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Liner Ship Fleet Deployment with Uncertain Demand

Shuaian Wang, Tingsong Wang, Xiaobo Gu, Zhiyuan Liu, and Sheng Jin

This paper points out that the deployment problem of the liner ship fleet with uncertain demand is different from other logistics problems with uncertain demand (e.g., truck transport and airlines) because container ships operate 24 ha day and 7 days a week. This difference is largely ignored in the literature. To address this problem, a multi-level optimization model is developed. In addition to liner ship fleet deployment, the model is applicable to other liner shipping decision problems, such as network design with uncertain demand, and to port operations planning problems, such as berth planning with uncertain ship arrival times.

A global liner container shipping company, such as Maersk Line or OOCL, the Orient Overseas Container Line, operates weekly ship routes with fixed schedules to transport containers (1–3). The company deploys suitable types of containerships on each route according to the container shipment demand over the planning horizon (e.g., of 6 months). Once a ship is deployed on a route, it usually serves the route for the whole planning horizon. In case of excess container shipment demand within the planning horizon, the liner shipping company may buy ship slots from other shipping companies to fulfill the excess demand. Hence, deployment of larger containerships provides more shipping capacity while incurring a higher ship-operating cost. Conversely, use of smaller ships reduces the ship-operating cost at the expense of buying more slots. Therefore, a liner shipping company must determine which type of ship to deploy on a route to minimize the total cost. This tactical-level planning issue is referred to as the liner ship fleet deployment problem (LSFDP) in the literature (4–7).

A major challenge for addressing the LSFDP is that containership demand, which is the most important input for the LSFDP, cannot be predicted accurately. As a consequence, researchers have developed stochastic optimization models by assuming that future demand in a week is a random variable with a known probability distribution function, to minimize, for example, the expected total operating cost (8–9). Nevertheless, this paper demonstrates that existing models have shortcomings in formulating the uncertain demand. New models are developed that overcome the shortcomings.

COMPARISON OF LSFDP AND OTHER LOGISTICS PROBLEMS

This section uses two examples—a truck fleet size problem (TFSP) and a LSFDP (both with uncertain demand)—to demonstrate the difference between the LSFDP and other logistics problems.

Consider a trucking company that transports cargo from a factory to a warehouse in a planning horizon of T days. The amount of cargo to transport on each day is a random variable. For ease of exposition, it is assumed that this random variable has finite support. W is the set of demand scenarios, and positive number p is the occurrence probability of scenario w ∈ W. The daily amount of cargo to be transported in scenario w ∈ W is q_w. The cost of operating a truck is the planning horizon C, and the amount of cargo that can be transported by a truck each day is v. The cargo demand on each day must be fulfilled. If the trucking company does not have enough trucks, it can hire other companies to transport the cargo at the cost of g per unit cargo. The trucking company needs to determine the optimal number of trucks to operate, denoted by z, to minimize the expected total cost. This problem is a TFSP.

Liner Ship Fleet Deployment Problem

Consider a liner shipping company that operates the weekly ship route shown in Figure 1. The journey times from Port a to Port b, from b to c, from c to d, and from d to a are each 1 week. Hence, four ships must be deployed to maintain weekly service. A planning horizon of T weeks is considered. There are two origin–destination (O-D) pairs: Port a to Port c and Port b to Port d. Suppose that there are two demand scenarios for the weekly container shipment demand of the two O-D pairs: W = {w_1, w_2} with p_w = p_1 = 0.5. In scenario w_1, the demand for O-D pair (a, c) is q^{w_1}_{ac} = 5,000 20-ft equivalent units (TEUs) and for O-D pair (b, d) q^{w_1}_{bd} = 5,000 TEUs. In scenario w_2, the demand is q^{w_2}_{ac} = 1,000 TEUs and q^{w_2}_{bd} = 5,000 TEUs. Suppose that there are two types of ships: V = {v_1 = 6,000-TEU ships, v_2 = 10,000-TEU ships}. The cost of deploying ships in type v ∈ V on the ship route in the planning horizon is denoted by C_v, C_v < C_{v_2}. The capacity of ships in type v ∈ V is denoted by E_v, E_v = 6,000, E_{v_2} = 10,000. The container shipment demand in each week must be fulfilled. If the liner shipping company does not have enough capacity, it can buy slots from other shipping companies at the cost of g_v > 0 per TEU for O-D pair (a, c) and g_{v_2} > 0 per TEU for O-D pair (b, d). Exactly one type of ship can be deployed on the route. The liner shipping company needs to determine the optimal type of ship to deploy to minimize the expected total cost. For simplicity, empty containers are not considered (10).

