A Simplified Method to Determine Shear Stiffness of Thin Walled Cold Formed Steel Storage Rack Frames

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Abstract: The shear stiffness of braced frames of thin-walled cold-formed steel storage racks was experimentally and numerically investigated in order to establish the effect of connection flexibility on the accuracy of different analysis methods. The analyses which included a detailed 3D Finite element model, a 2D frame analysis with beam elements and a simple hand calculation indicated significant variation of results compared with experimental values. A simplified modelling approach for 2D elastic analysis of braced frames was proposed. The approach is aimed at practical applications to account for the flexibility in bolted connections and leads to better approximation of the shear stiffness.

Keywords: *Experimental Tests, 3D Finite element model, Shear Stiffness, Thin-walled Storage Racks, Braced Frames, Connection Flexibility*

1. Introduction and Background

Industrial steel storage racks are normally made from thin-walled cold-formed perforated sections carrying very heavy pallet loads. Due to their lightness and slenderness, they are very susceptible to horizontal actions and therefore, 2nd order effects and stability analyses are important design considerations for those structures. Industrial steel storage rack structures typically comprise of two orthogonal sets of vertical plane frames arranged parallel and perpendicular to the aisles. To resist lateral forces and to provide stiffness and stability, steel storage racks utilise moment resisting frames in the along aisle direction and use braced frames in the transverse direction crossing the aisle. Bracing members used in cross aisle directions are usually 'Lipped Channel' sections that are bolted to the upright perforations providing pin-end bracing connections, which may lead to translational softness due to bolt bending and the bearing effects between bolts and the uprights. Therefore, when carrying out a global analysis of the cross-aisle frames, experimental tests are used to obtain realistic values

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for the shear stiffness of a given type of braced frame.

Conducting laboratory experiments can be costly and is not always practical. In the absence of experimental values, analyses that approximate real shear stiffness can have varying levels of accuracy and range from detailed 3D or 2D Finite Element simulations to simple hand calculations for a given type of braced frame. This paper compares the accuracy of different analysis methods with test results and proposes a simple analytical approach to approximate the connection flexibility in cross-aisle frames. The proposed method leads to stiffness correction factors, which can be incorporated in an elastic analysis to obtain better values for the shear stiffness of the entire frame.

A well-known hand calculation method is Timoshenko's (1961) theoretical equation for deriving shear stiffness of built-up columns [1]. Timoshenko's theoretical equation, which considers the width-depth aspect ratio of the bracing panel and the cross-section properties of the bracing members, can be safely adopted for hot rolled structures in which the joint flexibilities are negligible. However, for thin-walled cold-formed structures used for storage racks, the mentioned method may lead to unrealistic outcomes. RMI (2008) [2] and AS4084 (2012) [3] accept Timoshenko's theoretical formula to calculate the elastic buckling load "P_{cr}" for upright frames braced with diagonals when the connection flexibility is negligible. Few investigations have been reported on the shear stiffness of steel storage rack upright frames consisting of thin-walled cold-formed steel profiles and bolted connections. Rao et al. (2004) [4] and Sajja et al. (2006, 2008) [5,6] carried out numerical and experimental investigations on the shear stiffness of rack upright frames using various number of panels, aspect ratios, upright sizes, restraints and bracing configurations. Rao et al. (2004) [4] examined the inaccuracy of RMI specifications in the design of cross aisle braced frames by conducting an extensive experimental program including frames with different aspect ratios and different bracing arrangement. They concluded that Timoshenko and Gere (1961)'s theory can overestimate the shear stiffness by a factor up to 20. Furthermore, their linear numerical models were not able to accurately match the experimental results.

BS EN 15512 (2009) [7] explains, in detail the test set-up and how to conduct experimental tests to determine the longitudinal shear stiffness of upright frames (Figure 1). This test set up was originally recommended by Sajja et al. (2006) [5] to improve the old test method based on European Standard (FEM-2008) [8]. Australian Standard AS4084 (2012) [3] has been also updated by Gilbert and Rasmussen (2012) [9] who proposed an alternative test set up, shown in Figure 2, to account for both bending and shear effects. Experimental testing is a reliable method for determining the shear stiffness of low to mid-rise industrial steel storage rack structures but, as mentioned earlier, it can be time consuming and expensive and therefore there is a need for a practical method to determine the transverse shear stiffness of braced cold-formed thin-walled frames used in storage racks. In the remaining part of this paper, two sets of experimental shear stiffness values are presented in comparison with results obtained from finite element simulations and from Timoshenko's standard based approach. Finally, a simple method to better account for the connection flexibility is proposed and its effectiveness is compared with other approaches.

