

An adaptive weighted least square support vector regression for hysteresis in piezoelectric actuators

Xuefei Mao^a, Yijun Wang^a, Xiangdong Liu^a, Youguang Guo^b

a. School of Automation, Beijing Institute of Technology, Beijing 100081, China

b. School of Electrical, Mechanical and Mechatronic Systems, University of Technology Sydney, Sydney, NSW
2007, Australia

Abstract: To overcome the low positioning accuracy of piezoelectric actuators (PZAs) caused by the hysteresis nonlinearity, this paper proposes an adaptive weighted least squares support vector regression (AWLSSVR) to model the rate-dependent hysteresis of PZA. Firstly, the AWLSSVR hyperparameters are optimized by using particle swarm optimization. Then an adaptive weighting strategy is proposed to eliminate the effects of noises in the training dataset and reduce the sample size at the same time. Finally, the proposed approach is applied to predict the hysteresis of PZA. The results show that the proposed method is more accurate than other versions of least squares support vector regression for training samples with noises, and meanwhile reduces the sample size and speeds up calculation.

Keywords: piezoelectric actuator, hysteresis prediction, particle swarm optimization, adaptive weighted least squares support vector regression.

1 Introduction

It is known that piezoelectric actuators (PZAs) are widely used in micro-/nano-positioning systems thanks to their merits of small size, high positioning resolution, rapid response speed, and large driving force [1], [2]. Nevertheless, the major limitation of PZA is the low accuracy caused by the inherent hysteresis, which

can produce an open-loop positioning error as much as 10-15% of the motion range [3]. To improve the positioning accuracy, a hysteresis model should be established to identify the nonlinearity of PZA.

Due to the rate-dependent property, the characteristic of the piezoelectric hysteresis is dependent not only on the amplitude but also on the frequency of input voltage signals. Hence, modeling the hysteresis using the traditional rate-independent hysteresis models such as the Preisach model [4], [5], Prandtl-Ishlinskii (PI) model [6], [7] and Krasnoselskii-Pokrovskii (KP) model [8] could yield errors subject to dynamic inputs with different frequencies. To characterize the rate-dependent hysteresis, some models were put forward, such as the improved Preisach model [9], [10], improved PI model [11], [12], and time series similarity model [13] - [15]. However, these models have a lot of parameters to be determined, which complicate the modeling process. [16] -[17] employ artificial neural network (ANN) to model the hysteresis nonlinearity, whereas it has some shortcomings such as overfitting and easily getting into the local extremum. In contrast, the support vector machine (SVM), based on statistical theory and structural risk minimization principle [18], outperforms ANN in terms of global optimization and generalization capability [19] - [21]. SVM can be applied to nonlinear regression problem by solving a convex quadratic programming (QP) problem and it shows good performance in hysteresis modeling [22], [23].

As an extension of SVM, the least squares support vector regression (LSSVR) overcomes the defect of slow training speed in SVM by solving a linear equation set rather than a quadratic optimization problem. Considering that the hyperparameters in LSSVR substantially affect the regression accuracy, some parameters selection methods were put forward, such as the analytical selection method [24], grid-searching method [25], gradient method [26], and intelligent optimization algorithms [27] - [30]. The intelligent optimization algorithms have attracted great attention due to their advantages of fast convergence and efficient global optimization. Particle swarm optimization (PSO), as one of intelligent optimization algorithms, is widely used in the field of parameters optimization because of its easy operation and excellent convergence ability [31], [32].

Although LSSVR accelerates the training process, employing the sum of squared error (SSE) loss function leads to the degradation of robustness, which means that the model is sensitive to noises. To overcome this drawback, some methods were proposed to filter out the outliers according to the results of LSSVR [33], [34]. However, this outlier detection method brings some errors due to the low robustness of LSSVR itself. Therefore, Suykens et al. proposed a weighted least squares support vector machine (WLSSVM) to reduce the influence of outliers by assigning smaller weights to the samples with larger training errors [35]. The key of this method is to select the appropriate weights of sample points and some other weighting strategies were proposed. For example, Wen et al. assigned smaller weights to the samples with large distance from other samples [36]. Cui et al. proposed an adaptive weighting method combined with outlier detection [37]. Behnasr et al. proposed a weight selection method based on Myriad function [38]. Xing et al. proposed a weighting method based on Cauchy distribution [39]. These methods improve the robustness of the model to some extent, whereas they have not considered that the solution of LSSVR has considerable redundancy, which means that the sample size can be reduced without degrading the performance of the regression model. In this paper, an adaptive weighting strategy is proposed to eliminate the noise interference and reduce the size of the training samples at the same time. By this means, the model based on adaptive weighted least squares support vector regression (AWLSSVR) can be both robust and sparse.

