB}(\rho_{AB}) := \min_{\mathcal{E}_A, \mathcal{F}_B \in \text{QC}} (I(A : B)_{\rho_{AB}} - I(A : B)_{(\mathcal{E}_A \otimes \mathcal{F}_B)\rho_{AB}}), \quad (2)$$

respectively.

It is shown in [25] that the one-sided quantum discord is equal to the asymptotic average loss of correlation after the optimal broadcasting operation. Consider the following scenario. Alice and Bob, apart away from each other, share a bipartite quantum state ρ_{AB} . The information, or correlation, shared by them is measured by quantum mutual information in what follows. The goal of Alice is to broadcast the mutual information between them to many, say n , recipients, using local operation only. If the state is not classical on A , she cannot perform the task perfectly [5–7], and the mutual information between each recipient and Bob would decrease in general. Now her task naturally becomes to design a broadcasting operation in order to minimize the average loss of mutual information. Remarkably, the minimal average loss of correlation approaches quantum discord D_A of ρ_{AB} as n tends to infinity, as revealed in the following Proposition.

Proposition 1 ([25]) *Let ρ_{AB} be a bipartite state and D_A is defined by Eq. (1). Let $\Lambda_{A \rightarrow A_1 \dots A_n}$ be a cptp map and $\Lambda_j :=$*

$\text{Tr}_{\Lambda_j} \circ \Lambda$. Then

$$D_A(\rho_{AB}) = \lim_{n \rightarrow \infty} \min_{\Lambda_{A \rightarrow A_1 \dots A_n}} \frac{1}{n} \sum_{j=1}^n (I(A : B)_{\rho_{AB}} - I(A_j : B)_{(\Lambda_j \otimes \mathbb{1}_B) \rho_{AB}}).$$

B. Operational interpretation of two-sided quantum discord

We will give an operational interpretation of two-sided quantum discord in terms of bilocal broadcasting, analogous to the case of one-sided quantum discord (see Fig. 1).

Theorem 2 Let ρ_{AB} be a bipartite state, and the two-sided quantum discord $D_{AB}(\rho_{AB})$ is defined by Eq. (2). Let $\Lambda_{A \rightarrow A_1 \dots A_n}$ and $\Gamma_{B \rightarrow B_1 \dots B_n}$ be cptp maps, and denote $\Lambda_j := \text{Tr}_{\Lambda_j} \circ \Lambda$ and $\Gamma_j := \text{Tr}_{\Gamma_j} \circ \Gamma$. Then

$$D_{AB}(\rho_{AB}) = \lim_{n \rightarrow \infty} \min_{\substack{\Lambda_{A \rightarrow A_1 \dots A_n} \\ \Gamma_{B \rightarrow B_1 \dots B_n}}} \frac{1}{n} \sum_{j=1}^n (I(A : B)_{\rho_{AB}} - I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}}).$$

In order to prove this theorem, we need the following result.

Lemma 3 ([25]) Let $\Lambda : \mathcal{S}(A) \rightarrow \mathcal{S}(A_1 \otimes \dots \otimes A_n)$ be a cptp map. Denote $\Lambda_j := \text{Tr}_{\Lambda_j} \circ \Lambda$, and fix a number $0 < \delta < 1$. Then there exists a POVM $\{E_k\}_k$ and a set $S \subset [n]$ with $|S| \geq n(1 - \delta)$ such that for all $j \in S$,

$$\|\Lambda_j - \mathcal{E}_j\|_\diamond \leq 3 \left(\frac{\ln(2)|A|^6 \log_2 |A|}{n\delta^3} \right)^{1/3}, \quad (3)$$

with $\mathcal{E}_j(\cdot) := \sum_k \text{Tr}(E_k \cdot) \sigma_{j,k}$ for states $\sigma_{j,k} \in \mathcal{S}(A_j)$. Here $|A|$ is the dimension of the space A .

We also need the continuity bound of mutual information. Let ρ_{AB}, σ_{AB} be bipartite states on system A , of dimension $|A| \geq 2$, and system B . Assume $\gamma := \frac{1}{2} \|\rho_{AB} - \sigma_{AB}\|_1 \leq \frac{1}{2}$. Due to Fannes-Audenaert inequality [46, 47] and the fact that quantum operation cannot increase trace distance between two states, it holds $|H(A)_{\rho_{AB}} - H(A)_{\sigma_{AB}}| \leq \frac{1}{2} \|\rho_A - \sigma_A\|_1 \log_2(|A| - 1) + h_2(\frac{1}{2} \|\rho_A - \sigma_A\|_1) \leq \gamma \log_2(|A| - 1) + h_2(\gamma)$, where $h_2(x) := -x \log_2 x - (1 - x) \log_2(1 - x)$ is the binary entropy function. Due to Alicki-Fannes inequality [48] (see also [49] for a tighter continuity bound for conditional entropy), it holds $|H(A|B)_\rho - H(A|B)_\sigma| \leq 8\gamma \log_2 |A| + 2h_2(2\gamma)$. Therefore,

$$\begin{aligned} & |I(A : B)_\rho - I(A : B)_\sigma| \\ & \leq |H(A)_\rho - H(A)_\sigma| + |H(A|B)_\rho - H(A|B)_\sigma| \\ & \leq 8\gamma \log_2 |A| + \gamma \log_2(|A| - 1) + 2h_2(2\gamma) + h_2(\gamma). \end{aligned} \quad (4)$$

We are now in the position to prove Theorem 2.