2. Experimental Investigations

In this study, two types of upright frames, referred to in this paper as Type A and Type B, have been tested in order to establish their shear stiffness values. The difference between the two types of upright frames is the section properties of the upright members. Type B upright members are stiffer and larger in size compared with Type A uprights. A total number of six tests on upright frames, three frames for each type, have been considered in this study. The test setup, shown in Figure 4, was based on the Australian Standard AS4084-2012 [3] as depicted in Figure 3, whereby the distance (d) between uprights has been measured from the front faces of the uprights. Each upright frame was placed in the test rig with its plane in a horizontal orientation. In the out-of-plane (vertical) direction, the frame was supported on skates that allowed the uprights to slide freely along their axes. To prevent the frame from rotating or moving in the horizontal plane, the end of one upright was pinned (point A) while the diagonally opposite end (point B) of the other upright was roller supported. During the test, a compressive force (F) was applied at point B by means of a hydraulic jack along the centroid axis of the upright and the corresponding relative displacement between the uprights (δ) was determined. The required data was recorded using one load cell placed at point B between the jack and the upright, while two Linear Variable Displacement Transducers (LVDTs) were used to measure the displacements at points B and A along the axes of the uprights. The relative displacement (δ) was taken conservatively as the difference between the LVDT readings at A and B. During the test, the load was increased until a linear portion of the load deformation curve could be established and later used to derive the shear stiffness of the frame. The loads applied to the specimen were further increased until failure of the frame was reached. To prevent out-of-plane warping of the frames, as the load was increased, two further skates were placed above the upright flanges at the free ends at points C and D. Observed causes of failure were consistently the bending of the bolts followed by tearing of bolthole of one of the bracing members in the Type A frames, as depicted in Figure 5, while in the Type B frames the failure was due to the shearing of one of the bracing bolts. Force-Displacement curves obtained from tests on Type A uprights are shown in Figure 6. In Figure 6, a best-fit straight line for the linear portion of each experimental curve, identified by two points, was used to approximate the slope and hence the frame's stiffness k_{ti} in the longitudinal direction. The stiffness values k_{ti} obtained from the tests for Type A and Type B upright frames, respectively, are summarised in Tables 1 and 2. The averaged shear stiffness values for Type A and B upright frames are 3.46 kN/mm and 5.31 kN/mm, respectively.

3. Numerical Investigations

To compare the accuracy of different analysis approaches with test results, three numerical models are considered. The first analysis approach involves a 3D Finite Element model incorporating both geometric and material non-linearity with the aim of simulating the structural response of the frame as accurately as possible. Considering that such FE modelling can be time consuming and not always practical, a conventional 2D elastic frame analysis model, which would be familiar to most practicing engineers, was developed using the SAP 2000 program. Similar to what has been reported by Rao et al. (2004) [4] and Gilbert et al. (2012) [9], the results of the linear analysis and the experiment were found to be vastly different and therefore, as will be presented later in this paper, stiffness correction factors for the bracing members were developed and incorporated in the elastic analysis to obtain improved values for the stiffness of the frame.

3.1. Numerical Model in ABAQUS

The finite element model, adopted in this study, was developed as part of wider investigation on industrial racking and is described in detail by Firouzianhaji (2016) [12]. It should be noted that, the focus of the investigation was the frame's shear stiffness under service loads, when the response is normally assumed to remain in the linear domain. However due to the configuration of the bracing connection in the case of industrial racking, connection flexibility can significantly affect the shear stiffness. A detailed non-linear 3D FE model was therefore developed to enable a realistic simulation of the local bracing connection behaviour as closely as possible. The 3D FE model was developed using the ABAQUS software [10]. In the model, shown in Figure 7, all members were modelled by hexahedral elements (8-node brick element) whereby two elements were generated through the thickness of the upright. The FE model included the detailed geometry of the bolted connection and incorporated material, geometric and contact non-linearity, whereby a bi-linear stress-strain relationship was adopted for upright and bracing members. A soft contact method, available in ABAQUS, was used to model the flexibility of bolt surfaces when interacting with the bracing members and the uprights. Frictional interaction was used to model contacting surfaces in tangential direction. To model the "softened" contact, pressure-over-closure relationships were used to model the soft and thin layers on both surfaces. Adopting these relationships makes it easier to resolve the contact condition [10]. The "softened" contact relationships were specified in terms of over-closure (or clearance) versus contact pressure. A linear function with a relatively high stiffness was defined for pressure-clearance between the contact surfaces of the model to simulate the hard contact condition. The boundary conditions applied in the FE model are shown in Figure 7. The load was applied incrementally using a prescribed displacement of a rigid plate connected to the end of the upright.

Figure 6 shows the experimental force-displacement curves of Type A upright frames and the corresponding curve of the 3D numerical model. Due to initial slack observed in the test, which may be attributed to looseness of connections, the curve obtained from the FE analysis was slightly shifted to the right for ease of visual comparison. The shape of the curves compares reasonably well. Table 3 presents the numerical and experimental results that shows a 30% over estimate of the stiffness value obtained from FE analysis.