The rest of this paper is organized as follows. Section 2 provides a brief review of LSSVR and describes the hyperparameters optimization procedure based on PSO. Section 3 presents the WLSSVR method and the adaptive weighting strategy. In section 4, experimental results for modeling of hysteresis in a PZA based on AWLSSVR are presented. Conclusions are finally provided in section 5.

2 LSSVR and parameter optimization

2.1 LSSVR for hysteresis

In order to convert the hysteresis multivalued mapping into an one-to-one mapping, the nonlinear regression model is established to predict the current output by combining the current and previous inputs and previous outputs as exogenous inputs [40]. That is

$$y_k = f(\mathbf{x}_k) + \xi_k \quad (1)$$

with

$$\mathbf{x}_k = [u_k \quad u_{k-1} \quad \cdots \quad u_{k-n} \quad y_{k-1} \quad \cdots \quad y_{k-m}] \quad (2)$$

where u_k and y_k denote the input voltage and output displacement of the system at time instance k , ξ_k is the prediction error, $f(\cdot)$ represents the nonlinear regression model, and m and n define the system orders. It is found that as the system orders increase, the training error gradually decreases, while the testing error does not change monotonously [23]. To make a compromise between testing error and training error, m and n are both set to 3.

The LSSVR is employed to model the piezoelectric hysteresis and the model $f(\cdot)$ takes the form [41]

$$y(\mathbf{x}) = \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}) + b \quad (3)$$

where a nonlinear function $\boldsymbol{\varphi}(\mathbf{x})$ maps the input space into a high-dimensional space. $\{\mathbf{x}_k, y_k\}_{k=1}^N$ is given as the training set, where N is the sample size. $\boldsymbol{\omega}$ and b are the parameters which can be determined by solving the following optimization problem

$$\begin{aligned} \min_{\boldsymbol{\omega}, \xi, b} J(\boldsymbol{\omega}, \xi) &= \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{\omega} + \frac{1}{2} C \sum_{k=1}^N \xi_k^2 \\ \text{s.t. } y_k &= \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + \xi_k \end{aligned} \quad (4)$$

where C represents the regularization factor which balances the training error and model complexity. The Lagrangian function of problem (4) is then expressed as

$$L(\boldsymbol{\omega}, b, \xi, \alpha) = J(\boldsymbol{\omega}, \xi) - \sum_{k=1}^N \alpha_k [\boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + \xi_k - y_k] \quad (5)$$

where α_k are the Lagrangian multipliers. The optimal solutions meet the following conditions

$$\begin{cases} \frac{\partial L}{\partial \boldsymbol{\omega}} = 0 \rightarrow \boldsymbol{\omega} = \sum_{k=1}^N \alpha_k \boldsymbol{\varphi}(\mathbf{x}_k) \\ \frac{\partial L}{\partial \xi_k} = 0 \rightarrow \alpha_k = C \xi_k \\ \frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^N \alpha_k = 0 \\ \frac{\partial L}{\partial \alpha_k} = 0 \rightarrow \boldsymbol{\omega}^T \boldsymbol{\varphi}(\mathbf{x}_k) + b + \xi_k - y_k = 0 \end{cases} \quad (6)$$

Eliminating $\boldsymbol{\omega}$ and ξ , the solutions are given by the following linear equations

$$\begin{bmatrix} 0 & \mathbf{e}_{N \times 1}^T \\ \mathbf{e}_{N \times 1} & \boldsymbol{\Omega} + \mathbf{I}_N / C \end{bmatrix} \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{Y} \end{bmatrix} \quad (7)$$

where $\mathbf{e}_{N \times 1} = [1; 1; \dots; 1]$, $\boldsymbol{\alpha} = [\alpha_1; \alpha_2; \dots; \alpha_N]$, $\mathbf{Y} = [y_1; y_2; \dots; y_N]$, \mathbf{I}_N is an identity matrix, $\Omega_{ij} = \boldsymbol{\varphi}^T(\mathbf{x}_i) \cdot \boldsymbol{\varphi}(\mathbf{x}_j) = K(\mathbf{x}_i, \mathbf{x}_j)$. K is the kernel function and the radial basis function kernel is used in this paper

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\|\mathbf{x}_i - \mathbf{x}_j\|^2 / 2\sigma^2\right)$$

where σ is the kernel width parameter.

