Three-phase Optimal Power Flow for Smart Grids by Iterative Nonsmooth Optimization

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Abstract: Optimal power flow is important for operation and planning of smart grids. The paper considers the so called unbalanced thee-phase optimal power flow problem (TOPF) for smart grids, which involves multiple quadratic equality and indefinite quadratic inequality constraints to model the bus interconnections, hardware capacity and balance between power demand and supply. The existing Newton search based or interior point algorithms are often trapped by a local optimum while semidefinite programming relaxation (SDR) even fails to locate a feasible point. Following our previously developed nonsmooth optimization approach, computational solution for TOPF is provided. Namely, an iterative procedure for generating a sequence of improved points that converges to an optimal solution, is developed. Simulations for TOPF in unbalanced distributed networks are provided to demonstrate the practicability and efficiency of our approach.

1 INTRODUCTION

Optimal power flow (OPF) for minimizing the cost of power generation subject to operating constraints and meeting demands provides one of the most important applications of smart grids (Farhangi, 2010).

There are two types of modeling distribution networks in smart grid: balanced equivalent single-phase modelling, which aims at naively approximating the network by a balanced system of three decoupled single-phase subsystems, and unbalanced three-phase modelling, which preserves the unbalanced structure of the network for constructive power flow analysis (Yang and Li, 2016). In recent years, more attentions have been paid to the unbalanced three-phase modelling (Kersting, 2007).

The single-phase OPF problem in balanced transmission networks has been more or less well studied (see e.g. (Lavaei and Low, 2012; Bukhsh et al., 2013; Madani et al., 2015)). However, the unbalanced three-phase optimal power flow problem (TOPF) in unbalanced networks is still left open with no available efficient computational solution due its nonlinearity.

The nonlinear power balance equality constraints of TOPF have been linearized in (Deshmukh et al., 2012) using the first-order Taylor expansion. As a result, its found solution is not necessarily feasible for TOPF. On the other hand, (Abdelaziz et al., 2013) proposed to combine Newton-Raphson method and trust region method to handle these nonlinear constraints, which may lead to a local optimum only. Furthermore, (Dall'Anese et al., 2013) employed semi-definite programming relaxation (SDR) to address the TOPF. Namely, TOPF is equivalently expressed by a convex semi-definite program (SDP) with the additional nonconvex matrix rank-one constraint. The latter is then dropped for SDR. It has been claimed in (Dall'Anese et al., 2013) that the optimal solution of SDR is always turned-out to be rank-one so it provides the global optimal solution of TOPF. However, our simulation will show that it is not quite the case, i.e. the optimal solution of SDR is turned out to be high rank and as such it cannot provide even a feasible point for TOPF.

In this paper, we follow the approach of (Phan et al., 2012; Shi et al., 2015) to provide computational solution for TOPF. Namely, we develop an efficient iterative procedure, which invokes a SDP in each iteration to generate a sequence of infeasible points, which quickly converges to the optimal solution of TOPF.

The paper is structured as follows. Section II is devoted to the TOPF model formulation for smart grids. Section III provides the equivalent matrix optimization formulation. A nonsmooth optimization algorithm for its solution is developed in Section IV. Section V provides simulation to show the efficiency of our methods. The conclusions are drawn in Section VI.

The notations used in this paper are standard. Particularly, j denotes the imaginary unit, $(X)^*$ means element wise complex conjugate operation of vector/matrix X, $M \succeq 0$ means the Hermitian symmetric matrix M is positive semi-definite, $\mathsf{rank}(M)$ and $Tr(M)$ are the rank and trace of matrix M, respectively; $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex quantity. $a \leq b$ for two complex numbers a and b is componentwise understood, i.e. $\Re(a) \leq \Re(b)$ and $\Im(a) \leq \Im(b)$.