To address this LSFDP, binary variable z_{v} is defined. This variable equals 1 if ships in type v ∈ V are deployed and 0 otherwise. Variables
\( x_w^c \) and \( x_w^d \) are the volume of containers (TEUs) that are transported by other shipping companies for O-D pair \((a, c)\) and \((b, d)\), respectively, under scenario \( w \in W \). In work by Meng et al. (9), the problem is formulated as a single-level (SL) optimization model:

\[
\begin{align*}
\min \sum_{w \in W} \sum_{v \in V} C_{v} z_v + \sum_{w \in W} t_p^w (g_w^c x_w^c + g_w^d x_w^d) \\
\text{subject to} \\
(q_w^c - x_w^c) + (q_w^d - x_w^d) \leq \sum_{v \in V} E_v z_v, \quad w \in W \\
0 \leq x_w^c \leq q_w^c, \quad 0 \leq x_w^d \leq q_w^d, \quad w \in W \\
\sum_{v \in V} z_v = 1 \\
z_v \in \{0, 1\}, \quad v \in V
\end{align*}
\]  

(1) (2) (3) (4) (5)

The objective function (Equation 1) minimizes the expected total cost, which consists of the ship-operating cost and expected slot-purchasing cost. Equation 2 requires that the total volume of transported containers does not exceed the capacity of the ships deployed as all the containers are transported on the voyage leg from Port \( b \) to Port \( c \). Equation 3 enforces the lower and upper bounds of the volume of containers transported by other shipping companies. Equation 4 imposes that exactly one type of ship is deployed on the route. Equation 5 defines \( z_v \) as a binary decision variable. With the given parameters, deploying 6,000-TEU ships is an optimal solution.

**Difference Between TFSP and LSFDP**

At first glance, it seems that the TFSP and LSFDP are similar. Both need to make a tactical-level decision that affects the operational-level decisions over a given planning horizon, and the operational-level decisions need to incorporate uncertainty in demand forecasting. However, a closer examination reveals that for the TFSP, the operational-level decisions on a particular day do not affect the decisions in the subsequent days because trucks transport cargo only in the daytime and the next day is a new start. In the LSFDP, ships transport cargo 24 h a day, 7 days a week. As a result, the operational-level decisions made in a week affect the shipping capacities of subsequent weeks.

The space–time network in Figure 2 is used to illustrate this point. Because in practice most liner ship routes provide weekly service, a weekly service frequency for this ship route is assumed (11–14). In Figure 2, the location of each of the four ships at each time is plotted. For example, Ship 4 visits Port \( a \) in Week 1, Port \( b \) in Week 2, Port \( c \) in Week 3, Port \( d \) in Week 4, Port \( a \) again in Week 5, and so forth. Let \( \tilde{w} \) be a realization of the uncertain demand in all weeks on the planning horizon \( T \), and let \( q_{w,t}^a \) and \( q_{w,t}^d \) be the container shipment demand for O-D pairs \((a, c)\) and \((b, d)\) in week \( t = 1, 2, \ldots, T \), respectively, in scenario \( \tilde{w} \). The containers for the O-D pair \((a, c)\) in Week 1, whose volume is \( q_{w,t}^a \), are transported on Ship 4, containers for the O-D pair \((b, d)\) in Week 1 with a volume of \( q_{w,t}^d \), are transported on Ship 3, and containers for the O-D pair \((b, d)\) in Week 2 with a volume of \( q_{w,t}^d \) are transported on Ship 4. The containers for the two O-D pairs in the same week are not transported on the same ship, and the number of containers to transport in \( q_{w,t}^a \) affects the volume of containers that can be transported in \( q_{w,t}^d \) because these two batches of containers share the same slots of Ship 4 during the voyage from Port \( b \) to Port \( c \) in Week 2.

The authors now analyze why model SL is incorrect in handling the uncertain container shipment demand. The set of scenarios \( W \) represents possible scenarios of container shipment demand of

**FIGURE 2** Space–time network representation of ship route.
subject to
\[ \sum_{v \in V} z_v = 1 \quad \text{(11)} \]
\[ z_v \in \{0, 1\} \quad v \in V \quad \text{(12)} \]

where \((x^{w_0}_{a,c}, x^{w_1}_{a,c})\) is the optimal solution to ML-Week 1, \(\hat{w} \in \hat{W}\).