3.2. 2D Numerical Model in SAP2000

A simple 2D model was developed using SAP2000 software adopting beam elements to model the uprights and pin ended elements to model the bracing members. The results of the Type A and B simulations are presented in Table 3. The ratio of numerical / experimental frame stiffness value was found to be 5.1 and 4.3 for frame Types A and B, respectively. The main reason for the substantial discrepancy between the numerical results and experimental measurements is the inability of the 2D model to capture the flexibility and eccentricity of bolted brace-upright connections.

3.3. Standard-based Timoshenko's Equations

Appendix G of BS EN 15512:2009 [7] presents a theoretical approach based on Timoshenko's equations to determine shear stiffness of upright frames. This method can be used for "a frame in which the joint flexibility can be shown to be negligible or may be allowed for within the given expressions". Using Timoshenko's standard-based approach for a Class 2 frame, the shear stiffness per unit length in Eq.1 for frames Types A and B were determined according to:

$$S = A_d\, E \sin \Phi \cos^2 \Phi \qquad (1)$$

where, A_d is the section area of diagonal bracing members, E is the elastic modulus and Φ is the angle enclosed between the diagonal and the normal to the upright axis.

To be able to compare Timoshenko's stiffness values with other results presented in Table 3, the following relationship, which is given in EN 15512, was used to convert the transverse shear stiffness S to the longitudinal stiffnesses k_t which was established in the experiment.

$$S = k_t\, d^2 / h \qquad (2)$$

Where, d is the distance between centroidal axes of the uprights and h is the frame length as shown in Figure 3.

The comparison in Table 3 shows that stiffness values obtained using Timoshenko's equation overestimate the test results by a factor of 11.4 and 8.6 for frames A and B, respectively. This discrepancy may be attributed to the fact that Timoshenko's equation determines the shear stiffness of the frames solely based on the stiffness of the bracing members and ignores both the bending flexibility of the upright frames and joint flexibility of the brace-upright connections.

3.4. Effects of Bracing Members Configuration on Shear Stiffness

In shear frames used in industrial storage racks, bracing members can be arranged and bolted either back-to-back (Figure 8a) or front-to-front (lip to lip) as shown in Figure 8b. Experimental results reported by Rao et al. (2004) [4] indicated that the braced frames made of lipped C channels assembled in a 'back to back' arrangement show higher shear stiffness than those with 'lip to lip' bracing arrangement. In this study, in order to investigate the effects of adopting either of those configurations on the overall shear stiffness of the frames, the geometry of connection bolts was incorporated in the ABAQUS simulation. The results of this simulation, in which the model was loaded to failure, revealed that adopting either of these two arrangements can lead to a significantly different behaviour. As shown in Figure 9, for the back to back bracing arrangement, the failure mode occurred as a result of flexural plastic hinges being developed in the middle of the bolts (Figure 9a). In this configuration, the loads caused by the bracing member forces act at the centre of the bolts where the maximum bending moment is being mobilised. In contrast, the failure mode in the 'front to front' brace arrangement is caused predominantly by shear action as the bracing member forces are applied near the ends of the bolt shafts, close to the upright flanges and hence bending moment action is insignificant (Figure 9b). In addition, larger shear forces are transferred to the upright holes by the bolts and consequently larger bearing forces are applied to the perforated sections. As the loads are applied to the bolt ends, they can cause torsion in the upright members and consequently larger local deformation in the connection. Based on the numerical results, the determined stiffness values (K) for back to back and front to front bracing configurations are 4.75 kN/mm and 2.60 kN/mm, respectively. Therefore, for the frames investigated, the stiffness of the racking systems with back to back bracing configuration is significantly higher than the stiffness values of those with front to front configuration. Thus, it can be concluded that back to back bracing configuration is a preferable connection method. It should be noted that bolt tightening of the back to back connections was normal tightening and no torque control tightening was considered during the bolt installation as torque control tightening does not affect the connection load transfer mechanism between bracings and uprights.

4. Correction for Connection Flexibility

A theoretical analysis was carried out to estimate the stiffness of frames with back to back bracing configuration. As discussed in Section 3.4, it is assumed that, at the bolt locations, the resultant force of bracing members is directed along the axis of the upright. Consequently, the stiffness of each joint can be found by determining the bearing stiffness between the bolt shaft and the upright, the axial stiffness of the bracing member as well as the bolt bending stiffness. Based on the mathematical equation proposed by Zaharia et al. (2006) [11] for bearing stiffness of bolted joints of cold-formed steel trusses, the effect of bearing can be taken into consideration using Eq.3 [11] as follows:

$$K_{bearing} = 6.8 \frac{\sqrt{D}}{\frac{5}{t_b} + \frac{5}{t_u} - 1} \tag{3}$$

where, $K_{bearing}$ is the bearing stiffness of the connection in *kN/mm*, t_b is the bracing member thickness, t_u is the upright member thickness, and D is nominal diameter of the bolt.

