COMPREHENSIVE VELOCITY SENSITIVITY MODEL FOR SCANNING AND TRACKING LASER VIBROMETRY

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ABSTRACT

Recent work set out a comprehensive analysis of the velocity sensed by a single laser vibrometer beam incident in an arbitrary direction on a target that is of substantial interest in engineering – a rotating shaft requiring three translational and three rotational co-ordinates to describe its vibratory motion fully. Six separate "vibration sets", each a combination of motion parameters, appeared in the full expression for vibration velocity sensitivity and the difficulties associated with resolving individual vibration components within a complex motion were highlighted. One specific way in which the model has been extended is the subject of this paper.

The scanning laser Doppler vibrometer has become an increasingly popular instrument especially for experimental modal analysis. Continuously scanning strategies, in which the laser beam orientation is a continuous function of time, have received considerable attention. Researchers have reported use of several different types of scan profile including a tracking profile in which the probe laser beam remains fixed on a single point on a target such as a rotating disc. When the velocity sensitivity model was originally reported, it was stated that it could be used with such applications and this is shown explicitly, for the first time, in this paper.

NOMENCLATURE

$a(t)$	Distance between scanning laser beam incidence point and point normal to y_0
d_S	Beam steering mirror perpendicular separation distance
O	Origin of translating reference frame, xyz
P	Laser beam initial incidence point
r_S	Circular scan radius
U_m	Measured vibrometer output signal
xyz	Translating reference frame
x	x direction target displacement
y	y direction target displacement
z	z direction target displacement
$\dot{x}$, $\dot{y}$, $\dot{z}$	x, y, z, direction target velocity
x_0	x co-ordinate known point position
y_0, $y_0(t)$	y co-ordinate known point position
z_0	z co-ordinate known point position
$\overline{\Delta y_0}$	Maximum change in y_0 during typical circular scanning
$\Delta y_0(t)$	Change in y_0 during typical circular scanning
α	Laser beam orientation about z axis
β	Laser beam orientation about y axis
θ_x, $\dot{\theta}_x$	Angular vibration displacement and velocity about the x axis
θ_y, $\dot{\theta}_y$	Angular vibration displacement and velocity about the y axis
$\dot{\theta}_z$	Angular vibration velocity about the z axis
θ_{mx}	Scanning axis principal angular misalignment about the x axis
θ_{my}	Scanning axis principal angular misalignment about the y axis
Θ_m	Scanning axis total principal angular misalignment
$\rho(t)$	Time dependent variation in β during typical circular scanning
Ω	Target rotational angular velocity
Ω_S	Scan rotational angular velocity

1 INTRODUCTION

Laser Doppler Vibrometry (LDV) relies on the detection of the Doppler frequency shift in coherent light scattered from a moving target. By measuring this frequency shift, a time-resolved measurement of the target velocity is obtained. The non-contact nature of LDV offers significant advantages over traditional contacting vibration transducers and measurements on hot, light or rotating components are often cited as important applications.

A laser vibrometer measures target velocity in the direction of the incident laser beam; interpretation of the measurement in terms of the various target velocity components is essential. For rotating targets, pure axial vibration measurements are obtained by careful alignment of the laser beam with the rotation axis. Provided satisfactory consideration is given for the laser speckle effect, the measurement can be obtained in the same way as for on-axis translational surface vibration. For radial vibration measurements, however, the presence of a velocity component due to the rotation itself generates significant cross-sensitivities to speed fluctuation (including torsional oscillation) and motion components perpendicular to the intended measurement.

The velocity sensed by a single laser beam incident on a rotating shaft vibrating in all six degrees of freedom (d.o.f.s) is made up of six separate vibration "sets", each an inseparable combination of motion parameters [1]. By using a single laser vibrometer it is possible to isolate the translational vibration "sets" – two radial and one axial – but it was shown not to be possible using a single laser vibrometer to isolate the three rotational vibration "sets" – pitch and yaw (including bending vibration) and torsional oscillation (including whole body roll and/or speed fluctuation).

Multiple laser beam configurations are capable of measuring the rotational vibration "sets", with parallel beam arrangements being particularly useful [2].