2 TOPF STATEMENT

Consider a three-phase network with a set of nodes $\mathcal{N} := \{1, 2, \cdots, n\}$. The nodes are connected through a set of flow lines $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$, i.e. node k is connected to node m if and only if $(k,m) \in \mathcal{L}$. Accordingly, $\mathcal{N}(k)$ is the set of other nodes connected to node k. A subset $\mathcal{G} \subseteq \mathcal{N}$ of nodes is supposed to be connected to generators. Any node $k \in \mathcal{N} \setminus \mathcal{G}$ is thus not connected to generators. Denote by $\phi \in \{a,b,c\}$ the node phase. Accordingly V_k^ϕ and I_k^ϕ are the complex voltage and current at node k on phase ϕ.

Practically, all loads in smart grids are assumed constant, while the reactance between the neutral potentials and ground is assumed to be zero. Fig.1 depicts the $\pi-$equivalent model is used for this three-phase unbalanced network, which involves both self-impedance and mutual-impedance with other phase. Other forms of load models can be easily incorporated by introducing additional linear terms in the formulation.

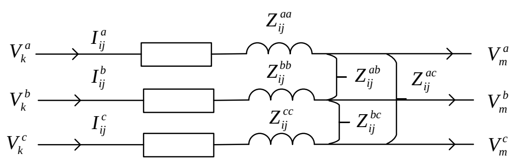

Figure 1: Three-phase distributed line model

Let $V_k = [V_k^a, V_k^b, V_k^c]^T$ be the three-phase complex voltage injected to node $k \in \mathcal{N}$, $I_{km} = [I_{km}^a, I_{km}^b, I_{km}^c]^T$ be the three-phase complex current in the power line $(k,m) \in \mathcal{L}$, and $y_{km} \in \mathbb{C}^{3\times 3}$ be three-phase admittance of line (k,m). Then,

$$\begin{bmatrix} I_{km}^a \\ I_{km}^b \\ I_{km}^c \end{bmatrix} = \begin{bmatrix} y_{km}^{aa} & y_{km}^{ab} & y_{km}^{ac} \\ y_{km}^{ba} & y_{km}^{bb} & y_{km}^{bc} \\ y_{km}^{ca} & y_{km}^{cb} & y_{km}^{cc} \end{bmatrix} \cdot \begin{bmatrix} V_k^a - V_m^a \\ V_k^b - V_m^a \\ V_k^c - V_m^a \end{bmatrix} \quad (1)$$

Other notations are:

- $S_{km} = [S_{km}^a, S_{km}^b, S_{km}^c]^T$ is three-phase apparent power transferred from node k to node m, $S_{km} = P_{km} + jQ_{km}$, where P_{km} and Q_{km} represent three-phase real and reactive line power, respectively;
- $S_{G_k} = [S_{G_k}^a, S_{G_k}^b, S_{G_k}^c]^T$ is three-phase apparent power injected by node $k \in \mathcal{G}$, $S_{G_k} = P_{G_k} + jQ_{G_k}$, where P_{G_k} and Q_{G_k} represent three-phase real and reactive generated power, respectively;
- $S_{L_k} = [S_{L_k}^a, S_{L_k}^b, S_{L_k}^c]^T$ is three-phase apparent power injected by node $k \in \mathcal{N} \setminus \mathcal{G}$, $S_{L_k} = P_{L_k} + jQ_{L_k}$, where P_{L_k} and Q_{L_k} represent three-phase real and reactive load power, respectively;

Let $[\cdot]_{diag}$ denote an operator that transport an $n \times 1$ vector to the diagonal of an $n \times n$ diagonal matrix. Then it is obvious that,

$$\begin{aligned} S_{G_k} - S_{L_k} &= P_{G_k} - P_{L_k} + j(Q_{G_k} - Q_{L_k}) \\ &= [V_k]_{diag} \sum_{m \in \mathcal{N}(k)} I_{km}^* \\ &= [V_k]_{diag} \sum_{m \in \mathcal{N}(k)} [y_{km}^*(V_k^* - V_m^*)]. \quad (2) \end{aligned}$$