Bolts can be modelled as simply supported beams with the span equal to the distance between upright flanges. Therefore, bolt stiffness (K_{bolt}) in *kN/mm* can be calculated using Eq. 4 as follows"

$$K_{bolt} = \frac{48EI_b}{L_b^3} \tag{4}$$

where, E is the elastic modulus, I_b is the moment of inertia of the bolt section, and L_b is the distance between the upright flanges.

Finally, modelling of the bolted connection can be simplified by as an assembly of two linear elastic springs connected in series and having the following combined joint stiffness (K_{joint}):

$$K_{joint} = \frac{1}{\frac{1}{K_{bolt}} + \frac{1}{K_{bearing}}} \quad (5)$$

The stiffness in Eq. (5) is defined along the line of action of the upright axis. For practical applications using conventional frame analysis software, it would be convenient to derive a reduction factor that can be applied to the bracing member stiffness in order to take into account the flexibility of the joint determined in Eq.5. The derivation of the factor can be achieved in two separate steps. Firstly, with reference to Figure 10 for one bracing member, considering force and displacement components of the bracing member in the direction of the upright, it can be shown that the effective joint stiffness along the bracing member axis, inclined at angle α is given by:

$$K'_{joint} = \frac{K_{joint}}{Sin^2\alpha} \quad (6)$$

where, K'_{joint} is the joint stiffness in the direction of bracing member.

In the next step, the stiffness of the bracing member and the inclined joint stiffness of the bolted connections at both ends are assembled. As depicted in Figure 10, this leads to an equivalent stiffness K^*_{member} which incorporates the connection flexibility as follows:

$$K^*_{memebr} = \beta K_{member} \quad (7)$$

where:

$$\beta = \frac{1}{1 + \frac{2K_{memeber} Sin^2\alpha}{K_{joint}}} \quad (8)$$

In practical applications, the proposed reduction factor "β" in Eq.8 can be used as the modification factor to reduce the cross sectional area of the bracing members prior to conducting the structural analysis as shown below:

$$A^*_{member} = \beta A_{member} \quad (9)$$

In practice, an elastic finite element model with beam elements is typically used to model the braced frame. The above equations can then be included in model as follows:

Step 1: Determine the bracing member axial stiffness

$$K_{memebr} = \frac{EA_{memebr}}{L} \tag{10}$$

where, E is the elastic modulus, A_{member} is the cross sectional area of the bracing member, and L is the length of the bracing member.

Step 2: Calculate K_{joint} using Eq.5 and Eq.8 to determine the modification factor "β".

Step 3: Apply "β" to the cross-section area of bracing members according to Eq.8 and adopt the modified Area (A^*_{member}) for each bracing of the analysis model.

The effectiveness of the above approach was examined by applying it to the 2D SAP2000 models of frame types A and B. As can be seen from Table 3, the results indicate a significant improvement in predicting the shear stiffness values whereby for frame Types A and B, the ratio of numerical to experimental stiffness was reduced from 5.1 to 1.5 and from 4.3 to 1.4, respectively. Therefore, it is noted that the results are much better than Timoshenko's equation where the corresponding overestimates in stiffness values were 11.4 and 8.6 for frame types A and B, respectively.

5. Discussion

Comparison of different analysis results with experimental values for the shear stiffness of braced frames used in industrial racking shows that analysis results can be grossly in error if the joint flexibility of bracing members is ignored. While in the absence of experimental data detailed finite element models can produce reasonable simulations, preparation of such FE models is usually time consuming and may not be able to be justified in engineering practice. In the examples investigated in this study, the computed shear stiffness showed a difference of 30% compared with the experimental data. In comparison, the application of Timoshenko's equation to frame Type A overestimated the shear stiffness by a factor of 11.4 while for the simple 2D elastic analysis with beam elements, the overestimate was around four to five fold. Considering the cost of obtaining experimental data and the effort and expertise required for preparing detailed finite element simulations, the proposed approach in Section 4 of this study could be an attractive approach for practicing engineers. For the frame type A and B tested in this study, the reduction in overestimates were respectively from 5.1 to 1.5 and from 4.3 to 1.4, which represents a significant improvement in predicting the shear stiffness.

