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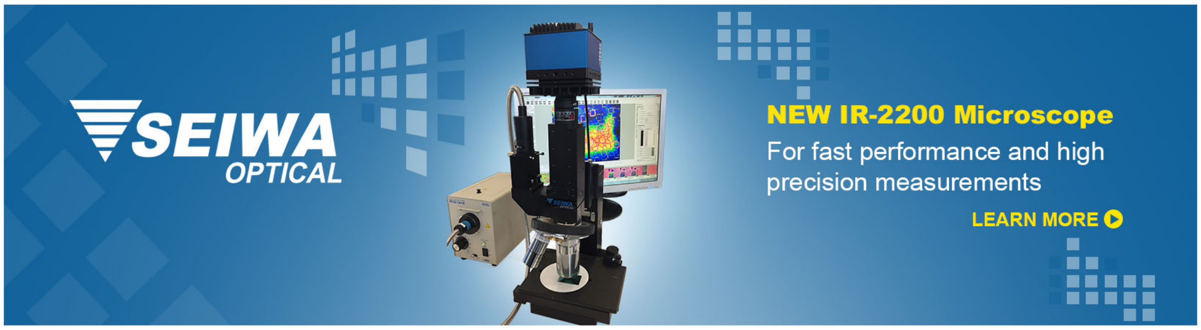

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Broadband isotropic μ-near-zero metamaterials

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Natural diamagnetism, while being a common phenomenon, is limited to permeability values close to unity. Artificial diamagnetics, to the contrary, can be engineered to provide much lower values and may even possess an effective permeability close to zero. In this letter, we provide an experimental confirmation of the possibility to obtain extremely low permeability values by manufacturing an isotropic metamaterial composed of conducting cubes. We show that the practical assembly is quite sensitive to fabrication tolerances and demonstrate that permeability of about $\mu = 0.15$ is realisable. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4832056]

An overwhelming flow of metamaterials research have had significant influence on a number of topics, many of which have led far from the initial flagship implications of metamaterials, such as negative refraction and perfect lens. On this track, research on various periodic structures thrived in a new turn of development. Indeed, not only resonant phenomena can benefit from one of the key highlights of metamaterials: collective response of artificial structural units. These days, most of the artificial dielectrics and diamagnetics, known in electrical engineering for ages,[1] would have been called metamaterials. New knowledge and approaches, however, brought even these simple concepts to a spectacular level of implementation and performance, typically exceeding earlier expectations.

In particular, *diamagnetism* of artificial media which has been extensively studied early[2,3] enjoyed a fresh attention recently.[4] Alike the fruitful application of wire media for artificial dielectrics,[5] we expect metamaterials to overcome the major limitation of natural diamagnetism suffering from permeability values close to unity and to create strongly diamagnetic materials with arbitrary permeability in the range from 0 to 1. Reported designs of diamagnetic metamaterials usually use either split-ring resonators above their resonance[6] or superconductors[7,8] for extremely low frequency operation. The latter topic was developed further towards experimental assessment[9,10] and was mostly driven by the search for high-index materials.[4]

The most straightforward structural element to create artificial diamagnetics is as simple as a closed conducting loop.[1] Preliminary studies on an array of such loops[11] indicated that a remarkable diamagnetism may be obtained. It was then natural to expect that metamaterials, with their dramatic sensitivity to the lattice parameters,[12] would offer a dramatic enhancement. Indeed, a detailed analysis and parametric study[13] revealed that, by choosing appropriate lattice constants and geometry, very strong artificial diamagnetism can be obtained in anisotropic arrays, with effective permeability values below 0.1 in a wide frequency range.

Unfortunately, closed loops, being so efficient for anisotropic response, are not suitable for constructing a *broadband* strong *isotropic diamagnetic*. Even if the closed loops are arranged into a fully isotropic system,[14] the required density along all the three orthogonal directions cannot be achieved due to obvious geometric constraints. We therefore seek an alternative structure, which is readily available as a lattice of conducting cubes (see Fig. 1). Alike those made of a superconductor,[7] simple metallic cubes will also exhibit an isotropic diamagnetic response above certain frequency threshold, analogous to what is observed in anisotropic system.[13] Note, there is no contradiction in having wideband low-frequency diamagnetism as it was recently shown in Refs. 15 and 16.