After obtaining b and $\boldsymbol{\alpha}$ from (7), the regression model of LSSVR becomes

$$y(\mathbf{x}) = \sum_{k=1}^N \alpha_k K(\mathbf{x}_k, \mathbf{x}) + b \quad (8)$$

2.2 Hyperparameters optimization based on PSO

The selection of hyperparameters C and σ in LSSVR is significant to obtain an accurate regression model. In this paper, PSO is adopted to optimize the hyperparameters due to its fast convergence and robustness.

PSO algorithm simulates the birds flock's behavior of preying on food and searching for the optimal position. PSO consists of a swarm of interacting particles searching in an L -dimensional search space of the problem's solutions (L is the size of hyperparameters). Each particle can be described by its current position and velocity. For instance, the position and velocity of particle i at iteration t can be expressed as $\mathbf{p}_i^t = \{p_{i1}^t, p_{i2}^t, \dots, p_{iL}^t\}$ and $\mathbf{v}_i^t = \{v_{i1}^t, v_{i2}^t, \dots, v_{iL}^t\}$. Each particle updates its speed and location by tracking individual best known position \mathbf{pbest}_i and swarm's best position \mathbf{gbest} . The velocity and position of particle i are updated according to the following two formulas

$$\begin{aligned} \mathbf{v}_i^{t+1} &= \eta \mathbf{v}_i^t + c_1 \times r_1 \times (\mathbf{pbest}_i - \mathbf{p}_i^t) + c_2 \times r_2 \times (\mathbf{gbest} - \mathbf{p}_i^t) \\ \mathbf{p}_i^{t+1} &= \mathbf{p}_i^t + \mathbf{v}_i^{t+1} \end{aligned} \quad (9)$$

where η denotes the inertia weight, c_1 and c_2 are the learning factors, and r_1 and r_2 are random numbers between 0 and 1.

The performance of each particle is evaluated by the prediction error using cross-validation. That is

$$f(C, \sigma) = \frac{1}{N_{\text{test}}} \sum_{k=1}^{N_{\text{test}}} (y_k - \hat{y}_k)^2 \quad (10)$$

where y_k and \hat{y}_k are the k th actual output and predicting output, and N_{test} is the size of test samples.

Thus, the algorithm steps of hyperparameters selection using PSO are provided as follows:

1. Establish PSO with a group of particles with random positions and velocities.
2. For each particle i , build the regression model and predict the outputs for test samples, and evaluate the particle's performance using (10).

3. Replace \mathbf{pbest}_i with the particle i if the latter is superior. Replace \mathbf{gbest} with the best particle of the population if the latter is superior.
4. Update the velocity and position of each particle based on (9).
5. Repeat steps 2-4 until the stop criteria are satisfied.

3 Adaptive WLSSVR

In this section, an adaptive WLSSVR is proposed to mitigate the effects of noises in the training samples. In this algorithm, the weight is assigned to each sample datum according to the training error of unweighted LSSVR. The proposed weighting method can not only eliminate the noise interference but also reduce the sample size.

3.1 WLSSVR

WLSSVR can be obtained by weighting the error terms ξ_k^2 in (4). Supposing the weights are $v_k (k=1,2,\dots,N)$, the optimization problem is changed as follows [35]

$$\begin{aligned} \min_{\omega, \xi, b} J(\omega, \xi) &= \frac{1}{2} \omega^T \omega + \frac{1}{2} C \sum_{k=1}^N v_k \xi_k^2 \\ \text{s.t. } y_k &= \omega^T \varphi(\mathbf{x}_k) + b + \xi_k \end{aligned} \quad (11)$$

The corresponding Lagrangian function takes the same form as (5). The optimal solutions are satisfied by

$$\begin{cases} \frac{\partial L}{\partial \omega} = 0 \rightarrow \omega = \sum_{k=1}^N \alpha_k \varphi(\mathbf{x}_k) \\ \frac{\partial L}{\partial \xi_k} = 0 \rightarrow \alpha_k = C v_k \xi_k \\ \frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^N \alpha_k = 0 \\ \frac{\partial L}{\partial \alpha_k} = 0 \rightarrow \omega^T \varphi(\mathbf{x}_k) + b + \xi_k - y_k = 0 \end{cases} \quad (12)$$

