Multicast Systems with Fair Scheduling in Non-Identically Distributed Fading Channels

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***Abstract*—Multicasting is emerging as an efficient method to deliver the same data to a group of users thereby saving network resources. The fairness between different multicast groups is an important quality of service (QoS) indication, but it has not been given significant attention. In this paper, we propose a normalized signal to noise ratio-based fair scheduling for multiple multicast groups in multicast systems. The system fairness and capacity are then analyzed and compared for both fair scheduling and greedy scheduling over independent but non-identically distributed (i.n.d.) fading channels. Closed-form expressions in terms of the system spectral efficiency, outage probability, system fairness and average bit error rate are derived in an uncoded/coded M-ary quadrature amplitude modulation-based adaptive transmission multicast system over i.n.d. Rayleigh fading channels. Numerical results show that compared with greedy scheduling, fair scheduling achieves considerably high fairness at the cost of slight system capacity loss regardless of the number of multicast groups. Our focus is on the physical layer without rate loss, but we also briefly discuss applications of the proposed scheduling in a cross-layer design subject to the loss rate QoS constraint.**

***Index Terms*—Multicast systems, fair scheduling, greedy scheduling, system capacity, system fairness, adaptive transmission, non-identically distributed fading channels.**

I. Introduction

In recent years, overwhelming demands and requirements for multimedia services such as mobile advertisements, stock prices, weather updates and multimedia entertainments are becoming urgent issues with the increasing data rate, power consumption and the number of supported mobile stations (MSs). Taking advantage of multicast to simultaneously transmit the same message via a single point-to-multipoint link instead of many point-to-point (unicast) links [1], [2], the limited system resources in the networks can be efficiently utilized. Meanwhile, the transmission power consumption at the base station (BS) can be minimized. Therefore, wireless multicast transmission has been widely adopted in 4G wireless communications standards (e.g., 3GPP LTE [3] and IEEE 802.16 [4]) as Evolved Multimedia Broadcast and Multicast Services (eMBMS) due to its higher spectral and power efficiencies, where the same data transmission across groups of adjacent cells is scheduled at the same carrier frequency synchronously. It greatly saves the radio resources to provide service towards other in-demand MSs. Currently, multicast scheduling has been investigated and considered as one attractive solution in the emerging 5G wireless networks [5], [6] to improve the network throughput, energy efficiency and reliability.

There has been much research on unicast scheduling in wireless networks [7], [8]. The basic idea for *unicast* is that, before downlink data transmission, each MS feeds a specified message indicating its maximum achievable rate back to the BS. Based on the feedback messages from different MSs, the BS determines the scheduled MS, and selects an appropriate rate for data transmission, where it is a point-to-point transmission. On the other hand, in *multicast* the BS transmits to one multicast group of MSs instead of a specified MS, where the transmit rate is chosen according to different schedulings. The major problem for multicast scheduling is the mismatch between the scheduling rate and the maximum achievable rate for each MS within a multicast group. For example, if the BS transmits at a higher rate than the maximum rate that some MSs can support, these MSs are incapable of decoding, thus leading to rate loss. Conversely, if the BS transmits at a lower rate requested by some MSs with worse channel condition, the data rate of other MSs with better channel condition will be limited. Therefore, the intractable problem of multicast is that, the BS schedules which group and at what rate to transmit.

Traditionally, by exploiting the characteristics of the time-varying channels in a wireless *unicast* system, the BS selects the MS with the best channel quality for data transmission to maximize system capacity, which is called *greedy scheduling* [9]. In [10], the system capacity performance of adaptive modulation with greedy scheduling was analyzed over independent but non-identically distributed (i.n.d.) Nakagami fading channels. In [11], [12], the fairness issue was taken into account, and capacity and fairness trade-off performance based on greedy scheduling and proportional fair (PF) scheduling was studied over i.n.d. Rayleigh fading channels. Greedy scheduling maximizes system capacity without any fairness consideration, whereas proportional *fair scheduling* [13], [14] achieves fairness among all MSs at the expense of system capacity loss. Recently, a proportional-fair resource allocation problem for downlink coordinated multi-point transmission systems is studied in [15], where multiple coordinated BSs serve their own MSs simultaneously. A parallel successive convex approximation-based algorithm is introduced in this work, which enables the proportional-fair metric maximization by updating optimization variables in parallel. However, this algorithm needs to iteratively obtain power allocation and

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transmitter precoder matrices to reduce the inter-cell interference, leading to large computational complexity. The work in [10]–[15] generally aims at the design of unicast systems.

