Matrix product state decomposition in machine learning and signal processing

Johann Anton Bengua

A thesis submitted for the degree of Doctor of Philosophy at

The University of Technology Sydney in 2016

Faculty of Engineering and Information Technology

Certificate of Original Authorship

I, **Johann Bengua**, certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature:

Date:

Author's Publications

The contents of this thesis are based on the following papers that have been published, accepted, or submitted to peer-reviewed journals and conferences.

Journal papers

[J1] Ho N. Phien, Johann A. Bengua, Hoang D. Tuan, Philippe Corboz, and Román Orús. Infinite projected entangled pair states algorithm improved: Fast full update and gauge fixing. *Phys. Rev. B,* 92:035142, July 2015.

[J2] Johann A. Bengua, Ho N. Phien, Hoang D. Tuan, and Minh N. Do. Efficient tensor completion for color image and video recovery: Low-rank tensor train, *IEEE Transactions on Image Processing*, Accepted, August 2016.

[J3] Johann A. Bengua, Ho N. Phien, Hoang D. Tuan, and Minh N. Do. Matrix Product State for Higher-Order Tensor Compression and Classification. *IEEE Transactions on Signal Processing*, Resubmitted, June 2016.

[J4] Johann A. Bengua, Hoang D. Tuan, Trung Q. Duong and H. Vincent Poor. Joint Sensor and Relay Power Control in Tracking Gaussian Mixture Targets by Wireless Sensor Networks, *IEEE Transactions on Signal Processing*, Submitted, July 2016.

Conference papers

[C1] Johann A. Bengua, Ho N. Phien and Hoang D. Tuan. Optimal Feature Extraction and Classification of Tensors via Matrix Product State Decomposition, *Proceedings of the 2015 IEEE International Congress on Big Data*, pp. 669-672, New York, NY, 2015.

[C2] Johann A. Bengua, Hoang D. Tuan, Ho N. Phien and Ha H. Kha. Two-hop Power-Relaying for Linear Wireless Sensor Networks, *Proceedings of the 2016 IEEE Sixth International Conference on Communications and Electronics*, pp. 1-5, Ha Long, Vietnam, 2016.

[C3] Johann A. Bengua, Hoang D. Tuan, Ho N. Phien and Minh N. Do. Concatenated image completion via tensor augmentation and completion, *10th International Conference on Signal Processing and Communication Systems*, Submitted, July 2016.

Acknowledgments

I would like to express my deepest gratitude to my supervisor, Prof. Tuan Hoang, for his unfaltering guidance and supervision. Thank you for your support and great insight. I also want to thank Dr. Phien Ho, who is now at Westpac, for teaching me many things related to the quantum world, for inspiring me and pushing me to try new algorithms and mathematical tools, which greatly increased my knowledge and abilities.

Furthermore, I would like to thank Prof. Minh N. Do (University of Illinois Urbana-Champaign, USA) and Prof. Vincent Poor (Princeton University, USA) for the immense support, input and collaboration in my work.

Additionally, a sincere appreciation to my colleagues in Prof. Tuan Hoang's research group at the University of Technology Sydney: Elong Che, Bao Truong and Tam Ho. Thank you for all the interesting discussions, and providing a fun and friendly research environment.

Finally, a tremendous gratitude to my mother, father and brother, for always encouraging me to seek knowledge. Most importantly, I would like to thank my beloved wife Dolly Sjafrial for always supporting me, for your patience and faith in me all these years, and for your love.

List of Abbreviations

MPS	Matrix product state
TT	Tensor train
CP	Canonical/Parallel factors
TD	Tucker decomposition
PARAFAC	Parallel factors
SiLRTC	Simple low-rank tensor completion
SiLRTC-TT	Simple low-rank tensor completion via tensor train
TMac	Tensor completion by parallel matrix factorization
TMac-TT	Parallel matrix factorization via tensor train
KA	Ket augmentation
ICTAC	Concatenated image completion via tensor augmentation and completion
TTPCA	Principal component analysis via tensor train
HOSVD	Higher-order singular value decomposition
HOOI	Higher-order orthogonal iteration
MPCA	Multilinear principal component analysis
FBCP	Fully Bayesian CP Factorization
STDC	Simultaneous tensor decomposition and completion
LRTC	Low-rank tensor completion
LRMC	Low-rank matrix completion
SVD	Singular value decomposition
ALS	Alternating least squares
PCA	Principal component analysis
LDA	Linear discriminant analysis
KNN	K-nearest neighbours

SPC-QV	Smooth PARAFAC tensor completion with quadratic variation
R-UMLDA	Uncorrelated multilinear discriminant analysis with regularization
WSN	Wireless sensor network
LSN	Linear sensor network
NSN	Nonlinear sensor network
GMM	Gaussian mixture model
MSE	Mean square error
MMSE	Minimum mean square error

Contents

List of Figures

List of Tables

Abstract

There has been a surge of interest in the study of multidimensional arrays, known as tensors. This is due to the fact that many real-world datasets can be represented as tensors. For example, colour images are naturally third-order tensors, which include two indices (or modes) for their spatial index, and one mode for colour. Also, a colour video is a fourth-order tensor comprised of frames, which are colour images, and an additional temporal index. Traditional tools for matrix analysis does not generalise so well in tensor analysis. The main issue is that tensors prescribe a natural structure, which is destroyed when they are vectorised. Many mathematical techniques such as principal component analysis (PCA) or linear discriminant analysis (LDA) used extensively in machine learning rely on vectorised samples of data. Additionally, since tensors may often be large in dimensionality and size, vectorising these samples and applying them to PCA or LDA may not lead to the most efficient results, and the computational time of the algorithms can increase significantly. This problem is known as the so-called curse of dimensionality.

Tensor decompositions and their interesting properties are needed to circumvent this problem. The Tucker (TD) or CANDECOMP/PARAFAC (CP) decompositions have been predominantly used for tensor-based machine learning and signal processing. Both utilise common factor matrices and a core tensor, which retains the dimensionality of the original tensor. A main problem with these type of decompositions is that they essentially rely on an unbalanced matricization scheme, which potentially converts a tensor to a highly unbalanced matrix, where the row size is attributed to always one mode and the column size is the product of the remaining modes. This method is not optimal for problems that rely on retaining as much correlations within the data, which is very important for tensor-based machine learning and signal processing.

In this thesis, we are interested in utilising the matrix product state (MPS) decomposition. MPS has the property that it can retain much of the correlations

within a tensor because it is based on a balanced matricization scheme, which consists of permutations of matrix sizes that can investigate the different correlations amongst all modes of a tensor. Several new algorithms are proposed for tensor object classification, which demonstrate an MPS-based approach as an efficient method against other tensor-based approaches. Additionally, new methods for colour image and video completion are introduced, which outperform the current state-of-the-art tensor completion algorithms.