α and b are obtained by the solving the following linear equations

$$\begin{bmatrix} 0 & \mathbf{e}_{1 \times N}^T \\ \mathbf{e}_{1 \times N} & \mathbf{\Omega} + \mathbf{V} / C \end{bmatrix} \begin{bmatrix} b \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{Y} \end{bmatrix} \quad (13)$$

with the diagonal matrix $\mathbf{V} = \text{diag}\{1/\nu_1, 1/\nu_2, \dots, 1/\nu_N\}$.

Finally, the WLSSVR is expressed in the same form as (8)

3.2 Adaptive weighting strategy

The core idea of WLSSVR is how to determine the weights ν_k to achieve a robust and accurate regression model. For a sample datum, the bigger the training error is, the more likely it is an outlier. Motivated by this idea, smaller weights are assigned to those sample data with large errors to eliminate the disturbance of the outliers.

In addition, the optimal condition $\alpha_k = C\nu_k \xi_k$ in (12) shows that the support values α_k are directly proportional to weights ν_k . Therefore, samples with relatively large training errors can be deleted in the regression model by assigning zero weights to them. This method can not only eliminate the effect of outliers on the model performance, but also prune some useless samples.

Hence, the adaptive weighting function is proposed as follows [35]

$$\nu_k = \begin{cases} e^{-k_1 \left(\frac{\xi_k}{\hat{s}} \right)^2} & \left| \frac{\xi_k}{\hat{s}} \right| < k_2 \\ 0 & \left| \frac{\xi_k}{\hat{s}} \right| \geq k_2 \end{cases} \quad (14)$$

with

$$\hat{s} = \frac{IQR}{2 \times 0.6745} \quad (15)$$

where ξ_k is the training error in the k th sample, k_1 and k_2 denote the parameters of weighting function. \hat{s} is a robust estimate of standard deviation of ξ_k and IQR is the difference between the 75th percentile and the 25th percentile.

The relationship between ν and ξ/\hat{s} is illustrated in Fig. 1. It is shown that the closer ξ/\hat{s} is to 0, the larger ν is, and if the absolute value of ξ/\hat{s} is larger than the threshold k_2 , ν is set to 0.

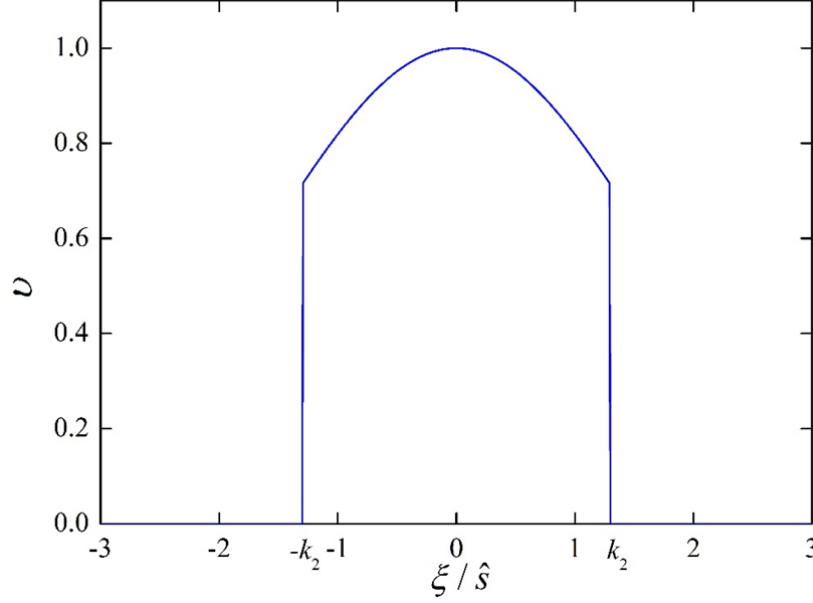


Fig. 1 Relationship between ν and ξ/\hat{s} .