Proof The desired statement is equivalent to that

$$\begin{aligned} & \max_{\mathcal{E}_A, \mathcal{F}_B \in \text{QC}} I(A : B)_{(\mathcal{E}_A \otimes \mathcal{F}_B) \rho_{AB}} \\ & = \lim_{n \rightarrow \infty} \max_{\Lambda, \Gamma} \frac{1}{n} \sum_{j=1}^n I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}}. \end{aligned}$$

Assume the POVMs that achieve $I_c(A : B) := \max_{\mathcal{E}_A, \mathcal{F}_B \in \text{QC}} I(A : B)_{(\mathcal{E}_A \otimes \mathcal{F}_B) \rho_{AB}}$ are $\{M_i\}_i$ on A and $\{N_i\}_i$ on B , then one can take $\Lambda(\cdot) = \sum_i \text{Tr}(M_i \cdot) |i\rangle\langle i|^{\otimes n}$ and $\Gamma(\cdot) = \sum_i \text{Tr}(N_i \cdot) |i\rangle\langle i|^{\otimes n}$. It follows that $I_c(A : B) \leq \max_{\Lambda, \Gamma} \frac{1}{n} \sum_{j=1}^n I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}}$, and it remains to show $I_c(A : B) \geq \lim_{n \rightarrow \infty} \max_{\Lambda, \Gamma} \frac{1}{n} \sum_{j=1}^n I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}}$.

Similarly to Lemma 3, let $\Gamma : \mathcal{S}(B) \rightarrow \mathcal{S}(B_1 \otimes \dots \otimes B_n)$ be an arbitrary cptp map, then there exists a POVM $\{F_k\}_k$ and a set $S' \subset [n]$ with $|S'| \geq n(1 - \delta)$ such that for all $j \in S'$,

$$\|\Gamma_j - \mathcal{F}_j\|_\diamond \leq 3 \left(\frac{\ln(2)|B|^6 \log_2 |B|}{n\delta^3} \right)^{1/3}, \quad (5)$$

with $\mathcal{F}_j(\cdot) := \sum_k \text{Tr}(F_k \cdot) \sigma'_{j,k}$ for states $\sigma'_{j,k} \in \mathcal{S}(B_j)$. Therefore for fixed $0 < \delta < 1$, there exists $S'' \subset [n]$ with $|S''| \geq n(1 - 2\delta)$ such that Eqs. (3) and (5) hold simultaneously for all $j \in S''$. Thus

$$\begin{aligned} \|\Lambda_j \otimes \Gamma_j - \mathcal{E}_j \otimes \mathcal{F}_j\|_\diamond & \leq \|\Lambda_j - \mathcal{E}_j\|_\diamond + \|\Gamma_j - \mathcal{F}_j\|_\diamond \\ & \leq 6 \left(\frac{\ln(2)d^6 \log_2 d}{n\delta^3} \right)^{1/3} =: \varepsilon, \end{aligned}$$

where $d := \max\{|A|, |B|\}$.

For any state ρ_{AB} , by definition of the diamond norm, we have

$$\|(\Lambda_j \otimes \Gamma_j) \rho_{AB} - (\mathcal{E}_j \otimes \mathcal{F}_j) \rho_{AB}\|_1 \leq \|\Lambda_j \otimes \Gamma_j - \mathcal{E}_j \otimes \mathcal{F}_j\|_\diamond \leq \varepsilon. \quad (6)$$

We now have

$$\begin{aligned} & I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}} \\ & \leq I(A_j : B_j)_{(\mathcal{E}_j \otimes \mathcal{F}_j) \rho_{AB}} + 4\varepsilon \log_2 |A_j| + \frac{\varepsilon}{2} \log_2(|A_j| - 1) \\ & \quad + 2h_2(\varepsilon) + h_2(\varepsilon/2) \\ & \leq I(A_j : B_j)_{(\tilde{\mathcal{E}}_j \otimes \tilde{\mathcal{F}}_j) \rho_{AB}} + 4\varepsilon \log_2 |A_j| \\ & \quad + \frac{\varepsilon}{2} \log_2(|A_j| - 1) + 2h_2(\varepsilon) + h_2(\varepsilon/2) \\ & \leq I_c(A : B) + 4\varepsilon \log_2 |A_j| + \frac{\varepsilon}{2} \log_2(|A_j| - 1) + 2h_2(\varepsilon) + h_2(\varepsilon/2) \\ & =: K, \end{aligned} \quad (7)$$

where $\tilde{\mathcal{E}}_j(\cdot) := \sum_k \text{Tr}(E_k \cdot) |k_j\rangle\langle k_j|$ and $\tilde{\mathcal{F}}_j(\cdot) := \sum_k \text{Tr}(F_k \cdot) |k'_j\rangle\langle k'_j|$, and $\{|k_j\rangle\}_k$ and $\{|k'_j\rangle\}_{k'}$ are orthonormal basis of system A_j and B_j respectively, Eq. (7) follows from the continuity bound Eq. (4), and Eq. (8) follows from the fact that local operations cannot increase mutual information.

Set $\delta = n^{-1/6}$, then as $n \rightarrow \infty$ one has $\delta, \varepsilon \rightarrow 0$ and $K \rightarrow I_c(A : B)$. It follows that

$$\begin{aligned} & \frac{1}{n} \sum_{j=1}^n I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}} \\ & \leq \frac{1}{n} ((1 - 2\delta)n \cdot K + 2\delta n \cdot 2 \log_2 d') \\ & \rightarrow K \rightarrow I_c(A : B) \text{ as } n \rightarrow \infty, \end{aligned}$$

where $d' := \max\{|A_j|, |B_j|\}_j$. That is,

$$\begin{aligned} & \lim_{n \rightarrow \infty} \max_{\Lambda, \Gamma} \frac{1}{n} \sum_{j=1}^n I(A_j : B_j)_{(\Lambda_j \otimes \Gamma_j) \rho_{AB}} \\ & \leq I_c(A : B), \end{aligned}$$

and we are done. \square

IV. OPTIMAL UNIVERSAL AND STATE-DEPENDENT BROADCASTING OF CORRELATIONS

We now turn to the non-asymptotic regime of the local broadcasting of quantum correlations. We first study the optimal universal unilocal broadcasting and then the optimal state-dependent unilocal broadcasting.