Realising an isotropic material with very small permeability values is an exciting possibility to implement μ-near-zero (MNZ) materials,[17] which were not yet reported in practice. These materials are magnetic analogues of ε-near-zero (ENZ) materials which attracted a lot of attention during last decades due to their extraordinary properties including capability to tunnel electromagnetic energy through subwavelength channels and bends,[18–20] matching capabilities,[21] and tailoring the radiation patterns.[22,23] The MNZ media[17] would demonstrate properties as exciting as the prominent ENZ materials and may also be employed for magnetic levitation.[24,25]

In theory,[26] effective permeability of an extremely dense pack of perfectly conducting cubes can be brought arbitrarily close to zero, though at a cost of an infinitely large permittivity, like it is the case for bulk metals. In practice, however, the response is complicated with finite conductivity, imperfections of the cube surface and the gaps between the cubes, and possible inhomogeneity of their arrangement.

The purpose of this letter is to provide a practical evaluation of the diamagnetic system of metallic cubes and to probe the frequency limitations and lowest magnitudes, likely to be feasible in practice. Below, we present the results of numerical simulations and experiments over an isotropic metamaterial composed as a lattice of metallic cubes. At the

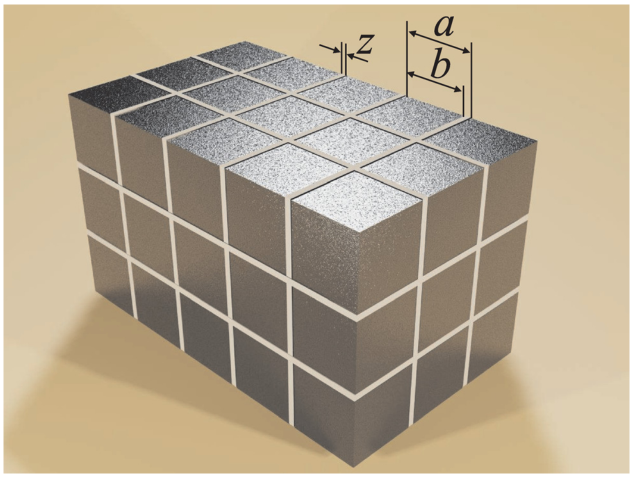

FIG. 1. A μ-near-zero metamaterial composed of metallic cubes. Three layers are shown; b is the cube size, z is the dielectric gap width, and $a = b + z$ is the unit cell size.

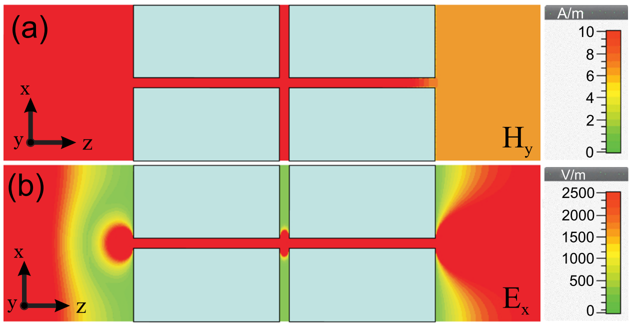

FIG. 2. Numerically simulated magnetic (a) and electric (b) fields distributions in the metamaterial layer containing 2 layers of cubes in the direction of propagation (left to right in these figures). Each plot shows one unit cell in the transverse direction, with a cut through the middle of the cubes.

same time, the system of cubes has a clear advantage over bulk metal pieces, as the effective permittivity can be, in principle, tailored to a desired value somewhat independently.

We consider the cubes of a size b to be separated with gaps of width z (see Fig. 1), so that the lattice constant of the array is $a = b + z$. Permeability of an infinitely large array of this kind can be estimated analytically[7,26] using the equations

$$n = \sqrt{2\varepsilon_h}, \quad Z = \sqrt{\frac{2}{\varepsilon_h}}\left(1 - \frac{b}{a}\right),$$

$$\mu = 1 - \frac{b^2}{a^2}, \quad \varepsilon = \frac{\varepsilon_h}{1 - b/a}, \tag{1}$$

where ε_h is the permittivity of the host medium.