It should be noted that the difference in accuracy when calculating the frame shear stiffness by different analyses approaches is attributed primarily to the modelling of the bracing-upright connection. While at service loads, material non-linearity may remain localised at the bearing between bracing bolt and upright hole, other contact and geometric non-linearity associated with local deformation of upright section as well as bending of the bolt will take place. Furthermore, the way two neighbouring bracing members are arranged when connected to the upright (front to front or back to back) can affect connection behaviour and deformation of upright section locally.

6. Conclusions and Recommendations

This study investigated two sets of experimental laboratory tests on braced cross-aisle frames of thin-walled cold-formed steel storage racks in order to determine their shear stiffness values. The experimental results were used to compare the accuracy of different methods of analysis for establishing the shear stiffness of those frames. The analyses considered included (i) a detailed 3D finite element model, (ii) a 2D FE model consisting of beam elements and (iii) a hand calculation method found in design standards for storage racks and finally (iv) a method developed in this paper to account for the flexibility of bracing member connections. The comparison of results showed significant discrepancies that were attributed primarily to the bracing member connection flexibility. The closest results to the experimental results were obtained from the 3D Finite Element model developed using the ABAQUS software which incorporated material, geometric and contact non-linearity. The numerical predictions of the 3D model showed an over-estimate of the shear stiffness of 30% and was considered in reasonable agreement when compared with other analysis results where five to eleven fold overestimates of the shear stiffness were observed. Furthermore, the method proposed in this paper, which considers the connection flexibility of bracing members when applied to the 2D Finite Element model, was able to predict the shear stiffness of the frames tested on average to within 40% to 50%. In the absence of experimental data, and considering the effort required to develop a detailed finite element model, the proposed method provides thereby an attractive practical approach to establish an initial estimate of the shear stiffness braced frames used in industrial storage racks.

Finally, the effect of two alternative methods of connecting the bracing members to the uprights on the shear stiffness of the frame was investigated using a detailed 3D Finite Element simulation. The analysis results revealed that the stiffness values of the racking systems with a back to back bracing configuration is significantly higher than the stiffness values of those with front to front configuration. Thus, for the frames investigated in this study, using a back to back bracing configuration in shear frames would be recommended when frame stability is critical.

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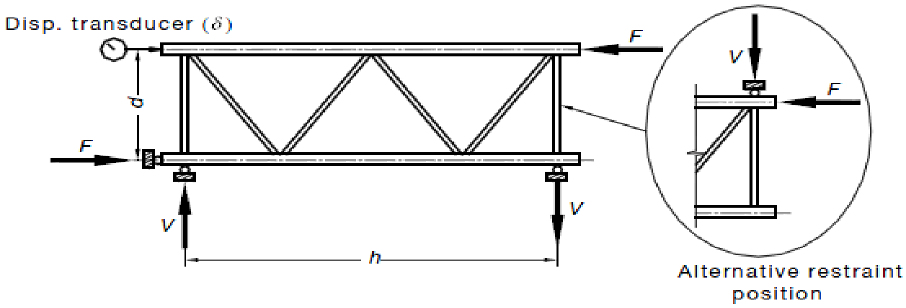


Figure1: Upright frame test set up for measuring the shear stiffness of upright frames [7]

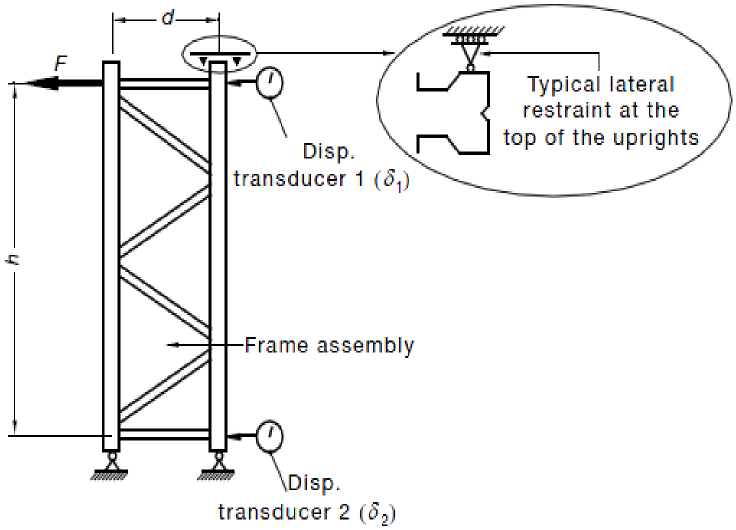


Figure 2: Test set up for measuring the combined shear and bending stiffness of upright frames [3]