A. Optimal universal unilocal broadcasting

We first give a general definition for the *unilocal n -broadcasting fidelity* of a bipartite state.

Definition 4 Given a bipartite state ρ_{AB} , the optimal unilocal n -broadcasting fidelity of ρ_{AB} on system A (see Fig. 1) is defined as the following optimal fidelity

$$\begin{aligned} f_n(\rho_{AB}) = \sup \left\{ \frac{1}{n} \sum_{j=1}^n F(\rho_{AB}, \text{Tr}_{A_j B} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_{AB})) : \right. \\ \left. \Lambda_{A \rightarrow A_1 \dots A_n} \text{ is a quantum channel} \right\}. \end{aligned} \quad (9)$$

Since the set of quantum channels is compact and the fidelity function is continuous [50], the supremum in Eq. (9) is attained. Define a unitary operator W_π on systems $A_1 \dots A_n$ for each permutation $\pi \in S_n$, by the action

$$W_\pi |j_1, j_2, \dots, j_n\rangle = |j_{\pi^{-1}(1)}, j_{\pi^{-1}(2)}, \dots, j_{\pi^{-1}(n)}\rangle$$

for any choice of $|j_1\rangle, |j_2\rangle, \dots, |j_n\rangle$. A quantum channel $\Lambda_{A \rightarrow A_1 \dots A_n}$ is called a *symmetric broadcasting channel*, if

$$\Lambda(\rho) = W_\pi(\Lambda(\rho))W_\pi^\dagger$$

for any $\rho \in \mathcal{S}(A)$ and $\pi \in S_n$.

We notice that for any channel $\Lambda_{A \rightarrow A_1 \dots A_n}$ and $\pi \in S_n$, $\Lambda(\cdot)$ and $W_\pi(\Lambda(\cdot))W_\pi^\dagger$ give the same average fidelity in Eq. (9), since

$$\begin{aligned} & \text{Tr}_{A_j B} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_{AB}) \\ & = \text{Tr}_{A_{\pi^{-1}(j)} B} W_\pi(\Lambda_{A \rightarrow A_1 \dots A_n}(\rho_{AB}))W_\pi^\dagger. \end{aligned}$$

Thus $\frac{1}{n!} \sum_{\pi \in S_n} W_\pi(\Lambda(\cdot))W_\pi^\dagger$, which is a symmetric broadcasting channel, also gives the same value. So we only need to consider the supremum over symmetric broadcasting channels. In Eq. (9), when Λ is a symmetric broadcasting channel, the summands are all the same.

Therefore, the optimal unilocal n -broadcasting fidelity of a bipartite state ρ_{AB} on A can be rewritten as

$$\begin{aligned} f_n(\rho_{AB}) = \max \{ F(\rho_{AB}, \text{Tr}_{A_1 B} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_{AB})) : \\ \Lambda_{A \rightarrow A_1 \dots A_n} \text{ is a symmetric broadcasting channel} \}. \end{aligned} \quad (10)$$

It is verified that $\Lambda_{A \rightarrow A_1 \dots A_n}$ is a symmetric broadcasting channel iff its Choi matrix J_Λ satisfies $J_\Lambda = W_\pi J_\Lambda W_\pi^\dagger$ for any π , i.e., $J_\Lambda = \frac{1}{n!} \sum_{\pi \in S_n} W_\pi J_\Lambda W_\pi^\dagger$. Using this symmetry, we give the SDP characterization for optimal unilocal broadcasting fidelity as follows.

Theorem 5 The optimal unilocal n -broadcasting fidelity of ρ_{AB} on A is given by the optimal solution of the following SDP,

$$\begin{aligned} f_n(\rho_{AB}) = \max \frac{1}{2} \text{Tr}(X_{AB} + X_{AB}^\dagger) \\ \text{s.t.} \quad \begin{pmatrix} \rho_{AB} & X_{AB} \\ X_{AB}^\dagger & \text{Tr}_{A_1 B}(J_{AA_1 \dots A_n}^T \rho_{AB}) \end{pmatrix} \geq 0, \\ J_{AA_1 \dots A_n} \geq 0, \text{Tr}_{A_1} J_{AA_1 \dots A_n} = \mathbb{1}_A, \\ J_{AA_1 \dots A_n} = \frac{1}{n!} \sum_{\pi \in S_n} W_\pi J_{AA_1 \dots A_n} W_\pi^\dagger, \end{aligned} \quad (11)$$

where W_π acts on $A_1 \dots A_n$.

Proof It suffices to consider the symmetric broadcasting channels only. Let $J_{AA_1 \dots A_n}$ be the Choi matrix of $\Lambda_{A \rightarrow A_1 \dots A_n}$, then for any ρ_A ,

$$\Lambda_{A \rightarrow A_1 \dots A_n}(\rho_A) = \text{Tr}_A(J_{AA_1 \dots A_n}^T \rho_A).$$

By linearity, for any ρ_{AB} ,

$$(\Lambda_{A \rightarrow A_1 \dots A_n} \otimes \mathbb{1}_B)\rho_{AB} = \text{Tr}_A(J_{AA_1 \dots A_n}^T \rho_{AB}),$$

and

$$\text{Tr}_{A_j B}(\Lambda \otimes \mathbb{1}_B)\rho_{AB} = \text{Tr}_{A_j B}(J_{AA_1 \dots A_n}^T \rho_{AB}).$$

Now we can rewrite the optimization problem in Eq. (10) in terms of the Choi matrix of Λ as

$$\begin{aligned} f_n(\rho_{AB}) = \max F(\rho_{AB}, \widehat{\rho}_{AB}) \\ \text{s.t.} \quad \widehat{\rho}_{AB} = \text{Tr}_{A_1 B}(J_{AA_1 \dots A_n}^T \rho_{AB}), \\ J_{AA_1 \dots A_n} \geq 0, \text{Tr}_{A_1} J_{AA_1 \dots A_n} = \mathbb{1}_A, \\ J_{AA_1 \dots A_n} = \frac{1}{n!} \sum_{\pi \in S_n} W_\pi J_{AA_1 \dots A_n} W_\pi^\dagger. \end{aligned} \quad (12)$$