Therefore, we can express the three-phase real generated power P_{G_k} and reactive generated power Q_{G_k} at node k as the following nonconvex quadratic functions of the node voltage V_k,

$$\begin{aligned} P_{G_k} &= P_{L_k} + \Re([V_k]_{diag} \sum_{m \in \mathcal{N}(k)} [y_{km}^*(V_k^* - V_m^*)]), \\ Q_{G_k} &= Q_{L_k} + \Im([V_k]_{diag} \sum_{m \in \mathcal{N}(k)} [y_{km}^*(V_k^* - V_m^*)]). \end{aligned} \quad (3)$$

The objective of TOPF is to minimize the following cost function of real active generated power P_G

$$f(P_G) = \sum_{k \in \mathcal{G}} \sum_{\phi \in \{a,b,c\}} (c_{k2}(P_{G_k}^\phi)^2 + c_{k1} P_{G_k}^\phi + c_{k0}), \quad (4)$$

where $(P_{G_k}^\phi)$ are the real generated power on phase ϕ, $\phi \in \{a,b,c\}$, $c_{k2} > 0, c_{k1}, c_{k0}$ are given. Substituting (3) in (4), the objective turns to be a function over bus voltages V:

$$\begin{aligned} f(V) \;=\; & \sum_{k \in \mathcal{G}} \sum_{\phi \in \{a,b,c\}} (c_{k2}(P_{L_k}^\phi + \Re([V_k]_{diag} \sum_{m \in \mathcal{N}(k)} \\ & [y_{km}^*(V_k^* - V_m^*)])^\phi)^2 + c_{k1}(P_{L_k}^\phi + \Re([V_k]_{diag} \\ & \sum_{m \in \mathcal{N}(k)} [y_{km}^*(V_k^* - V_m^*)])^\phi) + c_{k0}). \quad (5) \end{aligned}$$

Accordingly, TOPF problem is formulated as

$$\min_{V\in\mathbb{C}^n} f(V) \quad \text{s.t.} \tag{6a}$$

$$-P_{L_k} - jQ_{L_k} = [V_k]_{diag} \sum_{m\in\mathcal{N}(k)} [y^*_{km}(V^*_k - V^*_m)], \forall k \in \mathcal{N}\setminus\mathcal{G} \tag{6b}$$

$$P^{min}_{G_k} \le P_{L_k} + \Re([V_k]_{diag} \sum_{m\in\mathcal{N}(k)} [y^*_{km}(V^*_k - V^*_m)]) \le P^{max}_{G_k}, \forall k \in \mathcal{G} \tag{6c}$$

$$Q^{min}_{G_k} \le Q^{\phi}_{L_k} + \Im([V_k]_{diag} \sum_{m\in\mathcal{N}(k)} [y^*_{km}(V^*_k - V^*_m)]) \le Q^{max}_{G_k}, \forall k \in \mathcal{G} \tag{6d}$$

$$(V^{\phi}_k)^{min} \le |V^{\phi}_k| \le (V^{\phi}_k)^{max}, \forall k \in \mathcal{N} \tag{6e}$$

$$|S_{km}| = |[V_k]_{diag}[y^*_{km}(V^*_k - V^*_m)]| \le S^{max}_{km}, \forall (k,m) \in \mathcal{L} \tag{6f}$$

$$|V^{\phi}_k - V^{\phi}_m| \le (V^{\phi}_{km})^{max}, \forall (k,m) \in \mathcal{L} \tag{6g}$$

$$\forall \phi \in \{a,b,c\}.$$

where

- (6b) is the equation of the balance between the demand and supply power at the load node $k \in \mathcal{N}\setminus\mathcal{G}$;
- (6c)-(6d) are the power generation bounds, where $(P^{\phi}_{G_k})^{min}$, $(Q^{\phi}_{G_k})^{min}$ and $(P^{\phi}_{G_k})^{max}$, $(Q^{\phi}_{G_k})^{max}$ are the lower bound and upper bound of the real power reactive power generations on phase ϕ, respectively;
- (6e) are the voltage amplitude bounds;
- (6f)-(6g) are capacity limitations, where line currents between the connected nodes are constrained by (6f), while (6g) guarantees the voltage difference in terms of their magnitude (Zimmerman et al., 2011);