2 VELOCITY MEASURED BY A SINGLE LASER BEAM INCIDENT ON A ROTOR

With reference to Figure 1, the case considered is that of an axial element of a shaft of *arbitrary* cross-section, rotating about its spin axis whilst undergoing *arbitrary* vibration but this theory is equally applicable to any non-rotating vibrating structure. A translating reference frame, xyz, maintains its direction at all times and has its origin, O, fixed to a point on the shaft spin axis, with the undeflected shaft rotation axis defining the direction and position of the z axis.

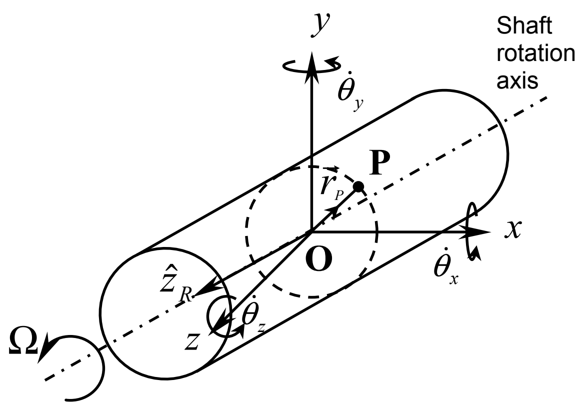

Figure 1 – Definition of axes and the point P on a vibrating and rotating structure

Provided that the illuminated axial element of the shaft can be assumed to be of *rigid* cross-section, the velocity measured by a laser beam, orientated according to the angles α and β (refer to Figure 2) and incident on the shaft surface, is given by [1]:

$$\begin{aligned}U_m &= \cos\beta\cos\alpha\left[\dot{x}+\left(\dot{\theta}_z+\Omega\right)y-\left(\dot{\theta}_y-\Omega\theta_x\right)z\right]\\&+\cos\beta\sin\alpha\left[\dot{y}-\left(\dot{\theta}_z+\Omega\right)x+\left(\dot{\theta}_x+\Omega\theta_y\right)z\right]\\&-\sin\beta\left[\dot{z}-\left(\dot{\theta}_x+\Omega\theta_y\right)y+\left(\dot{\theta}_y-\Omega\theta_x\right)x\right]\\&-\left(y_0\sin\beta+z_0\cos\beta\sin\alpha\right)\left[\dot{\theta}_x+\Omega\theta_y\right]\\&+\left(z_0\cos\beta\cos\alpha+x_0\sin\beta\right)\left[\dot{\theta}_y-\Omega\theta_x\right]\\&+\left(x_0\cos\beta\sin\alpha-y_0\cos\beta\cos\alpha\right)\left[\dot{\theta}_z+\Omega\right]\end{aligned} \quad (1)$$

where $\dot{x}$, $\dot{y}$, $\dot{z}$ and x, y, z are the translational vibration velocities and displacements of the origin, O, in the x, y, z directions, Ω is the total rotation speed of the axial shaft element (combining rotation speed with any torsional oscillation), θ_x, θ_y, $\dot{\theta}_x$, $\dot{\theta}_y$, $\dot{\theta}_z$ are the angular vibration displacements and velocities of the shaft around the x, y, z axes, referred to as pitch, yaw and roll, respectively, and (x_0, y_0, z_0) is the position of an arbitrary known point that lies along the line of the beam. It is usual to take (x_0, y_0, z_0) as the initial incidence point of the laser beam on the structure, in which case the point may be considered as lying in the "measurement plane".

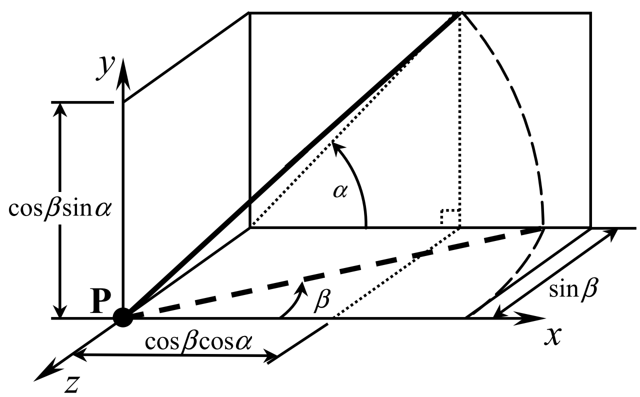

Figure 2 – Laser beam orientation, defining angles α and β

The original derivation of this important equation showed, more generally than in any previous study, that the velocity sensed by a laser vibrometer incident on a vibrating target is insensitive to the shape of the target, even when the axial and radial position of the incident beam on the target significantly alters. This immunity gives the instrument significant advantages over, for example, proximity probes, and the same immunity is obviously found for targets with less complex motions.