The fidelity function $F(\rho, \sigma)$ of two states ρ, σ is given by the optimal solution of the following SDP [44, 45],

$$\begin{aligned} F(\rho, \sigma) = \max & \frac{1}{2} \text{Tr}(X + X^\dagger) \\ \text{s.t.} & \begin{pmatrix} \rho & X \\ X^\dagger & \sigma \end{pmatrix} \geq 0. \end{aligned} \quad (13)$$

Combining Eqs. (12) and (13) gives the desired SDP (11). \square

Remark The only difference between the SDP (11) and that in Ref. [8] lies in the symmetry of the broadcasting channel, that is, $J = W_\pi J W_\pi^\dagger$ for any $\pi \in S_n$ is required in our SDP. In Ref. [8], it is required that $J = W_{\pi_1} J W_{\pi_2}^\dagger$ for any $\pi_1, \pi_2 \in S_n$ which makes sure that the output state lies in the symmetric subspace. These two SDPs are different generalization of perfect unilocal broadcasting. But the SDP (11) here has a more direct derivation, and it is clear that the optimal solution to SDP (11) is no less than that to the SDP in [8]. Numerical experiments show that the two SDPs give the same optimal solution for some cases of ρ_{AB} , but we do not know how to give a rigorous proof or disproof for general case up to now.

In the SDP (11), if $(J_{AA_1 \dots A_n}, X_{AB})$ is feasible solution of $f(\rho_{AB})$, then $((\mathbb{1}_A \otimes U^{\otimes n}) J_{AA_1 \dots A_n} (\mathbb{1}_A \otimes U^{\otimes n})^\dagger, (U_A \otimes V_B) X_{AB} (U_A \otimes V_B)^\dagger)$ is feasible solution of $f((U_A \otimes V_B) \rho_{AB} (U_A \otimes V_B)^\dagger)$ for any local unitaries U_A and V_B . In other words, the unilocal broadcasting fidelity f_n is invariant under local unitaries.

We now consider the unilocal broadcasting fidelity of a pure state $\psi_{AB} := |\psi\rangle\langle\psi|_{AB}$, and especially the maximally entangled state, under the action of the symmetric broadcasting channel. The optimal unilocal broadcasting fidelity f_n of a pure state ψ_{AB} can be written as

$$\begin{aligned} f_n(\psi_{AB}) = \max & \sqrt{\text{Tr}(\widehat{\rho}_{AB} \psi_{AB})} \\ \text{s.t.} & \widehat{\rho}_{AB} = \text{Tr}_{A_1 B} (J_{AA_1 \dots A_n}^{T_A} \psi_{AB}), \\ & J_{AA_1 \dots A_n} \geq 0, \text{Tr}_{A_1} J_{AA_1 \dots A_n} = \mathbb{1}_A, \\ & J_{AA_1 \dots A_n} = \frac{1}{n!} \sum_{\pi \in S_n} W_\pi J_{AA_1 \dots A_n} W_\pi^\dagger, \end{aligned} \quad (14)$$

where W_π acts on $A_1 \dots A_n$.

The corresponding dual SDP is

$$\begin{aligned} f_n(\psi_{AB}) = \min & \sqrt{\text{Tr} Y_A} \\ \text{s.t.} & Y_A, Z_{AA_1 \dots A_n} \text{ Hermitian,} \\ & \text{Tr}_B (\psi_{AB}^{T_A} \psi_{A_1 B}) - Y_A \\ & + Z - \frac{1}{n!} \sum_{\pi \in S_n} W_\pi^\dagger Z W_\pi \leq 0, \end{aligned} \quad (15)$$

where, again, W_π acts on $A_1 \dots A_n$.

It is verified that the strong duality holds by Slater's theorem since $J_{AA_1 \dots A_n} = \mathbb{1}/|A|^n$ is in the relative interior of the feasible region of SDP (14). That means the optimal solutions to SDPs (14) and (15) coincide.

Proposition 6 The optimal unilocal 2-broadcasting fidelity of the maximally entangled state $\Phi_d := |\Phi_d\rangle\langle\Phi_d|$ with $|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |jj\rangle$ on systems AB is given by

$$f_2(\Phi_d) = \sqrt{\frac{d+1}{2d}}.$$

Proof We prove this proposition by explicitly constructing feasible solutions in primal and dual problem both of which can achieve the value of $\sqrt{\frac{d+1}{2d}}$.

In the primal problem, we take

$$J_{AA_1 A_2} = \sum_{i=0}^{d-1} |v_i\rangle\langle v_i|, \quad (16)$$

where

$$|v_i\rangle = \frac{1}{\sqrt{2(d+1)}} (2|i\rangle \otimes |ii\rangle + \sum_{j \neq i} |j\rangle \otimes (|ij\rangle + |ji\rangle))$$

This operation is also known as the universal quantum copying machine (UQCM) [51, 52].

In the dual problem, we take

$$Y_A = \frac{d+1}{2d^2} \mathbb{1}_d, \quad Z_{AA_1 A_2} = -\frac{d+1}{d^3} (d\Phi_d - I_0) \otimes \mathbb{1}_d,$$

where $I_0 = \sum_{i=0}^{d-1} |ii\rangle\langle ii|$. \square

Remark It is interesting that the optimal unilocal 2-broadcasting channel of the maximally entangled state is the same as the UQCM which comes from the global broadcasting setting. There is much progress on quantum cloning machine that has been made in the past years (see, e.g., [53, 54]). For $d \otimes d$ bipartite maximally entangled state, its optimal unilocal 2-broadcasting channel is denoted as $\Upsilon_{A \rightarrow A_1 A_2}^d$ with Choi matrix (16) and

$$\text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(\rho_A) = \frac{d+2}{2d+2} \rho_A + \frac{1}{2d+2} \mathbb{1}_d,$$

is a depolarizing channel.