3 MATRIX RANK-ONE CONSTRAINED OPTIMIZATION FOR TOPF

Define

$$V := [V_1^T, \cdots, V_n^T]^T \in \mathbb{C}^{3n}$$

and

$$I := [I_1^T, \cdots, I_n^T]^T \in \mathbb{C}^{3n},$$

where V_n and I_n are the complex three-phase voltage and current respectively. Define a symmetric block matrix $Y \in \mathbb{C}^{3n\times 3n}$, with diagonal block $\sum_{m\in\mathcal{N}(k)} y_{km}$ and off-diagonal block $-y_{km}$. Set $y_{km} = 0$ if node k and m are not connected. The Ohm's law is written as

$$I = YV.$$

The voltage inserted at node k of phase ϕ can be expressed by

$$V^{\phi}_k = (e^{\phi}_k)^T V, \quad \phi \in a,b,c \tag{7}$$

where $e^{\phi}_k = [0_{1\times 3(k-1)}, \vec{e}^{\phi}_k, 0_{1\times 3(n-k)}]^T$, $\vec{e}^{\phi}$ denotes the canonical basis of $\mathbb{R}^3$.

Under the definition of the outer product matrix $W = VV^H$, for each phase ϕ, constraint (6b) becomes a linear function of W as follows,

$$\begin{aligned} -P^{\phi}_{L_k} - jQ^{\phi}_{L_k} &= V^{\phi}_k (I^{\phi}_k)^* \\ &= (V^H e^{\phi}_k (e^{\phi}_k)^T YV)^H \\ &= Tr(Y^{\phi}_k W), \end{aligned} \tag{8}$$

where $Y^{\phi}_k = e^{\phi}_k (e^{\phi}_k)^T Y$.

Similarly, the injected real and reactive powers corresponding to constraint (6c)and (6d) can be expressed by the following linear constraints in W:

$$P^{\phi}_{L_k} + \Re(V^{\phi}_k \sum_{m\in\mathcal{N}(k)} [y^*_{km}(V^*_k - V^*_m)]^{\phi}) = P^{\phi}_{L_k} + Tr(1/2(Y^{\phi}_k + (Y^{\phi}_k)^H)W) \tag{9}$$

$$Q^{\phi}_{L_k} + \Im(V^{\phi}_k \sum_{m\in\mathcal{N}(k)} [y^*_{km}(V^*_k - V^*_m)]^{\phi}) = Q^{\phi}_{L_k} + Tr(j/2(Y^{\phi}_k - (Y^{\phi}_k)^H)W) \tag{10}$$

Constraint (6e) is also linear in W because

$$\begin{aligned} |V^{\phi}_k|^2 &= (V^{\phi}_k)^* V^{\phi}_k \\ &= V^H e^{\phi}_k (e^{\phi}_k)^T V \\ &= Tr(e^{\phi}_k (e^{\phi}_k)^T W) \end{aligned} \tag{11}$$

Next, define complex matrix A_{km} and B_k as,

$$\begin{aligned} A_{km} : &= [0_{3\times 3(k-1)}, y_{km}, 0_{3\times 3(m-k-1)}, \\ & -y_{km}, 0_{3\times 3(n-m)}]_{3\times n} \\ B_k : &= [0_{3\times 3(k-1)}, 1_{3\times 3}, 0_{3\times 3(n-k)}]_{3\times n}. \end{aligned}$$

Then, it is obvious that $I^{\phi}_{km} = (y_{km}(V_k - V_m))^T \vec{e}^{\phi} = (A_{km}V)^T \vec{e}^{\phi}, V^{\phi}_k = (B_k V)^T \vec{e}^{\phi}$, $\vec{e}^{\phi}$ denotes the canonical base of $\mathbb{R}^3$, thus, $S^{\phi}_{km} = V^{\phi}_k (I^{\phi}_{km})^* = V^H B_k \vec{e}^{\phi} (\vec{e}^{\phi})^T A_{km} V = Tr(B_k \vec{e}^{\phi} (\vec{e}^{\phi})^T A_{km} W)$. Therefore, the line flow constraint (6f) can be re-expressed by

$$|S^{\phi}_{km}| = |Tr(B_k \vec{e}^{\phi} (\vec{e}^{\phi})^T A_{km} W)| \le (S_{km})^{max}, \forall (k,m) \in \mathcal{L} \tag{12}$$