The analysis is sufficiently versatile to give the velocity sensitivity in applications where the laser beam is *scanned* or where a single point on the target is *tracked*. This was discussed in the published work, [1] and [2], and is presented for the first time in this paper. It is also possible to extend the theory to axial elements with *flexible* cross-section and this will be described in a future publication.

3 APPLICATION TO CIRCULAR SCANNING LASER DOPPLER VIBROMETRY

Scanning LDV measurements are typically performed via the introduction of some form of laser beam deflection around two orthogonal axes [3]. This can be thought of as a transformation of the position of the laser vibrometer relative to some fixed reference frame. The potential effect of such motion on the velocity sensitivity model is the temporal variation of the beam orientation angles α and β, and the temporal variation of the arbitrary known point (x_0, y_0, z_0) that lies along the line of the beam.

A circular scanning profile is achieved by deflecting the laser beam around the two axes simultaneously through suitable angles. In the "ideal" scanning system, the deflection is controlled by a single optical element, e.g. a front reflecting mirror that can be rotated simultaneously in two perpendicular directions about axes coincident with the reflective surface of the mirror, as shown schematically in Figure 3.

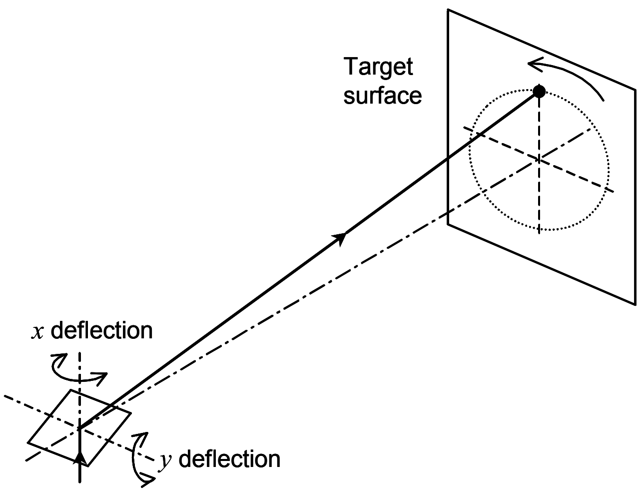

Figure 3 – The "ideal" scanning arrangement

The convenience of such a system is that the known point (x_0, y_0, z_0) used in the derivation of equation (1), can be taken as the initial incidence point of the laser vibrometer beam on the scanning mirror, since its position will remain constant in time, thus considerably simplifying the analysis. The laser beam scanning can therefore be conveniently accounted for in the velocity sensitivity model by defining α and β as functions of time.

In some commercially available scanning laser vibrometers, however, the method of x and y laser beam deflection is achieved by the introduction of two orthogonally aligned mirrors, separated by some distance, d_S, into the laser vibrometer beam path, as illustrated schematically in Figure 4.

For such an arrangement the position of the known point (x_0, y_0, z_0) no longer remains constant in time. Here it is more convenient to think of the known point moving along the rotation axis of the x mirror, i.e. x_0 and z_0 are constant, but y_0, α and β are functions of time.

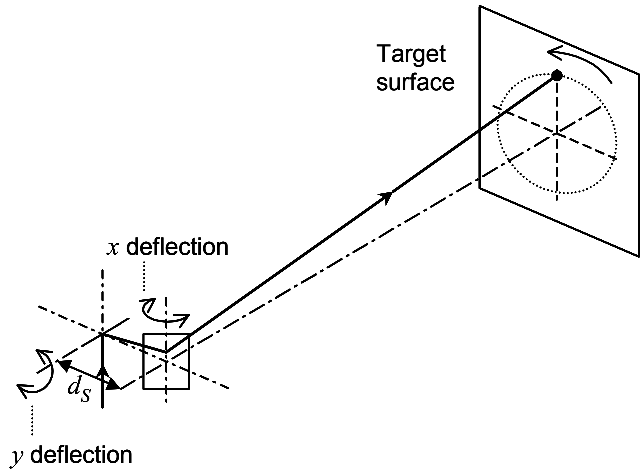

Figure 4 – The scanning arrangement incorporating two orthogonally aligned mirrors