A popular way[27] to determine the effective parameters of metamaterial samples is to measure the complex S-parameters in a waveguide and use the Nicolson-Ross-Weir (NRW) technique afterwards.[28–32] Although, in general, effective parameter retrieval from the scattering parameters is not reliable, especially for resonant structures (see Ref. 33 for an exhaustive treatment), this method may provide suitable results for relatively simple, highly symmetric, and non-resonant structures.[34,35]

To estimate the material parameters of the metamaterial composed as a lattice of metallic cubes, we first obtain the S-parameters numerically by employing CST Microwave Studio commercial software. In our simulations, a unit cell of metamaterial is set up with periodic boundary conditions in the directions, transverse to the wave propagation. Metamaterial samples containing of 1, 2, and 4 layers of cubes (in the direction of propagation) have been studied. In the simulation, we have set the cube size as $b = 1.5$ mm, cube material as aluminium with conductivity $\sigma = 3.56 \times 10^{13}$ S/m, and 0.1 mm gaps were filled with dielectric having permittivity $\varepsilon = 4 + 0.1\mathrm{i}$. The unit cell size $a = 1.6$ mm is about $\lambda_0/20$ with subwavelength enough to be considered in quasistatic approximation.[36] Typical magnetic and electric fields distributions are shown in Fig. 2 for the slab of metamaterial consisting of 2 layers of cubes in the direction of propagation. It is clearly visible that the electric field is strongly localised in the gaps between the cubes in the direction of propagation, whereas the magnetic field uniformly fills the gaps without any enhancement.

From the simulated S-parameters, we extract the effective materials parameters using the NRW technique. The results are plotted in Fig. 3. In the frequency range 6–12 GHz, the real part and imaginary part of the refractive index were about 2.8 and 0.09, respectively. The real part of the permittivity changes from 50 to 55, while the imaginary part is less than 5 in the mentioned frequency range. The real part of the permeability of about 0.15 with the imaginary part close to 0.1 was obtained. In general, the value of the permeability of 0.15 obtained during the simulations is really close to the value 0.121, estimated analytically using Eq. (1) with the corresponding parameters.

We note that the resonant appearance of permeability retrieved from the scattering parameters can occur due to numerous factors, including the effects of spatial dispersion,[37,38] which are not relevant in our case, as well as the effects of the finite size of the sample, which imposes Fabry-Perot resonance (half-wavelength) or leads to remarkable boundary effects.[39–41] In our case, the anti-resonance of metamaterial composed of the 4 layers is observed and can be explained by the presence of Fabry-Perot resonance. Such resonances can be corrected by the refined retrival techniques.[39,40,42]

To confirm the numerical results, several metamaterial samples with 1, 2, and 4 layers have been fabricated. The cubes of size 1.6 mm have been made from aluminium to provide a thin gap between the cubes, and, at the same time, to fix the cubes together, we have used a glue. The measurements of the thin layers of the glue have shown that it has the permittivity $\varepsilon = 3.9 + 0.3\mathrm{i}$ in the frequency range relevant for our results. The gaps between cubes were then found to be in the range of 0.1–0.2 mm. An array of 11×5 cubes has been tightly fit into WR-90 waveguide ($22.86 \times 10.16\,\mathrm{mm}^2$ cross-section, operating in the X-band, 8.5–12 GHz). The S-parameters have been measured using an E8362C PNA Vector Network Analyser.