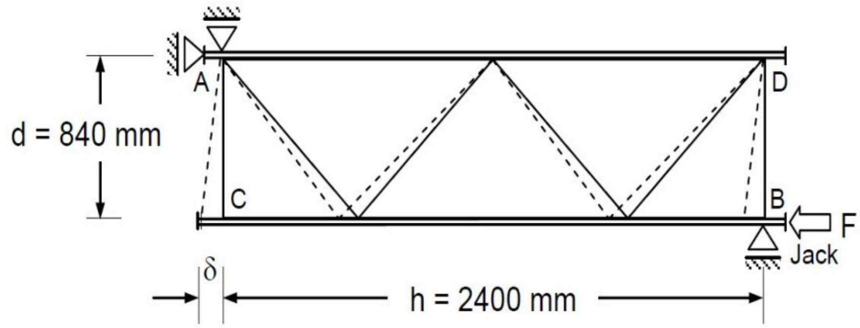


Figure 3: Schematic Test Setup

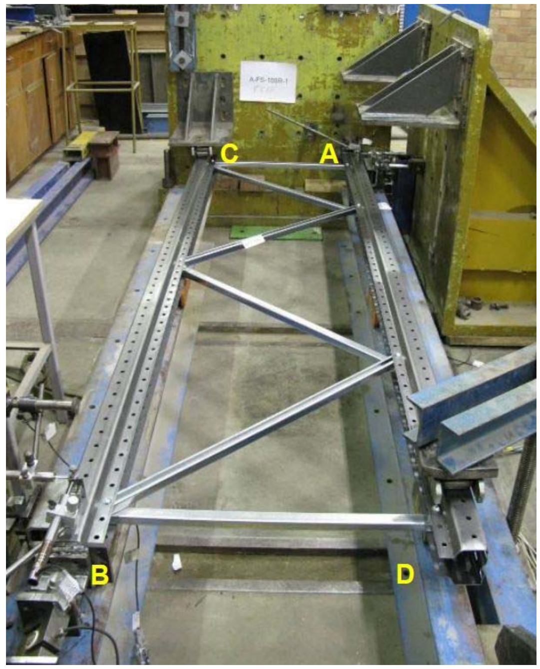


Figure 4: Test rig

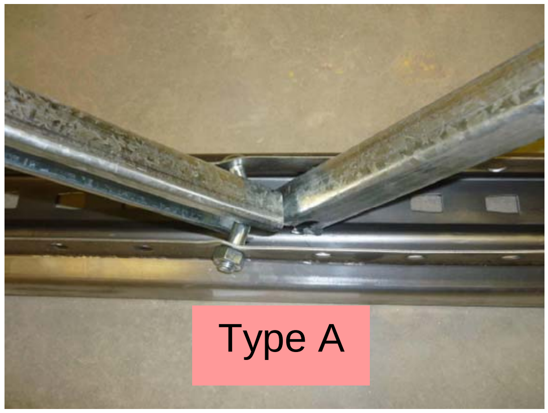


Figure 5: Bending of the bolts followed by tearing of bolthole

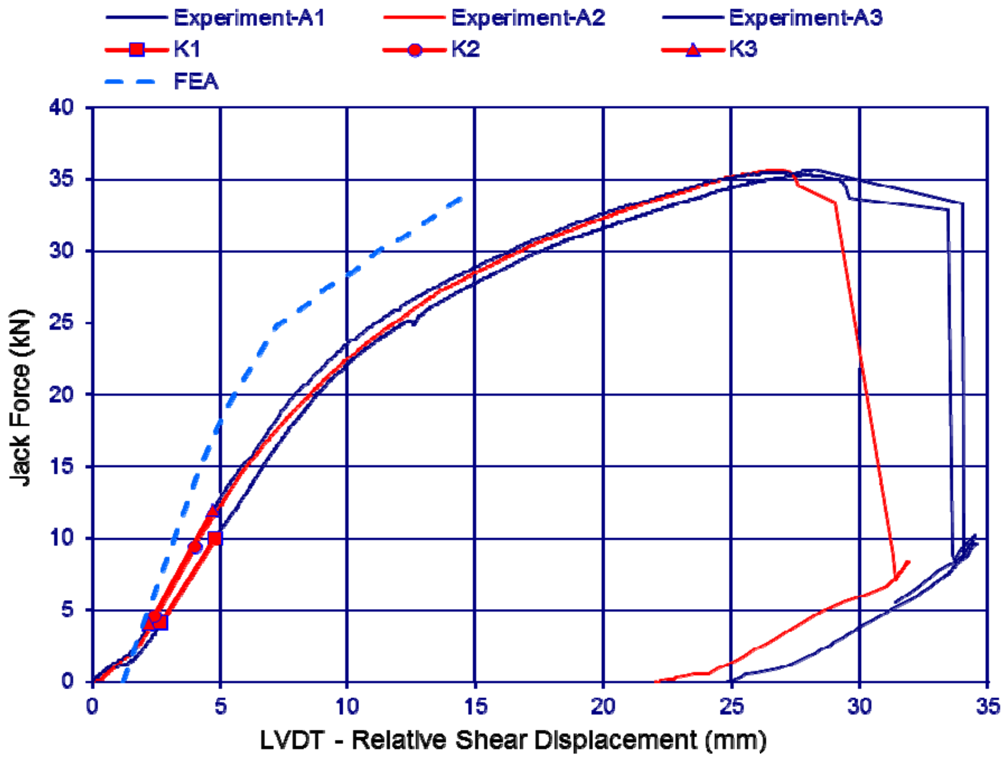


Figure 6: Experimental Force-Displacement curves of Type A frames and corresponding curve of 3D numerical FE model analysis results