Similarly, the line flow constraint (6g) can be re-expressed by

$$\begin{aligned} |V^{\phi}_k - V^{\phi}_m|^2 &= V^H (B_k - B_m)^H \vec{e}^{\phi} (\vec{e}^{\phi})^T (B_k - B_m) V \\ &= Tr((B_k - B_m)^H \vec{e}^{\phi} (\vec{e}^{\phi})^T (B_k - B_m) W) \\ &\le (V^{\phi}_{km})^{max}, \forall (k,m) \in \mathcal{L} \end{aligned} \tag{13}$$

In summary, TOPF (6) is reformulated by the following optimization problem in matrix $W \in \mathbb{C}^{3n\times 3n}$,

$$\min_{W\in\mathbb{C}^{3n\times 3n}} F(W) \quad \text{s.t. (14a)}$$

$$-P_{L_k} - jQ_{L_k} = Tr(Y_k^{\phi} W), \forall k \in \mathcal{N}\setminus\mathcal{G}, \text{(14b)}$$

$$(P_{G_k}^{\phi})^{min} \le P_{L_k}^{\phi} + Tr(1/2(Y_k^{\phi} + (Y_k^{\phi})^H)W) \le (P_{G_k}^{\phi})^{max}, \forall k \in \mathcal{G} \text{ (14c)}$$

$$Q_{G_k}^{min} \le Q_{L_k}^{\phi} + Tr(j/2(Y_k^{\phi} - (Y_k^{\phi})^H)W) \le Q_{G_k}^{max}, \forall k \in \mathcal{G} \text{ (14d)}$$

$$(V_k^{min})^2 \le Tr(\bar{e}_k^{\phi}(\bar{e}_k^{\phi})^T W) \le (V_k^{max})^2, \forall k \in \mathcal{N} \text{ (14e)}$$

$$|Tr(B_k e^{\phi}(e^{\phi})^T A_{km} W)| \le (S_{km})^{max}, \forall (k,m) \in \mathcal{L} \text{ (14f)}$$

$$Tr((B_k - B_m)^H \bar{e}^{\phi}(\bar{e}^{\phi})^T (B_k - B_m) W) \le (V_{km}^{max})^2, \forall (k,m) \in \mathcal{L} \text{ (14g)}$$

$$W \succeq 0, \text{(14h)}$$

$$\mathsf{rank}(W) = 1, \text{ (14i)}$$

where

$$F(W) = \sum_{k\in\mathcal{G}} [c_{k2}(P_{L_k}^{\phi} + Tr(1/2(Y_k^{\phi} + (Y_k^{\phi})^H)W))^2 + c_{k1}(P_{L_k}^{\phi} + Tr(1/2(Y_k^{\phi} + (Y_k^{\phi})^H)W)) + c_{k0}], \quad (15)$$

which is convex quadratic in W.

As all constraints (14b)-(14h) are linear, the difficulty of (14) is now concentrated at the nonconvex matrix rank-one constraint (14i). The existing SDRs, such as (Lavaei and Low, 2012) and (Dall'Anese et al., 2013) simply drop the only nonconvex constraint (14i) to have the SDP (14a)-(14h). If the optimal solution of this SDR is of rank-one, i.e. it satisfies the nonconvex rank-one constraint (14i) then it obviously leads to the global optimal solution of the nonconvex program (14). Otherwise, SDR cannot provide even feasible point to the original TOPF (6). In the next section we will provide an efficient computational nonsmooth algorithm for the optimal solution of the nonconvex problem (14).

4 NONSMOOTH OPTIMIZATION ALGORITHM FOR TOPF

In this section, followed by our previous work (Shi et al., 2015), a nonsmooth optimization algorithm is proposed to deal with the nonconvex rank-one constraint (14i) in program (14). Firstly, the rank-one constraint (14i) is equivalently expressed by the following spectral constraint

$$\mathsf{Trace}(W) - \lambda_{\max}(W) \le 0, \quad (16)$$

where $\lambda_{\max}(W)$ stands for the maximal eigenvalue of W.