3.3 The adaptive WLSSVR algorithm

Firstly, the optimal hyperparameters C and σ are selected using PSO. The initial fitting errors $\xi'_k (k=1,2,\dots,N)$ are obtained based upon the un-weighted LSSVR solutions. However, due to the low robustness of the LSSVR, the initial errors ξ'_k are not accurate enough. Thus, the iterative WLSSVR is used to obtain the error variables ξ'_k . This method is realized using WLSSVR iteratively based on training samples and the weights are updated according to the fitting errors and weighting function until the error variables ξ_k are converged. Finally, the weights are determined according to the weight function (14) and the errors ξ_k . The weighting function during iteration is the non-zero part of function (14): $\nu_k = \exp\left(-k_1 \left(\xi_k / \hat{s}\right)^2\right)$.

The adaptive WLSSVR approach is as follows:

- 1) Obtain the optimal hyperparameters C and σ by PSO.
- 2) LSSVR is applied to the model based on the training samples, and the initial fitting errors $\bar{\xi}_k = \bar{\alpha}_k / C$ are obtained.
- 3) Calculate the weights $\bar{\nu}_k = \exp\left(-k_1 \left(\bar{\xi}_k / \hat{s}\right)^2\right)$.
- 4) Build the model by WLSSVR and calculate the fitting errors $\xi_k = \alpha_k / \bar{\nu}_k C$.
- 5) If the difference between ξ_k and $\bar{\xi}_k$ is not small enough, let $\bar{\xi}_k = \xi_k$ and go back to step 3; or if the error variables ξ_k are converged, determine the weights ν_k based upon (14).
- 6) Develop the adaptive WLSSVR model with the weights ν_k .

4 Application for modeling hysteresis in PZA

4.1 Experimental setup

The experiments are carried out on a piezoelectric actuator MPT-1JRL002 (withstand-voltage range: -30 to 150 V and displacement resolution: 0.01 μm). Fig. 2 shows the system devices. The hardware-in-the-loop simulation system produces an analogy voltage output which is then amplified by a power amplifier to drive the PZA. The output displacement of PZA is measured by a resistance strain gauge sensor which is installed within the PZA as a micrometer and then transmitted back to the hardware-in-the-loop simulation system. Fig. 3 shows the schematic diagram of the experimental system.

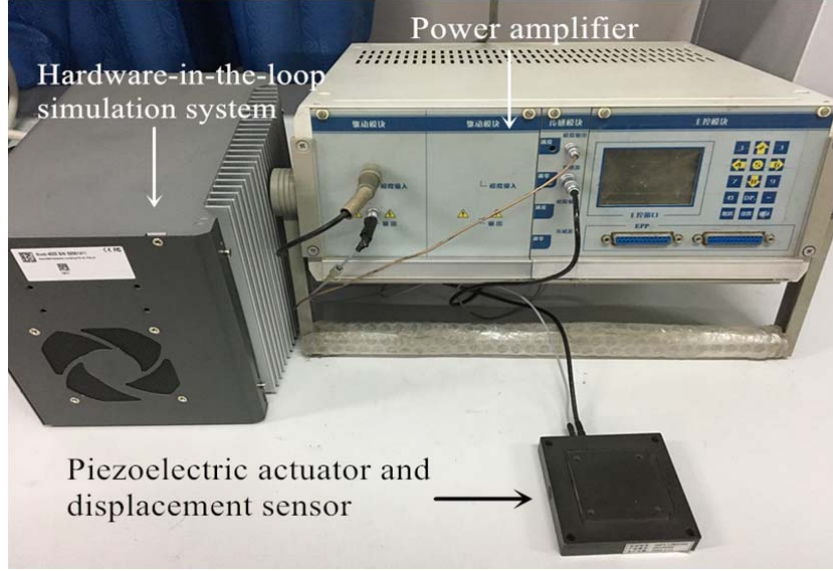


Fig. 2 Piezoelectric actuator experiment devices.

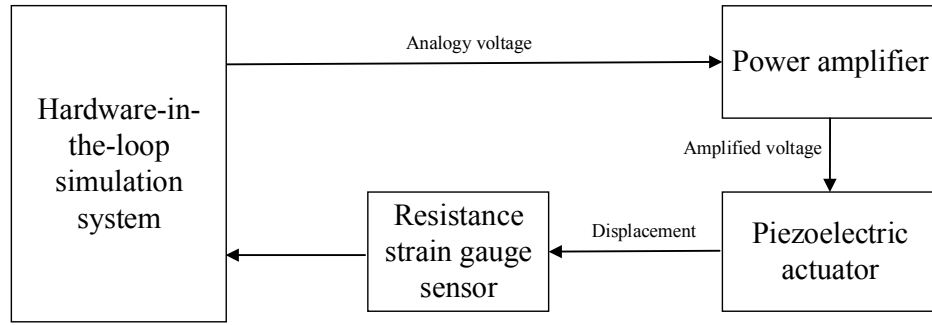


Fig. 3 Schematic diagram of piezoelectric actuator based experimental system.