Figures 5 is similar to Figure 4, but with greater detail of the scan axis translational and angular misalignments – x_m, y_m, and Θ_m, respectively. A circular scan profile is achieved by setting β (the laser beam rotation around the y axis) and α (the laser beam rotation around the z axis) as follows:

$$\beta(t) = \frac{3\pi}{2} - \rho(t) \tag{2a}$$

and

$$\alpha(t) = \Omega_S t\,, \tag{2b}$$

where Ω_S is the scan rotational angular velocity. Figure 6 shows the various important dimensions and angles associated with the laser beam scanning the end face of the target shown in Figure 5. Assuming that the distance between the laser vibrometer and the target, z_0, is large, relative to the circular scan radius, r_S:

$$\rho(t) \approx \frac{a(t)}{z_0}\,, \tag{3}$$

in which:

$$\begin{aligned} a(t) &= \sqrt{\left(r_S \sin\Omega_S t - \Delta y_0(t)\right)^2 + \left(r_S \cos\Omega_S t\right)^2} \\ &= r_S\left[1 + \frac{\left(\Delta y_0(t)\right)^2}{r_S^2} - \frac{2\Delta y_0(t)}{r_S}\sin\Omega_S t\right]^{1/2}, \end{aligned} \tag{4}$$

where

$$\Delta y_0(t) = \overline{\Delta y_0}\sin\Omega_S t = \frac{r_S d_S}{z_0 + d_S}\sin\Omega_S t\,. \tag{5}$$