Next, we will introduce a worst-case quantifier for the performance of unilocal broadcasting of a symmetric channel.

Definition 7 For any symmetric broadcasting channel $\Lambda_{A \rightarrow A_1 \dots A_n}$, we define the unilocal broadcasting power $\mathcal{P}(\Lambda)$ of Λ as

$$\mathcal{P}(\Lambda) := \inf_{\rho_{AB} \in \mathcal{S}(AB)} F(\rho_{AB}, \text{Tr}_{A_1 B} \Lambda(\rho_{AB})) \quad (17)$$

The unilocal broadcasting power of a symmetric broadcasting channel gives a measure of the universal unilocal broadcasting ability for symmetric broadcasting channels. The universality means it is independent of the input state. The channel with a larger value of unilocal broadcasting power is more capable of unilocal broadcasting quantum states in a universal sense.

Based on the result of optimal unilocal 2-broadcasting fidelity of maximally entangled state, we will prove that the optimal unilocal 2-broadcasting channel $\Upsilon_{A \rightarrow A_1 A_2}^d$ for the maximally entangled state has the greatest power for unilocal 2-broadcasting.

Lemma 8 For any $d \otimes d$ pure state $|\psi\rangle$,

$$f_2(|\psi\rangle\langle\psi|) \geq F(|\psi\rangle\langle\psi|, \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(|\psi\rangle\langle\psi|)) \geq \sqrt{\frac{d+1}{2d}}. \quad (18)$$

Proof Consider the Schmidt decomposition $|\psi\rangle = \sum_i \lambda_i |i\rangle_A |i\rangle_B$, where $\{|i\rangle_A\}_i$ and $\{|i\rangle_B\}_i$ are some orthonormal bases. Thus,

$$\begin{aligned} \rho_{out} &= \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(|\psi\rangle\langle\psi|) \\ &= \sum_{ij} \lambda_i \lambda_j |i\rangle\langle j| \otimes \left(\frac{d+2}{2d+2} |i\rangle\langle j| + \frac{1}{2d+2} \mathbb{1}_d \right) \\ &= \frac{d+2}{2d+2} |\psi\rangle\langle\psi| + \sum_{ij} \frac{\lambda_i \lambda_j}{2d+2} |i\rangle\langle j| \otimes \mathbb{1}_d. \end{aligned} \quad (19)$$

Then the second inequality in Eq. (18) follows from

$$\begin{aligned} &F^2(|\psi\rangle\langle\psi|, \rho_{out}) \\ &= F^2\left(|\psi\rangle\langle\psi|, \frac{d+2}{2d+2} |\psi\rangle\langle\psi| + \sum_{ij} \frac{\lambda_i \lambda_j}{2d+2} |i\rangle\langle j| \otimes \mathbb{1}_d\right) \\ &= \frac{d+2}{2d+2} + \sum_{ij} \frac{\lambda_i \lambda_j}{2d+2} \langle\psi|(|i\rangle\langle j| \otimes \mathbb{1}_d)|\psi\rangle \\ &= \frac{d+2}{2d+2} + \frac{\sum_i d \lambda_i^4}{(2d+2)d} \\ &\geq \frac{d+2}{2d+2} + \frac{(\sum_i \lambda_i^2)^2}{(2d+2)d} = \frac{d+1}{2d}. \end{aligned} \quad (20)$$

□

Proposition 9 For any $d \otimes d$ mixed state ρ ,

$$f_2(\rho) \geq F(\rho, \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(\rho)) \geq \sqrt{\frac{d+1}{2d}}.$$

Proof Suppose $\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$ is a pure state decomposition of ρ and $\hat{\rho}_j = \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(|\psi_j\rangle\langle\psi_j|)$, then we have

$$\begin{aligned} \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(\rho) &= \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d\left(\sum_j p_j |\psi_j\rangle\langle\psi_j|\right) \\ &= \sum_j p_j \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(|\psi_j\rangle\langle\psi_j|) \\ &= \sum_j p_j \hat{\rho}_j \end{aligned} \quad (21)$$

Employing the joint concavity of fidelity, we have that

$$\begin{aligned} F(\rho, \text{Tr}_{A_2} \Upsilon_{A \rightarrow A_1 A_2}^d(\rho)) &= F\left(\sum_j p_j |\psi_j\rangle\langle\psi_j|, \sum_j p_j \hat{\rho}_j\right) \\ &\geq \sum_j p_j F(|\psi_j\rangle\langle\psi_j|, \hat{\rho}_j) \\ &\geq \sum_j p_j \sqrt{\frac{d+1}{2d}} = \sqrt{\frac{d+1}{2d}}, \end{aligned} \quad (22)$$

where the last inequality uses the result in Lemma 8. □

Theorem 10 $\Upsilon_{A \rightarrow A_1 A_2}^d$ has the strongest power for unilocal 2-broadcasting in $d \otimes d$ system, i.e.,

$$\max_{\Lambda} \mathcal{P}(\Lambda) = \mathcal{P}(\Upsilon_{A \rightarrow A_1 A_2}^d),$$

where the maximum is taken over all symmetric broadcasting channels.