Instead of dealing with the nonconvex constraint (16), we incorporate it into the objective, leading to the following formulation

$$\min_{W\in\mathbb{C}^{3n\times 3n}} F(W) + \mu(\mathsf{Trace}(W) - \lambda_{\max}(W)) \quad \text{s.t.} \quad (14b)-(14h), \quad (17)$$

where $\mu > 0$ is a penalty parameter. The above penalization is exact because the constraint (16) can be satisfied by a minimizer of (14) with a finite value of μ. On the other hand, any feasible W to (14) is also feasible to (17), implying that the optimal value of (14) for any $\mu > 0$ is upper bounded by the optimal value of (17).

Function $\lambda_{\max}(W)$ is nonsmooth but is lower bounded by a linear function as the following relation shows (Tuan et al., 2000):

$$\begin{aligned}\lambda_{\max}(W) &\ge \lambda_{\max}(W^{(\kappa)}) + (w_{\max}^{(\kappa)})^H (W - W^{(\kappa)}) w_{\max}^{(\kappa)} \\ &= (w_{\max}^{(\kappa)})^H W w_{\max}^{(\kappa)}, \quad \forall W \succeq 0. \end{aligned} \quad (18)$$

Here, $w_{\max}^{(\kappa)}$ is the eigenvector corresponding to the eigenvalue $\lambda_{\max}(W^{(\kappa)})$.

Therefore, for any $W^{(\kappa)}$ feasible to convex constraints (14b)-(14h), the following SDP yields an upper bound for nonconvex program (17)

$$\min_{W\in C^{3n\times 3n}} F^{(\kappa)}(W) := F(W) + \mu[\mathsf{Trace}(W) - (w_{\max}^{(\kappa)})^H W w_{\max}^{(\kappa)}] \quad \text{s.t.} \quad (14b)-(14h) \quad (19)$$

because

$$F^{(\kappa)}(W) \ge F(W) + \mu(\mathsf{Trace}(W) - \lambda_{\max}(W)) \quad \forall W \succeq 0$$

according to (18).

By Algorithm 1, we provide an iterative computational procedure for computing (14). Its initial step is to solve SDP (20), which is a SDR for (14) so its optimal value is a lower bound for (14).

Suppose that $W^{(\kappa+1)}$ is the optimal solution of SDP (19). Since $W^{(\kappa)}$ is also feasible to (19), it is true that

$$\begin{aligned} F(W^{(\kappa)}) + \mu(\mathsf{Trace}(W^{(\kappa)}) - \lambda_{\max}(W^{(\kappa)})) &= \\ F^{(\kappa)}(W^{(\kappa)}) &\ge \\ F^{(\kappa)}(W^{(\kappa+1)}) &= \\ F(W^{(\kappa+1)}) + \mu(\mathsf{Trace}(W^{(\kappa+1)}) - \lambda_{\max}(W^{(\kappa+1)})), \end{aligned}$$

so $W^{(\kappa+1)}$ is better solution of (17) than $W^{(\kappa)}$.

5 SIMULATION RESULTS

The hardware and software facilities for our computational implementation are:

Algorithm 1 Nonsmooth Optimization Algorithm for the unbalanced TOPF problem

1: Initialize $\kappa := 0$ and solve the SDP

$$\min_{W \in \mathbb{C}^{3n\times 3n}} F(W) \quad \text{s.t.} \quad (14b)-(14h) \tag{20}$$

to find its optimal solution $W^{(\kappa)}$. Stop the algorithm if

$$\text{Trace}(W^{(\kappa)}) - (w_{\max}^{(\kappa)})^H W^{(\kappa)} w_{\max}^{(\kappa)} \le \varepsilon \tag{21}$$

and accept $W^{(\kappa)}$ as the optimal solution of the nonconvex program (6).

2: **repeat**

3: Solve the convex program (19), to find the optimal solution $W^{(\kappa+1)}$

4: Set $\kappa := \kappa + 1$.