4.2 AWLSSVR model training and testing

Considering the rate-dependent behavior of piezoelectric hysteresis, the training data must reflect the effects of input signal frequency on the response of the hysteresis. Thus, 10002 sample points shown in Fig. 4(a) is used for training, which is composed of several parts with change-rate of 2V/s, 5V/s, 10V/s, 20V/s and 40V/s. The corresponding output displacement is shown in Fig. 4(b), of which 524 points randomly selected is added by Gaussian noises ($\mu = 0$, $\sigma = 0.2 \mu\text{m}$) to represent the samples contaminated by outliers.

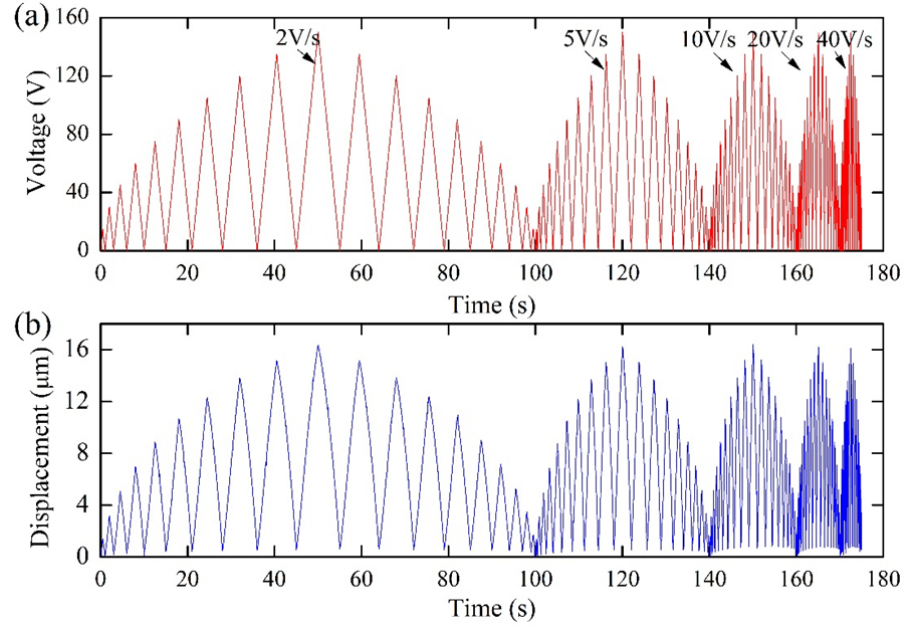


Fig. 4 The training data set. (a) The input voltage. (b) The output displacement.

Given the training data set, the hyperparameters are set as $C = 8.832 \times 10^5$ and $\sigma = 0.5716$ by PSO. Then the effects of the weighting function parameters k_1 and k_2 on model performance are investigated. It is found that with the increase of k_1 , the predictive accuracy of the model increases first and then decreases; as k_2 increases, the prediction capability of the model rises slowly, whereas the sparseness degrades. To make a trade-off between prediction ability and sparseness, parameters are chosen as $k_1 = 0.2$ and $k_2 = 1$.

Testing datasets with the capacity of 1378 and 1118 are generated by input voltage signals with two different waveforms as shown in Fig. 5 and each waveform corresponds to a series of test inputs with change-rate of 1V/s, 2V/s, 5V/s, 10V/s, 20V/s, 30V/s and 40V/s respectively. The testing waveforms are chosen as random triangular waves, which are consistent with the training waveforms.