First, we measure the S-parameters of the samples with 1 layer in the direction of propagation and use the NRW extraction procedure to retrieve the material parameters. The

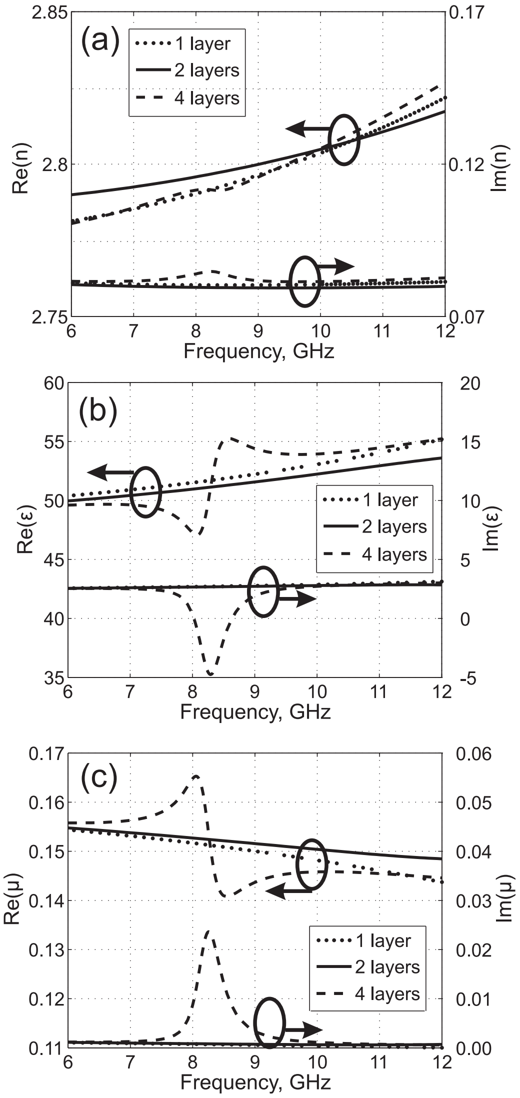

FIG. 3. Refractive index (a), permittivity (b), and permeability (c) extracted from the numerical results for the structures having 1, 2, or 4 layers of cubes in the direction of propagation.

obtained values of the refractive index, permittivity, and permeability as the functions of frequency are shown in Figs. 4(a)–4(c), respectively. We present the mean values averaged over 8 measurements (4 samples with two orientations for each sample), with the deviation as shown by error-bars. The mean value of the refractive index yields about 3 for the real part and about 0.5 for the imaginary part within the frequency range of 8.5–12 GHz. The permittivity then is found to vary from 60 to 50 with the imaginary part around 30. The high value of the imaginary part can be explained by strong dissipation in the glue as well as mainly due to sample roughness which was not taken into account in the simulation. The permeability is found to have a real part about 0.15 with the imaginary part close to 0.01.

To check the convergence of the extracted parameters, metamaterial samples with 2 layers in the direction of propagation have been studied in the same frequency range of 8.5–12 GHz. The extracted effective parameters averaged over 4 measurements (2 samples with two different orientations) are depicted in Figs. 4(d)–4(f). Again, the mean value of the refractive index is about 3 for the real part and between 0.5 and 1 for the imaginary part. The mean value of the real part of the permittivity varies from 60 to 10, while the mean value of the imaginary part is about 50. On the other hand, the real part of permeability just slightly changes in the comparison to the data of 1 layer samples, having the real part about 0.15 and the imaginary part below 0.1. The higher value of the imaginary parts of the effective parameters in comparison to the previous case can be attributed to an increase of disorder caused by double scattering from the rough surfaces in the sample.

Finally, we perform the experimental investigations of the sample with 4 layers (see the inset in Fig. 5). The data for the refractive index and permittivity real parts were found to be generally the same as in the case of the 1 layer and 2 layers samples. The imaginary parts are larger due to an increase of scattering losses. However, the extracted permeability (Fig. 5) still demonstrates the real part of about 0.15