Proof For any symmetric broadcasting channel $\Lambda_{A \rightarrow A_1 A_2}$, we have

$$\begin{aligned} \mathcal{P}(\Lambda) &= \inf_{\rho_{AB} \in \mathcal{S}(AB)} F(\rho_{AB}, \text{Tr}_{A_1 B} \Lambda(\rho_{AB})) \\ &\leq F(\Phi_d, \text{Tr}_{A_1 B} \Lambda(\Phi_d)) \\ &\leq F(\Phi_d, \text{Tr}_{A_1 B} \Upsilon_{A \rightarrow A_1 A_2}^d(\Phi_d)) \\ &= \sqrt{\frac{d+1}{2d}}, \end{aligned} \quad (23)$$

where Φ_d is the maximally entangled state. The second inequality holds since $\Upsilon_{A \rightarrow A_1 A_2}^d$ is the optimal unilocal 2-broadcasting channel for Φ_d .

For the unilocal 2-broadcasting operation $\Upsilon_{A \rightarrow A_1 A_2}^d$, from Proposition 9, we have that

$$\mathcal{P}(\Upsilon_{A \rightarrow A_1 A_2}^d) = \sqrt{\frac{d+1}{2d}}. \quad (24)$$

Combining Eqs. (23) and (24), it is clear that $\Upsilon_{A \rightarrow A_1 A_2}^d$ maximizes the unilocal broadcasting power \mathcal{P} . Thus, it is optimal under the setting of universal unilocal 2-broadcasting.

□

B. Optimal unilocal broadcasting for two-qubit pure state

In the following theorem, we give analytical solution of optimal unilocal 2-broadcasting fidelity for two-qubit pure state. Since f_n is invariant under local unitary, we only need to consider two-qubit pure state in the form of $|\psi_\theta\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, $\theta \in (0, \pi/4]$ without lose of generality.

Theorem 11 For two-qubit pure state $\psi_\theta = |\psi_\theta\rangle\langle\psi_\theta|$ with $|\psi_\theta\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, $\theta \in (0, \pi/4]$, its optimal unilocal 2-broadcasting fidelity is given by

$$f_2(\psi_\theta) = \begin{cases} \cos^2\theta + (\sin^2\theta)/\sqrt{2}, & \theta \in (0, \arctan(2^{-1/4})] \\ ((\frac{3}{2}(\cos^4\theta + \sin^4\theta))^{1/2}), & \theta \in (\arctan(2^{-1/4}), \pi/4] \end{cases}$$

Proof We prove this theorem by explicitly constructing a feasible solution in both primal and dual problem which achieves $f_2(\psi_\theta)$.

Case 1: If $\theta \in (0, \arctan(2^{-1/4})]$, in the primal problem, we construct feasible solution

$$J_{AA_1A_2} = |v\rangle\langle v|, \quad (25)$$

where

$$|v\rangle = |000\rangle + \frac{1}{\sqrt{2}}|101\rangle + \frac{1}{\sqrt{2}}|110\rangle.$$

In the dual problem, we construct feasible solution

$$Y_A = p \begin{pmatrix} \sqrt{2}\cos^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}, \text{ where } p = \frac{\sqrt{2}\cos^2\theta + \sin^2\theta}{2}.$$

$$Z_{AA_1A_2} = x(|000\rangle\langle 110| + |110\rangle\langle 000| + |001\rangle\langle 111| + |111\rangle\langle 001|),$$

where $x = \sqrt{2}p \cdot \sin^2\theta$. It is easy to check that $J_{AA_1A_2}$ and $\{Y_A, Z_{AA_1A_2}\}$ are feasible solutions to SDP (14) and (15).

Case 2: If $\theta \in (\arctan(2^{-1/4}), \frac{\pi}{4})$, in the primal problem, we construct a feasible solution

$$J_{AA_1A_2} = |v_1\rangle\langle v_1| + |v_2\rangle\langle v_2|,$$

where

$$|v_1\rangle = \sqrt{\frac{2\tan^4\theta - 1}{6}}(|001\rangle + |010\rangle) + \sqrt{\frac{4 - 2\cot^4\theta}{3}}|111\rangle,$$

$$|v_2\rangle = \sqrt{\frac{2\cot^4\theta - 1}{6}}(|101\rangle + |110\rangle) + \sqrt{\frac{4 - 2\tan^4\theta}{3}}|000\rangle.$$

In the dual problem, let us choose

$$Y_A = \frac{3}{2} \begin{pmatrix} \cos^4\theta & 0 \\ 0 & \sin^4\theta \end{pmatrix},$$

$$Z_{AA_1A_2} = x(|000\rangle\langle 110| + |110\rangle\langle 000| + |001\rangle\langle 111| + |111\rangle\langle 001|),$$

where $x = -\frac{3}{2}\sin^2\theta\cos^2\theta$. It is also easy to check that $J_{AA_1A_2}$ and $\{Y_A, Z_{AA_1A_2}\}$ are feasible solutions to SDP (14) and (15). \square

From the above proof, we can see that the optimal unilocal 2-broadcasting channel is independent of parameter θ in the first piece, that is, $\theta \in (0, \arctan(2^{-1/4})]$. We denote this channel as Ξ with Choi matrix $J_{AA_1A_2}$ (25).

We show the difference between fidelity of unilocal 2-broadcasting via channel Υ and Ξ , denoted as $f_{2,\Upsilon}(\psi_\theta)$, $f_{2,\Xi}(\psi_\theta)$ respectively, and the optimal unilocal 2-broadcasting fidelity $f_2(\psi_\theta)$ in the following Fig. 2.

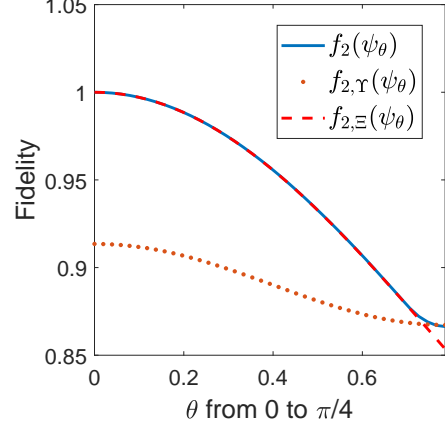


FIG. 2: The solid line depicts the optimal unilocal 2-broadcasting fidelity $f_2(\psi_\theta)$, the dashed line depicts the fidelity of unilocal 2-broadcasting via channel Ξ , $f_{2,\Xi}(\psi_\theta)$, which almost coincides with $f_2(\psi_\theta)$ except when θ is close to $\pi/4$, and the dotted line depicts the fidelity of unilocal 2-broadcasting via channel Υ , $f_{2,\Upsilon}(\psi_\theta)$.