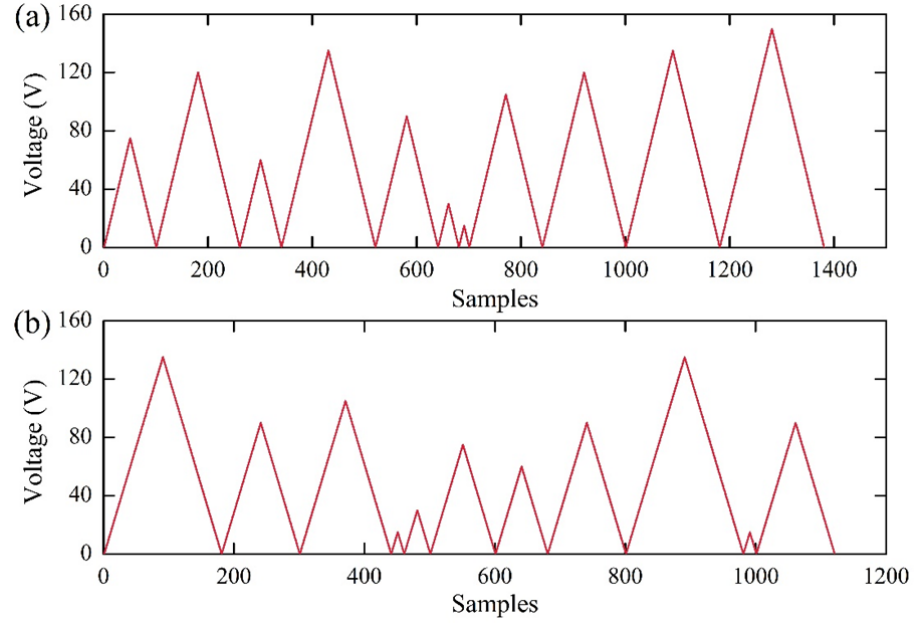


Fig. 5 The input voltage waveforms. (a) Test waveform 1. (b) Test waveform 2.

Taking the test data excited by the 10V/s-change-rate input voltage signals as an example, the prediction results with two different waveforms are shown in Fig. 6. The output results subject to voltage signals with all the other change-rates follow the similar pattern as Fig. 6. The results show that the proposed algorithm can achieve accurate regression for the PZA hysteresis under random triangular input excitation.

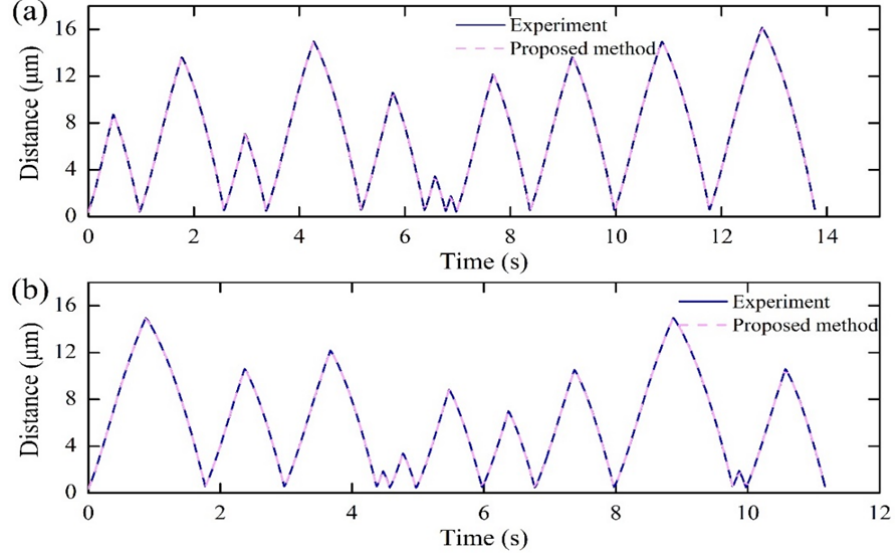


Fig. 6 Hysteresis modeling results under 10V/s-change-rate input excitation. (a) Test waveform 1. (b) Test waveform 2.

To validate the proposed model, LSSVR and Suykens' WLSSVR are also adopted to model the PZA hysteresis as comparison. The root mean squared error (RMSE) and maximal absolute error (MAXE) are employed to evaluate the robustness of regression models, which are respectively expressed as

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (y_k - \hat{y}_k)^2} \quad (16)$$

$$MAXE = \max(|y_k - \hat{y}_k|)$$

where y_k and \hat{y}_k are the true value and predicted value, respectively.