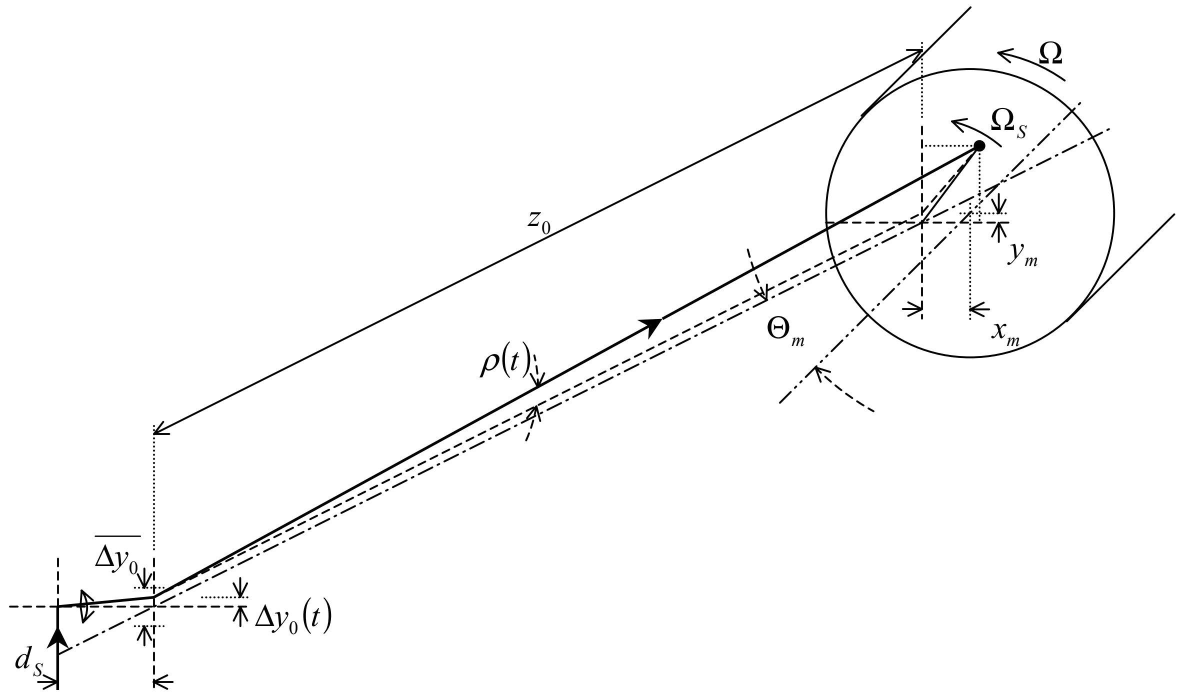

Figure 5 – Laser beam orientation for circular scanning profiles

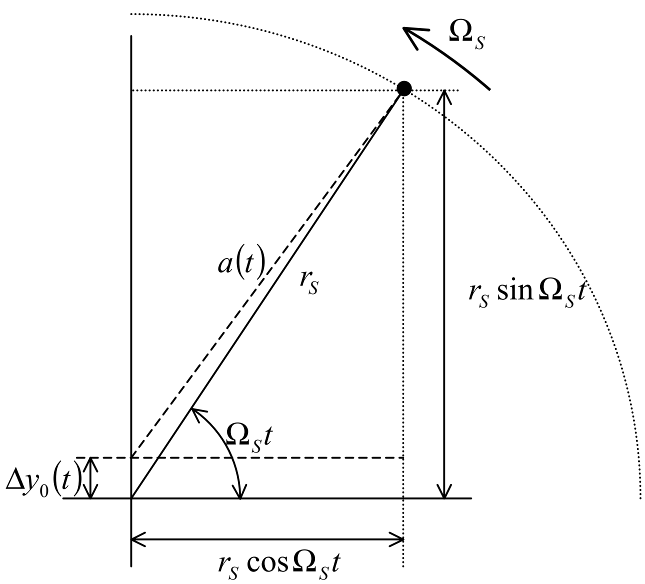

Figure 6 – Circular scanning profile projected onto a target

Figure 7 shows the variation of *a*(*t*), during one scan, for an arrangement typical of a conventional circular scanning LDV system, i.e. z_0 = 3m, r_S = 100mm and d_S = 30mm.

It is possible to express y_0 as a function of time, i.e.:

$$y_0(t) = y_0 + \Delta y_0(t) = y_0 + \frac{r_S d_S}{z_0 + d_S}\sin\Omega_S t \,. \tag{6}$$

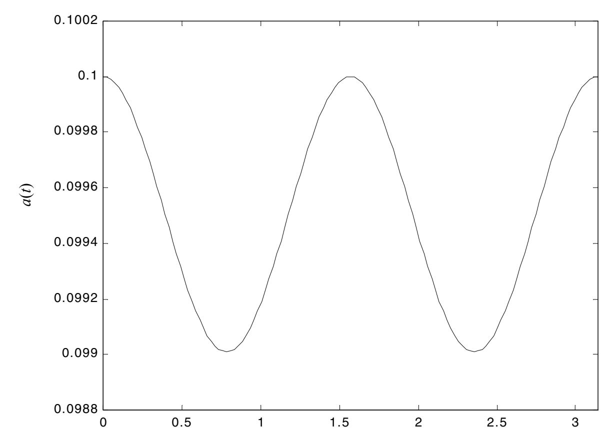

Figure 7 – Variation of *a*(*t*) during a circular scan

Equation 1 can therefore be re-written to represent the total velocity sensed by a circular scanning laser beam incident on a rigid axial element of a vibrating, rotating target. Using a small angle approximation, since z_0 is large relative to $a(t)$:

$$\begin{aligned} U_m = {} & \frac{-a(t)}{z_0}\cos\Omega_S t\begin{bmatrix} \dot{x} + \left(\dot{\theta}_z + \Omega\right)\left(y - y_0(t)\right) \\ -\left(\dot{\theta}_y - \Omega\theta_x\right)\left(z - z_0\right) \end{bmatrix} \\ & - \frac{a(t)}{z_0}\sin\Omega_S t\begin{bmatrix} \dot{y} - \left(\dot{\theta}_z + \Omega\right)\left(x - x_0\right) \\ +\left(\dot{\theta}_x + \Omega\theta_y\right)\left(z - z_0\right) \end{bmatrix} \\ & + \begin{bmatrix} \dot{z} - \left(\dot{\theta}_x + \Omega\theta_y\right)\left(y - y_0(t)\right) \\ +\left(\dot{\theta}_y - \Omega\theta_x\right)\left(x - x_0\right) \end{bmatrix} \end{aligned} \tag{7}$$

4 APPLICATION TO CIRCULAR TRACKING LASER DOPPLER VIBROMETRY

When a scanning LDV system is configured to track a single point on a target such as a rotating disc, the scan rotational angular velocity is set at the rotational speed, i.e. $\Omega_S = \Omega$. Translational and/or angular misalignment between the dual-mirror scanning system axis and the target spin axis results in systematic errors in the measured vibration velocity, manifested by additional vibration information at integer multiples of the rotation/scan speed. One source of the first order errors has been correctly attributed to angular misalignment, due to the changing laser beam path length, [3] and [4], but there has been no comprehensive analysis of the origins of the other first order components and the mechanism by which the higher order errors occur until now.