V. APPROXIMATE BROADCASTING OF A SET OF QUANTUM STATES

A. Fidelity of broadcasting a set of quantum states

The no-go theorem for simultaneously broadcasting quantum states [3] says that we cannot perfectly broadcast two arbitrary noncommuting states. It is natural to ask how well we can do the task approximately. Generally, given m states ρ_i with respective prior probability p_i , how large average fidelity can we achieve when broadcasting these states via the same channel? Mathematically, assuming the given states ρ_i are on the system A , we study how to optimize the n -broadcasting fidelity $g_n(\eta)$ of an ensemble $\eta := \{p_i, \rho_i\}_{i=1}^m$, which is defined as

$$\begin{aligned} g_n(\eta) &:= \sup \sum_{i=1}^m \sum_{j=1}^n \frac{1}{n} p_i F(\rho_i, \widehat{\rho}_{ij}) \\ \text{s.t. } \widehat{\rho}_{ij} &= \text{Tr}_{A_j} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_i), \\ \Lambda &\text{ is a quantum channel.} \end{aligned} \quad (26)$$

Using the idea in the derivation of Eq. (10), namely, exploiting the symmetry in the broadcasting channel Λ , we can simplify this definition. The n -broadcasting fidelity g_n of an ensemble $\eta := \{p_i, \rho_i\}_{i=1}^m$ can be rewritten as

$$\begin{aligned} g_n(\eta) &= \sup \sum_{i=1}^m p_i F(\rho_i, \widehat{\rho}_{i1}) \\ \text{s.t. } \widehat{\rho}_{i1} &= \text{Tr}_{A_1} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_i), \\ \Lambda &\text{ is a symmetric broadcasting channel.} \end{aligned} \quad (27)$$

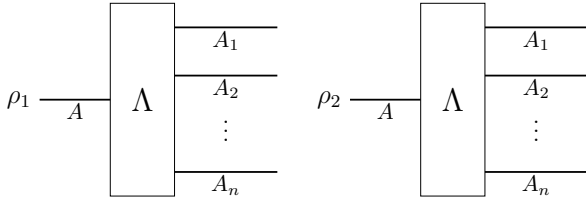


FIG. 3: Broadcasting states ρ_1, ρ_2 via the same channel Λ .

Theorem 12 *The n -broadcasting fidelity $g_n(\eta)$ of an ensemble $\eta = \{p_i, \rho_i\}_{i=1}^m$ is given by the optimal solution of the SDP in (29).*

Proof The output state on system A_1 of broadcasting ρ_i is

$$\hat{\rho}_{i1} = \text{Tr}_{\setminus A_1} \Lambda_{A \rightarrow A_1 \dots A_n}(\rho_i) = \text{Tr}_{\setminus A_1} (J_{AA_1 \dots A_n} \rho_i^T), \quad (28)$$

where $J_{AA_1 \dots A_n}$ is the Choi matrix of $\Lambda_{A \rightarrow A_1 \dots A_n}$.

By using the SDP characterization of fidelity function, we then have

$$\begin{aligned} g_n(\eta) = \max \quad & \sum_{i=1}^m \frac{1}{2} p_i \text{Tr}(X_i + X_i^\dagger) \\ \text{s.t.} \quad & \begin{pmatrix} \rho_i & X_i \\ X_i^\dagger & \text{Tr}_{\setminus A_1}(J_{AA_1 \dots A_n} \rho_i^T) \end{pmatrix} \geq 0, \forall i \in [m], \\ & J_{AA_1 \dots A_n} \geq 0, \text{Tr}_{\setminus A} J_{AA_1 \dots A_n} = \mathbb{1}_A, \\ & J_{AA_1 \dots A_n} = \frac{1}{n!} \sum_{\pi \in S_n} W_\pi J_{AA_1 \dots A_n} W_\pi^\dagger, \end{aligned} \quad (29)$$

where W_π acts on $A_1 \dots A_n$. \square

VI. CONCLUSIONS AND DISCUSSION

In summary, we have studied the approximate broadcasting of quantum correlations from several aspects. Firstly, we extend the operational characterization of one-sided quantum discord to two-sided one, that is, the asymptotic optimal average mutual information loss after the action of two local broadcasting channels is equal to the two-sided quantum discord. Then we give an alternate derivation for the SDP characterization of the unilocal broadcasting fidelity, based on which we analyze some properties of unilocal broadcasting. We show that the universal quantum clone machine (UQCM) is also the optimal universal unilocal broadcasting operation. Moreover, the optimal state-dependent unilocal broadcasting operation for pure two-qubit states is analytically solved. Finally, we also formulate the broadcasting of a finite set of quantum states as an SDP. It would be of interest to study other topics related to broadcasting and correlations, such as the broadcasting of Gaussian state and correlation, and the relation between Gaussian quantum broadcasting and Gaussian quantum discord. One can also study the asymptotic behavior of the n -broadcasting fidelity of a finite set of quantum states in the large n limit.

ACKNOWLEDGEMENTS

The authors are grateful to Kun Wang for useful discussion. This work was partly supported by the Australian Research Council under Grant Nos. DP120103776 and FT120100449.