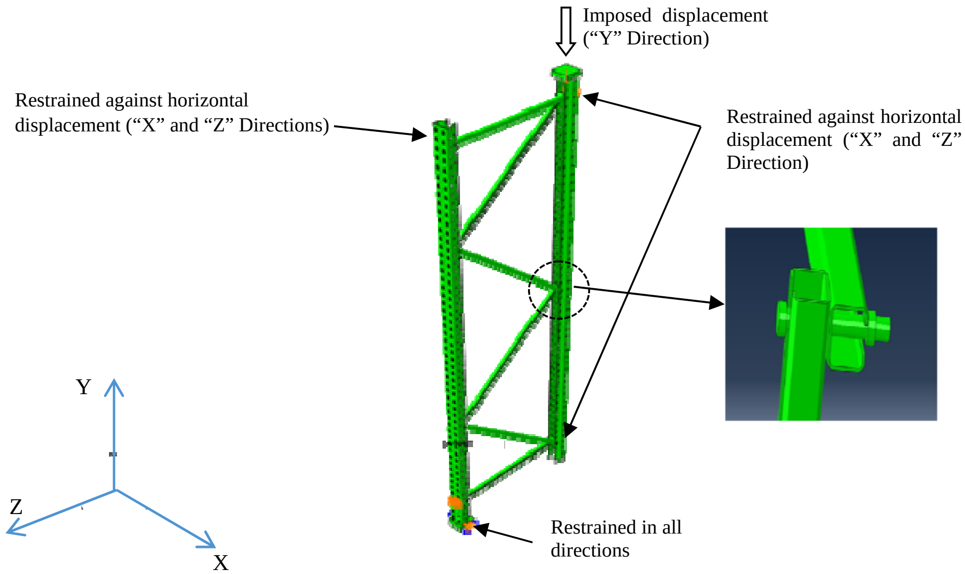
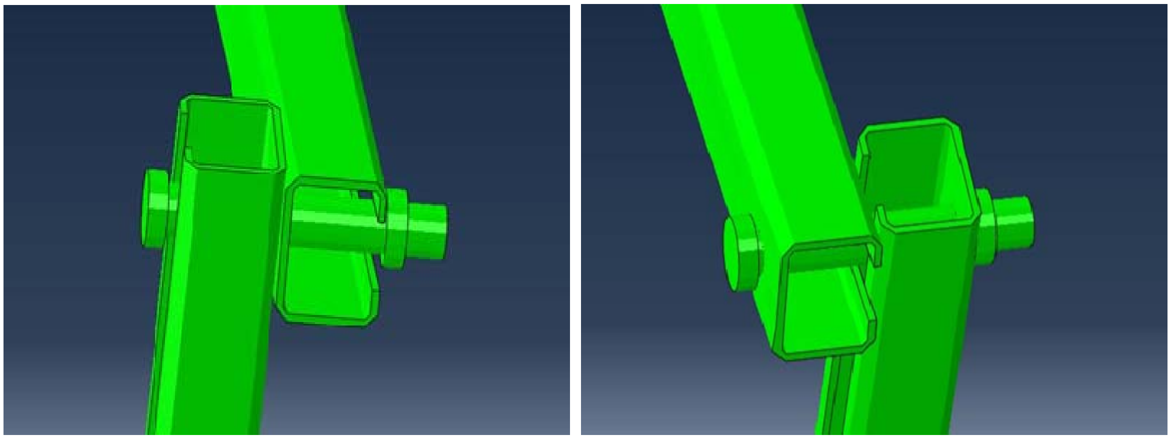
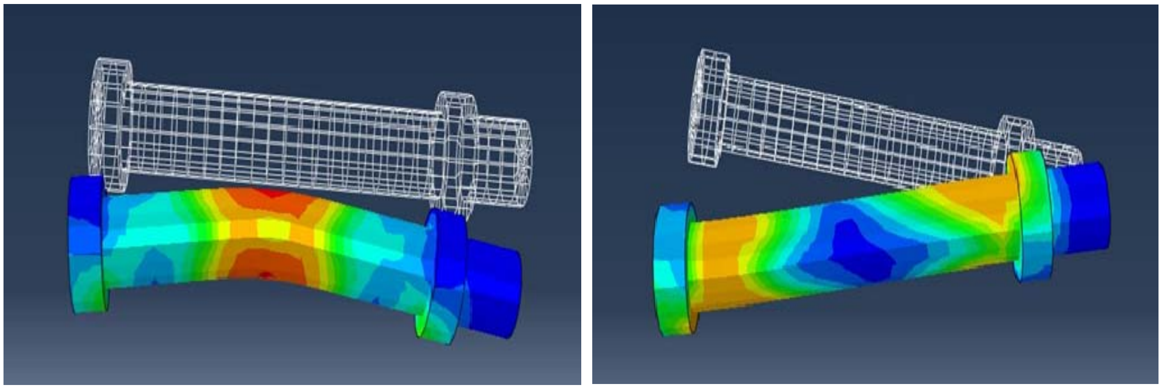


