Application of recurrence plot quantification to mineralising systems in geology

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Outline

- General motivation
- Models and experimental data
  - Benchmark models
    - Hénon map
    - Gray-Scott reaction diffusion
  - Experimental Pilot: Drill core data (minerals)
- Results
- Conclusions
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General Motivation

• About 40% of exploration projects are concerned with finding new profitable ore deposits.
• Exploration of economically significant gold deposits becomes increasingly expensive.
• Huge pressure on exploration companies.
• Traditionally models vs. nonlinear thinking.

Question: Can we detect stable/unstable periodic orbits using RPQA?

1. Non-stationary (spatio-temporal) data
2. Effect of noise and...
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- Question: Can we detect stable/unstable periodic orbits using RPQA?\(^1\)
  1. Non-stationary (spatio-temporal) data
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\(^1\) Bradley, E. and Mantilla, R., Recurrence plots and unstable periodic orbits, Chaos, 12: 596-600, 2003
Methods used - Power spectra over recurrences; return time probability

- Wiener-Khinchin theorem \(^2\)
  \[ S_x(\omega) = \frac{1}{N} \left| \sum_{i=0}^{N-1} x(i)e^{-j\omega i} \right|^2 = \sum_{\tau=-\infty}^{\infty} C_x(\tau)e^{-j\omega\tau} \]

- Auto-covariance of \(x(n)\) (with zero mean)
  \[ C_x(\tau) = \frac{1}{N} \sum_{i=0}^{N-1-\tau} x(i)x^*(i+\tau) = \frac{m}{N} \sum_{i=0}^{N-1-\tau} x(i)x(i+\tau) \]

- The RR = average of all recurrences f(distance matrix D)
  \[ D(i, j) = ||x(i) - x(j)||, \text{ with } x \in \mathbb{R}^m \]

- Average distance
  \[ d(\tau) = \frac{1}{N} \sum_i D(i, i + \tau) \]

- Relation between \(D\) and \(C_x\)
  \[ C_x(\tau) = \sigma^2 - \frac{d^2(\tau)}{2} \]

Systems analysed

- Hénon map\(^3\)
  \[X_{i+1} = 1 - aX_i^2 + Y_i\]
  \[Y_{i+1} = bX_i\]

- Gray-Scott reaction-diffusion
  \[U + 2V \rightarrow 3V\]
  \[V \rightarrow P\]
  \[\frac{\partial u}{\partial t} = ru \nabla^2 u - uv^2 + f(1 - u)\]
  \[\frac{\partial v}{\partial t} = rv \nabla^2 v - uv^2 - (f - k)v\]

- Experimental data: Amphibole, Chlorite, Sericite

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\(^3\) Oberst S, Marburg S, Hoffmann N, Determining periodic orbits via nonlinear filtering and recurrence spectra in the presence of noise, EuroDyn 2017, 10 – 13 Sep, Rome, Italy.
Observation function

(1) Additive noise applied to Hénon map $^3$

$$\bar{X} = X + p \cdot wn \circ 1$$

(2) Multiplicative noise applied to Gray-Scott model

$$\bar{X} = X + p \cdot wn \circ X$$

$^3$Oberst S, Marburg S, Hoffmann N, Determining periodic orbits via nonlinear filtering and recurrence spectra in the presence of noise, EuroDyn 2017, 10 – 13 Sep, Rome, Italy.
Nonlinear Filtering *ghkss* (TISEAN) $^4,5$

- Locally projective noise reduction scheme: For each embedding vector exists a small correction so that $s_n - \Theta_n \in \mathcal{M}$ with a correction being orthogonal to a low dimensional manifold.

- At end-pieces ‘diverge’ also a result of the dynamics.

- Correction of centre part of delay vectors; the end pieces are left unchanged (here the influence of the negative and the positive Lyapunov exponents is the largest).

- Mostly it is sufficient to fix only the first and the last component of a delay vector (metric tensor $P_{ij} = \{0,1\}$).

- Minimisation problem

$$\sum (\Theta_i P^{-1} \Theta_i) \rightarrow min$$

With constraints

$$a_n^i (s_n - \Theta_n) + b_n^i = 0$$

And

$$a_n^i P a_n^i = \delta_{ij}$$

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$^4$ Hegger R, Kantz H, Schreiber T, Practical implementation of nonlinear time series methods: The TISEAN package, CHAOS, 9:413, 1999

Results ‘Hénon map’
Results: ‘Gray-Scott reaction diffusion’ (1/4)
Results: ‘Gray-Scott reaction diffusion’ (2/4)
Results: ‘Gray-Scott reaction diffusion’ (2/4)
Results: ‘Gray-Scott reaction diffusion’ (3/4)
Results: ‘Gray-Scott reaction diffusion’ (2/4)
Results: ‘Drill Core Data’ (1/3)
Results ‘Drill Core Data’ (2/3)

\[ m=5, \ d=1, \ \text{fan-norm} \]
Results ‘Drill Core Data’ (3/3)

Amphibole

Carbonate

Sericite
Summary & Conclusions

- The application of recurrence plots is here conducted for the first time to mineralising data; we assumed that a mineralising system behaves similar to a *flow-driven hydrothermal chemical reactor* with dynamics far from equilibrium.

- The Hénon map as a discrete nonlinear dynamical – *stationary* - system and the Gray-Scott reaction diffusion equation as continuous spatio-temporal nonlinear dynamical- *non-stationary*, system have been used as benchmarks.

- We considered additive noise (Hénon) and multiplicative noise (Gray-Scott) to form the observer function, nonlinear filtering worked fine with those noisy systems (see also Oberst et al. 2017, EuroDyn, Rome, Italy).

- The power spectral density estimates based on recurrence measures perform better than a classical Welch spectrum: In particular for short time series the resolution is better, for non-stationary data (GSM), the RP based power spectra perform also better.

- Future work aims at taking into account a higher sampling rate and more minerals as well as chemical (digestive) data.
Thank you for your attention

Fordite, Michigan (US) 😆
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