The performance comparisons of three algorithms are illustrated in Fig. 7. Results indicate that when the training samples are contaminated by noises, the LSSVR produces about twice as large RMSEs as WLSSVR. Although WLSSVR is more accurate than LSSVR, it still brings 50%–100% larger RMSEs than the proposed algorithm. The proposed algorithm yields the best RMSEs of about 0.005 μm , accounting for 0.0312% of the motion range. As for MAXE, the performance of LSSVR is comparable to that of WLSSVR and both of them produce over 4 times larger MAXEs compared with the proposed method. In contrast, MAXEs of the proposed method are within 0.1 μm , accounting for 0.625% of the motion range.

Therefore, it can be concluded that the classical LSSVR is sensitive to noises, indicated by its relatively large RMSEs and MAXEs. WLSSVR produces better RMSEs than LSSVR but it can bring more than 1.9% MAXEs at some points. The reason lies in that WLSSVR relies on the training errors of LSSVR, which are less robust. On the contrary, the proposed algorithm obtains the best RMSEs and MAXEs, indicating that it is more robust than other models, regardless of dataset or evaluation criterion.

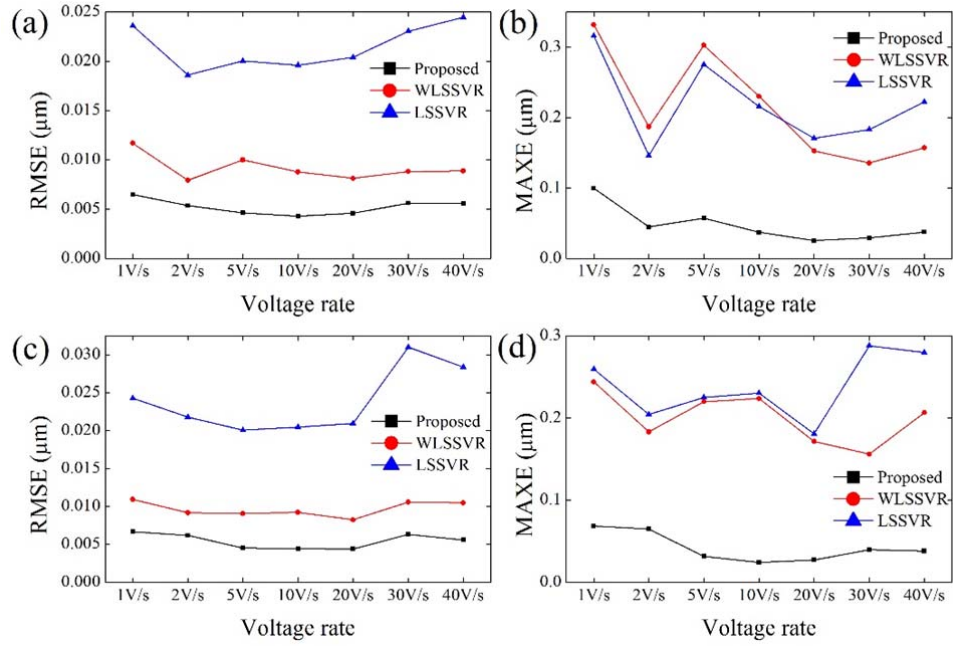


Fig. 7 Performance comparisons of LSSVR, Suykens' WLSSVR and the proposed method for hysteresis modeling. (a) and (b) RMSE and MAXE with test waveform 1. (c) and (d) RMSE and MAXE with test waveform 2.

Furthermore, the proposed algorithm reduces the sample size by 37.7% from 10,002 to 6,232. Consequently, the average time to calculate 1,378 samples for test waveform 1 is reduced from 0.271 s to 0.188 s, and the average time to calculate 1,118 samples for test waveform 2 is reduced from 0.226 s to 0.152 s. Thus, the adaptive WLSSVR proposed in this paper can mitigate the effects of noises in the training samples while reducing the computational cost by 32% on average.

5 Conclusion

In this paper, an adaptive WLSSVR is proposed to model the hysteresis of piezoelectric actuators when the training data contain noises. Firstly, PSO is used to select the optimal hyperparameters. Then, the weights of the samples are determined according to the adaptive weighting function and sample errors obtained by iterating WLSSVR. Test data with two different input waveforms at different change-rates are adopted to validate the robustness of the proposed method. The results show that the proposed method is more accurate than WLSSVR and unweighted LSSVR for regression with noises. Moreover, the proposed model can reduce the sample size and accelerate calculation with satisfactory regression results.

In the future work, some pruning algorithms will be investigated to further improve the sparseness of the model so that it can be applied to large-scale data.

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