Figure 7: Details and boundary conditions of the 3D ABAQUS model



(a) **(b)**

Figure 8: Different bracing member arrangements in the open upright section; a) back-to-back; b) front-to-front



(a) **(b)**

Figure 9: Connection bolts in different bracing configurations before and after failure; a) back to back bracing configuration; b) front to front bracing configuration

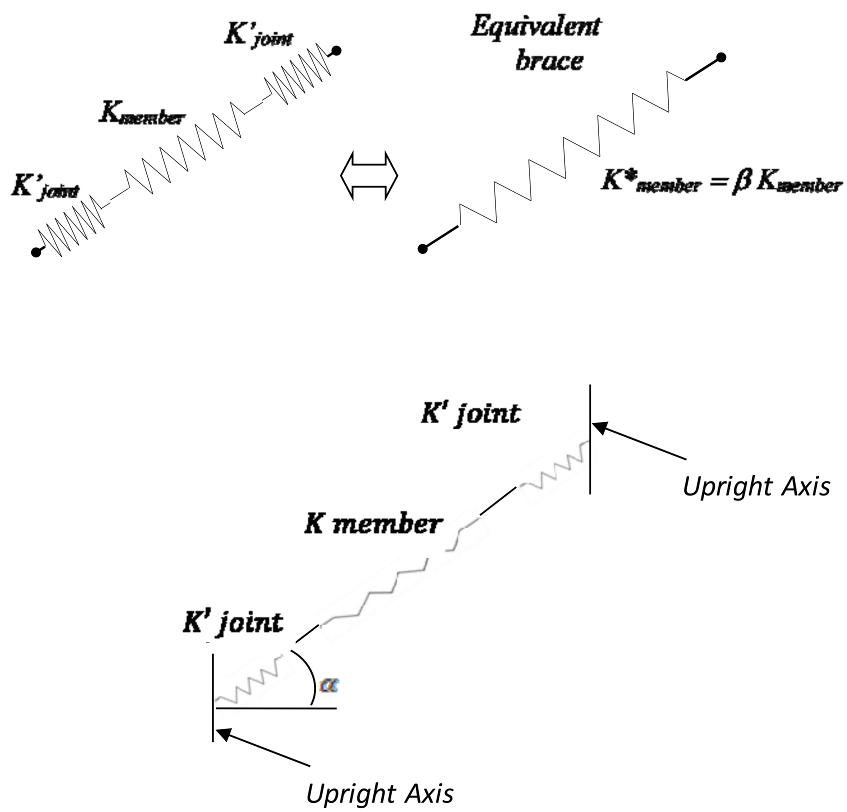


Figure 10: Equivalent stiffness model for bolted bracing members

Table.1 : The transverse shear stiffness values resulted from the tests for Type A upright frames

Specimen ID	Stiffness k_{ti} (kN/mm)
Type A-1	3.15
Type A-2	3.97
Type A-3	3.25
Average	**3.46**

Table.2 : The transverse shear stiffness values resulted from the tests for Type B upright frames

Specimen ID	Stiffness k_{ti} (kN/mm)
Type B-1	5.33
Type B-2	5.03
Type B-3	5.58
Average	**5.31**

Table 3 : **S**hear stiffness values resulted from modified 2D SAP2000 models vs other approches

Upright Frame Type	Method	Stiffness k (kN/mm)	Ratio k / k_t (Method / Test)
Type A	Experimental Testing	3.46	1.0
	3D ABAQUS Simulation	4.40	1.3
	Modified 2D SAP2000 model	5.12	1.5
	2D SAP2000 model	23.00	5.1
	Timoshenko's Equations	39.49	11.4
Type B	Experimental Testing	5.19	1.0
	Modified 2D SAP2000 model	7.56	1.4
	2D SAP2000 model	35.60	4.3
	Timoshenko's Equations	16.60	8.6