-
- [1] W. K. Wootters and W. H. Zurek, *Nature* **299**, 802 (1982).
 - [2] D. Dieks, *Physics Letters A* **92**, 271 (1982).
 - [3] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, *Physical Review Letters* **76**, 2818 (1996).
 - [4] M. Piani, P. Horodecki, and R. Horodecki, *Physical Review Letters* **100**, 090502 (2008).
 - [5] S. Luo and W. Sun, *Physical Review A* **82**, 012338 (2010).
 - [6] S. Luo, *Letters in Mathematical Physics* **92**, 143 (2010).
 - [7] M. Piani, in *Lectures on General Quantum Correlations and their Applications* (Springer, 2017) pp. 201–216.
 - [8] M. Piani, *Physical Review Letters* **117**, 080401 (2016).
 - [9] S. Boyd and L. Vandenberghe, *Convex optimization* (Cambridge university press, 2004).
 - [10] A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, *Physical Review Letters* **88**, 187904 (2002).
 - [11] E. M. Rains, *IEEE Transactions on Information Theory* **47**, 2921 (2001).
 - [12] X. Wang and R. Duan, *Physical Review A* **94**, 050301 (2016).
 - [13] R. Jain, Z. Ji, S. Upadhyay, and J. Watrous, *Journal of the ACM (JACM)* **58**, 30 (2011).
 - [14] X. Wang and R. Duan, arXiv preprint arXiv:1606.09421 (2016).
 - [15] M. Berta and M. Tomamichel, *IEEE Transactions on Information Theory* **62**, 1758 (2016).
 - [16] Y. Li, X. Wang, and R. Duan, *Physical Review A* **95**, 052346 (2017).
 - [17] J. Kempe, O. Regev, and B. Toner, *SIAM Journal on Computing* **39**, 3207 (2010).
 - [18] X. Wang, W. Xie, and R. Duan, arXiv preprint arXiv:1610.06381 (2016).
 - [19] P. Skrzypczyk, M. Navascués, and D. Cavalcanti, *Physical Review Letters* **112**, 180404 (2014).
 - [20] M. Navascués, S. Pironio, and A. Acín, *New Journal of Physics* **10**, 073013 (2008).
 - [21] C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, and G. Adesso, *Physical Review Letters* **116**, 150502 (2016).
 - [22] H. Ollivier and W. H. Zurek, *Physical Review Letters* **88**, 017901 (2001).
 - [23] L. Henderson and V. Vedral, *Journal of Physics A: Mathematical and General* **34**, 6899 (2001).
 - [24] A. Datta, A. Shaji, and C. M. Caves, *Physical Review Letters* **100**, 050502 (2008).

- [25] F. G. Brandão, M. Piani, and P. Horodecki, *Nature Communications* **6** (2015).
- [26] E. Knill and R. Laflamme, *Physical Review Letters* **81**, 5672 (1998).
- [27] M. Piani, V. Narasimhachar, and J. Calsamiglia, *New Journal of Physics* **16**, 113001 (2014).
- [28] S. Boixo, L. Aolita, D. Cavalcanti, K. Modi, and A. Winter, *International Journal of Quantum Information* **9**, 1643 (2011).
- [29] T. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, and M. Piani, *Physical Review Letters* **109**, 070501 (2012).
- [30] A. Streltsov, H. Kampermann, and D. Bruß, *Physical Review Letters* **108**, 250501 (2012).
- [31] I. Devetak and A. Winter, *IEEE Transactions on Information Theory* **50**, 3183 (2004).
- [32] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, *Physical Review A* **83**, 032324 (2011).
- [33] V. Madhok and A. Datta, *Physical Review A* **83**, 032323 (2011).
- [34] V. Madhok and A. Datta, *International Journal of Modern Physics B* **27**, 1345041 (2013).
- [35] A. Streltsov, H. Kampermann, and D. Bruß, *Physical Review Letters* **106**, 160401 (2011).
- [36] D. Girolami, A. M. Souza, V. Giovannetti, T. Tufarelli, J. G. Filgueiras, R. S. Sarthour, D. O. Soares-Pinto, I. S. Oliveira, and G. Adesso, *Physical Review Letters* **112**, 210401 (2014).
- [37] S. Pirandola, *Scientific reports* **4** (2014).
- [38] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, *Reviews of Modern Physics* **84**, 1655 (2012).
- [39] G. Adesso, T. R. Bromley, and M. Cianciaruso, *Journal of Physics A: Mathematical and Theoretical* **49**, 473001 (2016).
- [40] A. Streltsov and W. H. Zurek, *Physical Review Letters* **111**, 040401 (2013).
- [41] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, 10th ed. (Cambridge University Press, New York, NY, USA, 2011).
- [42] A. Jamiolkowski, *Reports on Mathematical Physics* **3**, 275 (1972).
- [43] M.-D. Choi, *Linear Algebra and its Applications* **10**, 285 (1975).
- [44] N. Killoran, *Entanglement quantification and quantum benchmarking of optical communication devices*, Ph.D. thesis (2012).
- [45] J. Watrous, arXiv preprint arXiv:1207.5726 (2012).
- [46] M. Fannes, *Communications in Mathematical Physics* **31**, 291 (1973).
- [47] K. M. Audenaert, *Journal of Physics A: Mathematical and Theoretical* **40**, 8127 (2007).
- [48] R. Alicki and M. Fannes, *Journal of Physics A: Mathematical and General* **37**, L55 (2004).
- [49] A. Winter, *Communications in Mathematical Physics* **347**, 291 (2016).
- [50] J. Watrous, University of Waterloo Fall **128** (2011).
- [51] V. Bužek and M. Hillery, *Physical Review A* **54**, 1844 (1996).
- [52] V. Bužek and M. Hillery, *Physical Review Letters* **81**, 5003 (1998).
- [53] V. Scarani, S. Iblisdir, N. Gisin, and A. Acin, *Reviews of Modern Physics* **77**, 1225 (2005).
- [54] H. Fan, Y.-N. Wang, L. Jing, J.-D. Yue, H.-D. Shi, Y.-L. Zhang, and L.-Z. Mu, *Physics Reports* **544**, 241 (2014).