The derivation of equation 7 is of great significance since it enables these systematic errors to be quantified for the first time. With reference to Figure 5, consider the basic case of a non-vibrating, i.e. $\dot{x} = \dot{y} = \dot{z} = 0$ and $\dot{\theta}_x = \dot{\theta}_y = \dot{\theta}_z = 0$, rotating target. The angular misalignment between the scanning system axis and the target spin axis, Θ_m, can be represented by the principal angular misalignments θ_{mx} and θ_{my} around the x and y axes respectively, such that the velocity measured by the tracking laser vibrometer is given by:

$$\begin{aligned} U_m &= \Omega \frac{a(t)}{z_0} \cos \Omega t \left(y_0(t) + \theta_{mx} z_0 \right) \\ &+ \Omega \frac{a(t)}{z_0} \sin \Omega t \left(-x_0 + \theta_{my} z_0 \right) \\ &+ \Omega \left(\theta_{mx} x_0 + \theta_{my} y_0(t) \right). \end{aligned} \tag{8}$$

The frequency spectrum of such a laser vibrometer output will contain components at the rotational speed and subsequent harmonics due to the sum and difference of the sinusoidal functions contained within $a(t)$ and $y_0(t)$. A simulation of this effect for an arrangement typical of a circular tracking LDV system (the same as for the circular scanning LDV system earlier) resulted in the frequency spectrum illustrated in Figure 8. In this case translational and angular misalignments of $x_0 = y_0 =$ 5mm, θ_{mx} = 5mrad and θ_{my} = 0rad, and a rotation frequency of 50Hz, were used.

Here the DC component occurs as a result of the combination of translational and angular misalignment, whilst the component at synchronous frequency contains contributions resulting from both the translational and the angular misalignment. The component at twice the rotation frequency originates primarily from the dual-mirror arrangement, clearly manifested in $a(t)$ (see Figure 7). All components, however, contain different contributions from the various sources; the extent of each is the subject of ongoing research and will be discussed in a further publication.

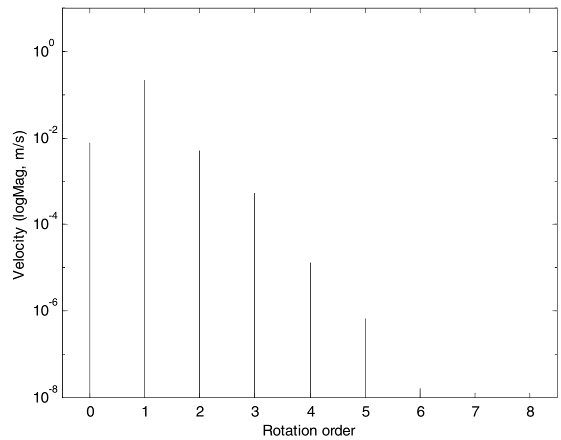

Figure 8 – Simulated frequency spectrum due to translational and angular misalignment in a circular tracking LDV system

CONCLUSIONS

The use of laser vibrometers incorporating some form of manipulation of the laser beam orientation, typically using two orthogonally aligned mirrors, has become increasingly popular in recent years, particularly for the rapid acquisition of multi-point surface vibration information. Considerable attention has now been directed towards the operation of such scanning laser vibrometers in continuous scanning mode, in which the probe laser beam position is a continuous function of time. Researchers have reported the use of several different types of scan profile including a tracking profile in which the probe laser beam remains fixed on a single point on a target such as a rotating disc. This is clearly a technique of importance in many different industrial sectors for design, development and monitoring measurements on rotating shafts, discs and blades. The velocity sensitivity model presented here is well suited to the analysis of such LDV applications.

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