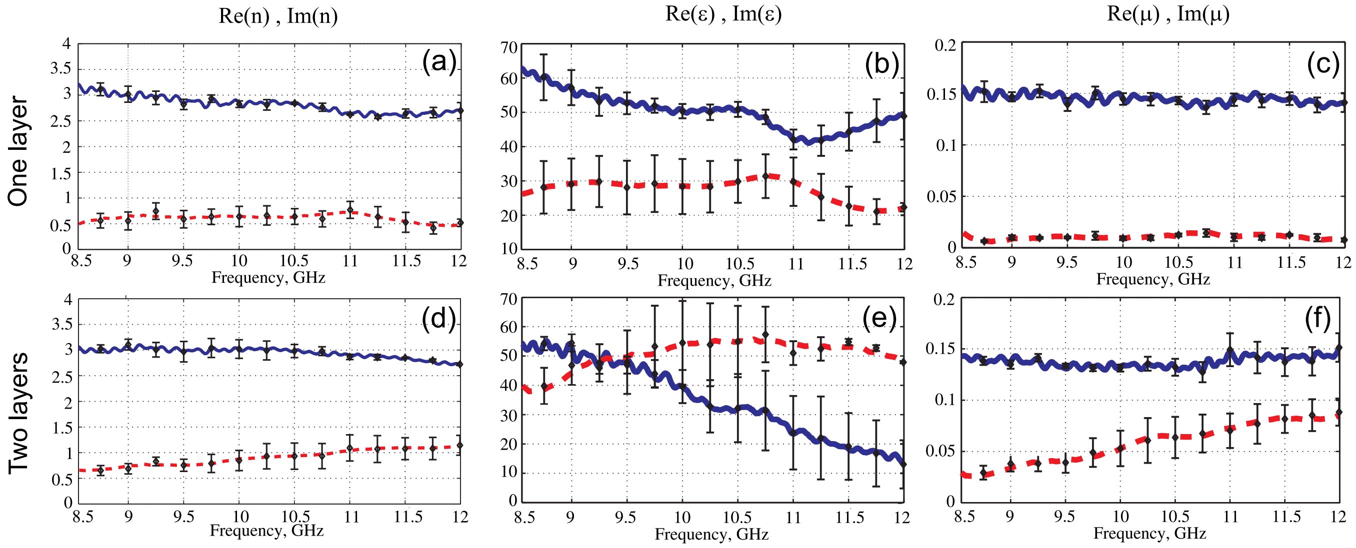

FIG. 4. Refractive index (a), (d), permittivity (b), (e), and permeability (d), (f) extracted from the experimental data for the samples with 1 and 2 layers of the cubes. Real parts are represented by solid lines; imaginary parts are represented by dashed lines, with the uncertainty shown by error-bars.

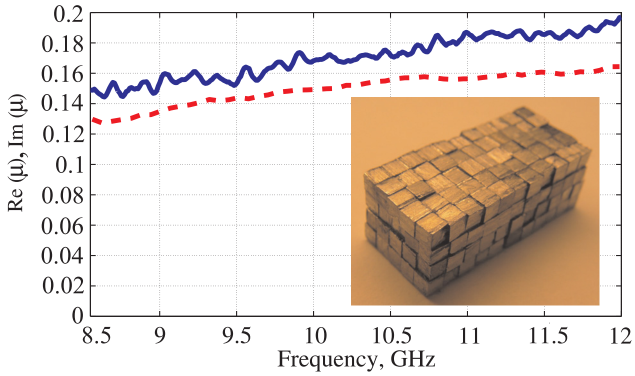

FIG. 5. Permeability extracted from the experimental data measured for the 4 layers sample. Solid line shows the real part; dashed line shows the imaginary part of the permeability. The inset shows the photograph of the 4 layers sample.

with a slightly smaller imaginary part. Therefore, a sufficient stability of the experimental data with respect to the real part of the extracted permeability was observed among the samples with 1, 2, or 4 layers. This supports a possibility to implement MNZ materials using lattices of densely packed cubes.

In summary, we have analysed experimentally and numerically the feasible values of the effective permeability which could be obtained in a practical sample of artificial diamagnetic material. For the reported system, we conclude that the effective permeability down to 0.15 may be achieved, which is almost as good as the theoretical predictions of minimum practically realisable permeability for anisotropic diamagnetics.[13] We also notice that more accurate sample fabrication will help to reduce the losses. We believe that our findings will be useful for further development of artificial diamagnetics and provide a promising forecast for application of diamagnetics for magnetic levitation and MNZ materials.

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