

Pricing and Hedging of Long-Dated Commodity Derivatives

A Thesis Submitted for the Degree of
Doctor of Philosophy

by

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Certificate of Authorship and Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signed

Date

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Abstract

Commodity markets have grown substantially over the last decade and significantly contribute to all major financial sectors such as hedge funds, investment funds and insurance. Crude oil derivatives, in particular, are the most actively traded commodity derivative in which the market for long-dated contracts have tripled over the last 10 years. Given the rapid development and increasing importance of long-dated commodity derivatives contracts, models that can accurately evaluate and hedge this type of contracts become of critical importance.

Early commodity pricing models proposed in the literature are spot price models with convenience yields either modelled as a function of the spot price or as a correlated stochastic process. These models may have desired features of commodity prices such as mean-reversion and seasonality. However, futures prices from this type of models are endogenously derived. Consequently, futures prices of different maturities are highly correlated. Multi-factor spot price models may remedy this issue. Aiming to model the entire term structure of commodity futures price curve, several authors have proposed commodity pricing models within the Heath, Jarrow & Morton (1992) (hereafter HJM) framework, with different levels of generality. These models, albeit having captured empirically observed features of commodity derivatives, such as unspanned stochastic volatility and hump volatility structures, may not be suitable to price and/or hedge long-dated commodity derivatives as they assume deterministic interest rates. Models featuring stochastic volatility and stochastic interest rates have been studied for equity and FX markets, known as hybrid models, and yet the research in commodity derivatives markets is limited.

The main contributions of this thesis include:

- ▷ *Pricing of long-dated commodity derivatives with stochastic volatility and stochastic interest rates – Chapter 2.* This chapter develops a class of forward price

models within the HJM framework for commodity derivatives that incorporates stochastic volatility and stochastic interest rates and allows a correlation structure between the underlying processes. The functional form of the futures price volatility is specified, so that the model admits finite dimensional realisations and retains affine representations; henceforth, quasi-analytical European futures option pricing formulae can be obtained. A sensitivity analysis of the model parameters on pricing long-dated contracts is conducted, and the results are discussed.

▷ *Empirical pricing performance on long-dated crude oil derivatives – Chapter 3.*

This chapter conducts an empirical study on the pricing performance of stochastic volatility/stochastic interest-rate models on long-dated crude oil derivatives. Forward price stochastic volatility models for commodity derivatives with deterministic and stochastic interest-rate specifications are considered that allow for a full correlation structure. By using historical crude oil futures and option prices, the proposed models are estimated, and the associated computational issues and results are discussed.

▷ *Hedging of futures options with stochastic interest rates – Chapter 4.* This chapter studies hedging of long-dated futures options with spot price models incorporating stochastic interest rates, a modified version of the Rabinovitch (1989) model. Several hedging schemes are considered including delta hedging and interest-rate hedging. The impact of the model parameters, such as the volatility of the interest rates, the long-term level of the interest rates, and the correlation on the hedging performance is investigated. Hedging long-dated futures options with shorter maturity derivatives is also considered.

▷ *Empirical hedging performance on long-dated crude oil derivatives – Chapter 5.*

This chapter conducts an empirical study on hedging long-dated crude oil derivatives with the stochastic volatility/stochastic interest-rate models developed in Chapter 2. Delta hedging, gamma hedging, vega hedging and interest-rate hedging are considered, and the corresponding hedge ratios are computed by using factor hedging. The hedging performance of long-dated crude oil options is assessed with a variety of hedging instruments, such as futures and options with shorter maturities.