5: **until**

$$\text{Trace}(W^{(\kappa)}) - (w_{\max}^{(\kappa)})^H W^{(\kappa)} w_{\max}^{(\kappa)} \le \varepsilon. \tag{22}$$

6: Accept $W^{(\kappa)}$ as a found solution of (6).

- Processor: Intel(R) Core(TM) i5-3470 CPU @3.20GHz;
- Software and toolbox : Matlab version R2015b; CVX (Grant and Boyd, 2014) with Sedumi (Sturm, 1999) to solve SDP (19).
- tolerance: $\varepsilon = 10^{-4}$ is set for the stop criterion (22) of Algorithm 1, which is applied to solutions of all cases.

To demonstrate the efficiency of our nonsmooth optimization algorithm, the following two cases are tested.

5.1 Six-node Network

This six-node three-phase network is a modification from the unbalanced network from (Sanseverino et al., 2015), which is depicted by Fig. 2. There are six nodes with three distributed generators and five lines, which lead to three nonlinear equality constraint in (6b). The size of the Hermitian symmetric matrix variable W is 18×18. The coefficients of the power cost are set by $c_{k2} = 0$, $c_{k1} = 4$ and $c_{k0} = 10$ for each node and phase, respectively. The minimum and maximum capacity of service voltage are set by $V_k^{min} = 0.95pu$, $V_k^{min} = 1.05pu$ for all nodes. The initial iteration of Algorithm 1 found $\text{rank}(W^{(0)}) = 8$ with power cost 1086 (\$/h), which is only a lower bound of TOPF (14). SDR thus can not lead to feasible point for the original TOPF (6). After 5 iterations, Algorithm 1 yields a rank-one solution with the power cost 1125 (\$/h), with a 3.5% increase compared to the lower bound 1086 (\$/h).

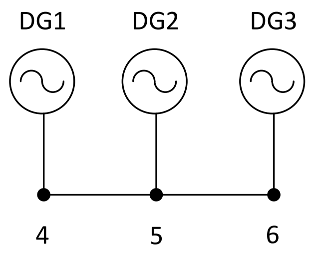

Figure 2: Topology of the 6-node three-phase network

5.2 Ten-node Network

This ten-node three-phase network is a modification of the unbalanced network modified from (Dall'Anese et al., 2013), which is depicted by Fig. 3. There are ten nodes with two distributed generators and nine lines, which lead to eight nonlinear equality constraint in (6b). The size of the Hermitian symmetric matrix variable W is 30×30. The coefficients of the power cost are set by $c_{k2} = 0$, $c_{k1} = 6$ and $c_{k0} = 30$ for each node and phase, respectively. The minimum and maximum capacity of service voltage are set by $V_k^{min} = 0.95pu$, $V_k^{min} = 1.05pu$ for all nodes. The initial iteration of Algorithm 1 found $\text{rank}(W^{(0)}) = 12$ with power cost 1573 (\$/h), which is only a lower bound of TOPF (14). Again SDR can not find even a feasible point for original TOPF (6). After ten iterations, the nonsmooth optimization Algorithm 1 yields a rank-one matrix solution with the power cost 1652 (\$/h), which is a 5.0% increase compared to the lower bound 1573 (\$/h).

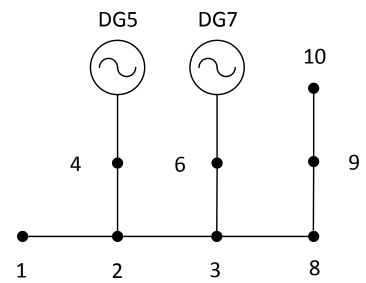

Figure 3: Topology of the 10-node three-phase network

6 CONCLUSIONS

TOPF is a very computationally difficult problem as it involves multiple quadratic equality and indefinite quadratic inequality constraints of the bus interconnections, hardware operating capacity and balance between power demand and supply. We have proposed an iterative nonsmooth algorithm for its computational solution. The provided simulations demonstrate its merit. Its applications to larger scale TOPFs are currently under consideration.

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