\[ \min g_w x^{w}_{a,c} + g_w x^{w}_{a,c} \quad \text{(13)} \]

subject to
\[ q^{w}_{a,c} - x^{w}_{a,c} \leq \sum_{w \in W} E_z, \quad \text{(14)} \]
\[ q^{w}_{a,c} - x^{w}_{a,c} \leq \sum_{w \in W} E_z, \quad \text{(15)} \]

\[ 0 \leq x^{w}_{a,c} \leq q^{w}_{a,c}, \quad 0 \leq q^{w}_{a,c} \leq q^{w}_{a,c} \quad \text{(16)} \]

and \((x^{w}_{a,c}, x^{w}_{a,c})\), \(t = 2, 3, \ldots, T\) are the optimal solutions to ML-Week \(t\), \(t = 2, 3, \ldots, T\), \(\hat{w} \in \hat{W}\).

\[ \min g_w x^{w}_{a,c} + g_w x^{w}_{a,c} \quad \text{(17)} \]

subject to
\[ q^{w}_{a,c} - x^{w}_{a,c} \leq \sum_{w \in W} E_z, \quad \text{(18)} \]
\[ (q^{w-1}_{a,c} - x^{w-1}_{a,c}) + (q^{w}_{a,c} - x^{w}_{a,c}) \leq \sum_{w \in W} E_z, \quad \text{(19)} \]

\[ 0 \leq x^{w}_{a,c} \leq q^{w}_{a,c}, \quad 0 \leq q^{w}_{a,c} \leq q^{w}_{a,c} \quad \text{(20)} \]

ML-Week 1 imposes that the liner shipping company makes operational-level decisions in Week 1 exclusively on the demand of Week 1. ML-Week \(t\) enforces that the company makes operational-level decisions in week \(t\) according to the demand in week \(t\) and the demands and decisions before week \(t\). Given the fleet deployment decisions, for each scenario \(w \in W\), the models ML-Week \(t\) should be solved sequentially for \(t = 1, 2, 3, \ldots, T\) to obtain the optimal operational-level decisions. Hence, models ML-Week \(t\) successfully incorporate the nonanticipativity constraints. Meng and Wang also captured the nonanticipativity constraints, but they did not incorporate demand uncertainty (31).

The model ML can be transformed to a mixed-integer linear programming model using the duality theorems and Big M method. The resulting model could be solved by commercial solvers.

**NUMERICAL EXPERIMENTS**

To evaluate the proposed models, numerical experiments are conducted on the basis of randomly generated data. The number of ports on a ship route is \(N \in \{5, 6, \ldots, 10\}\). The distance between two ports is in \([1,000, 3,000]\) n miles. Each ship route has an outbound and an inbound direction. In each direction, the probability that an O-D pair has demand is 0.5, and the demand is in \([0, 1,200]\)
TABLE 1 Computational Results

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The slot-purchasing cost \( g_e \) ($/TEU) is equal to 1,000 plus the distance multiplied by a random number in (0, 0.05). Considered are 20 types of ships, from Type 1: 500 TEU, Type 2: 1,000 TEU, through Type 20: 10,000 TEU. The operational cost of a ship is assumed to be proportional to the square root of its size. The time horizon \( T = 12 \) weeks. The number of demand scenarios \( |W| = 5 \). The models are coded with MATLAB and solved by CPLEX 12.2 on a PC with Windows 7 operating system, 16-core 3.5 GHz processor, and 16 GB of memory.

For each port number \( N \), five random instances are tested, for a total of 30 instances. All the instances are solved to optimality in 3 min. Table 1 reports the number of ports \( N \), the instance ID, the average total demand (TEUs) to be transported over the scenarios in \( W \), the optimal type of ship to deploy, and the average number of slots (TEUs) purchased over the scenarios in \( W \). Table 1 indicates that model ML has successfully captured the nonanticipativity nature of the problem and obtained the optimal decisions.

**CONCLUSIONS**

This paper has pointed out that the LSFPD with uncertain demand is different from other logistics problems with uncertain demand because containerships operate 24 h a day, 7 days a week. This difference is largely ignored in the literature. To address this problem, an ML optimization model is developed. Numerical experiments demonstrate the applicability of the proposed model.

The model ML has successfully captured demand uncertainty. It should be mentioned that whether the additional efforts of incorporating demand uncertainty are worthwhile compared with using an average demand value depends on the volatility of demand. In reality, a few major customers contribute a large proportion of the demand to global liner shipping companies, and the demand follows some known seasonality according to historical data. For example, the demand from Asia to Europe is larger a few weeks before Christmas. Therefore, the part of demand that cannot be accurately predicted may not be dramatic. Still, the proposed models are applicable whether the demand fluctuates significantly or insignificantly, or remains constant. If empirical data are available, the uncertainty of demand can be further explored.

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**REFERENCES**


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