Motivated by the greedy scheduling [9] applied in unicast systems, a greedy scheduling based scheme [16] was proposed for a wireless *multicast* system, where the BS needs to select the multicast group with the best channel quality for data transmission. The MSs in the same group are interested in the same information stream from the BS, and the affordable data rate is determined by the minimum received signal to noise ratio (SNR) among the MSs in a group. Therefore, the objective of greedy scheduling is to maximize the minimum received SNR of the selected group. In [16], multicast system capacity performance was evaluated only for greedy scheduling, where all MSs' channels were assumed to follow independent and identically distributed (i.i.d.) fading. However, in a practical wireless environment where the channels often follow i.n.d. fading, fairness issue among different groups should be appropriately treated since the groups close to the BS (i.e., equivalently with large average received SNR) tend to monopolize the system resource. An opportunistic multicast scheduling with resource fairness constraints was proposed in [17], where the system throughput was maximized by selecting the optimum transmission rate for each group based on the current channel condition and average received throughput of each MS, thereby achieving tradeoff between throughput and fairness.

Besides, the authors in [18] proposed a cross-layer design based modelling for QoS-driven multimedia multicast service to efficiently satisfy the diverse QoS requirement. The basic idea is to achieve high system throughput subject to QoS constraint from upper layer protocol. Particularly at the physical layer, a dynamic rate adaptation scheme was presented to optimize the average throughput subject to the loss rate, and the scenarios with i.i.d and i.n.d fading channels were investigated, respectively. However, this scheduling is only applicable for single multicast group, and hence multiple-multicast-group diversity and fairness among different groups are not considered.

Our main contributions in this paper are summarized as follows.

- We propose a normalized SNR-based fair scheduling for multiple multicast groups in multicast systems, where the BS selects the group with the best "normalized" channel (i.e., the channel with the largest received SNR compared with the group's average received SNR), rather than selecting the group with the "absolute" best channel quality.
- We evaluate and compare the system capacity and fairness performance of the multicast system with fair and greedy schedulings respectively over i.n.d. fading channels, and particularly present closed-form system capacity and fairness expressions over i.n.d. Rayleigh fading channels, which are verified by numerical results.
- By applying the adaptive transmission in multicast systems with fair and greedy schedulings, we study the performance of constant-power variable-rate uncoded/coded M-ary quadrature amplitude modulation (M-QAM) in terms of the system spectral efficiency, outage probability, system fairness and average bit error rate (BER). Numerical and simulation results show that the proposed fair scheduling maintains the fairness at the expense of some performance loss in comparison with greedy scheduling, which provides a tradeoff between system capacity and fairness.
- Inspired by [18] that only considers multicast systems with single multicast group, we further present a general framework based on the proposed scheduling for multicast systems with multiple multicast groups, where a cross-layer design subject to the loss rate QoS constraint specified from upper-layer protocol is considered. Two rate selection schedulings (fixed scheduling and dynamic scheduling) provide effective approaches to improve the system fairness, but also maximize the system capacity with each MS's loss rate constraint.

The remainder of this paper is organized as follows. Section II describes the fair and greedy schedulings-based multicast system models and assumptions. Section III presents the performance analysis of the proposed scheduling in terms of system capacity and fairness under i.n.d. fading channels. Section IV applies the proposed scheduling in an adaptive transmission based multicast system, then gives closed-form expressions for some performance measures over i.n.d. Rayleigh fading channels. Section V applies the proposed scheduling in a cross-layer design subject to the loss rate QoS constraint and presents two feasible schedulings. In Section VI, numerical and simulation results demonstrating the performance of the proposed scheduling are given, before concluding the paper in Section VII.

II. System Model

A. Downlink Transmission and Channel Models

We consider a downlink single-cell interference-free multicast network consisting of a BS and K active MSs subdivided into M multicast groups. Let K_m be the number of MSs in group m, $m = 1, \dots, M$, and we have $\sum_{m=1}^{M} K_m = K$. In each scheduling, the BS only serves one selected multicast group for data transmission during any given slot. We make the following assumptions [16], [19]:

(1) All active MSs are ideally synchronized to the BS, and the feedback channel state information (CSI) can be always received without delay and error.

(2) The data for each multicast group are infinitely backlogged, therefore the queueing dynamics is not considered.

(3) The channel is frequency flat fading, blockwise time invariant for each slot, and changes independently from slot to slot. This allows adaptive modulation and coding (AMC) to adjust on a frame-by-frame basis.

(4) The average transmit powers of the transmitted signals $\{x_{m,k}\}$ from the BS to MS k in group m are normalized (i.e., $\mathbb{E}\left\{|x_{m,k}|^2\right\} = 1$). The noise at the receiver is an i.i.d. complex Gaussian noise with zero mean and σ^2 variance.

(5) The fading channel gains $\left\{|h_{m,k}|^2\right\}$ from the BS to MS k in group m are i.n.d.. Therefore, the instantaneous received SNRs $\{\gamma_{m,k}\}$ are also i.n.d..

In this paper, we adopt the Rayleigh fading model to statistically characterize small scale fading channels for the simplicity in the performance analysis, but it can be extended to practical wireless fading channel models including path loss and shadowing effects [10]. As a result, the instantaneous received SNRs $\{\gamma_{m,k}\}$ follow exponential distribution with the probability density function (pdf) and cumulative distribution function (cdf) given by

$$f_{\gamma_{m,k}}(x) = \frac{1}{\bar{\gamma}_{m,k}} \exp\left(-\frac{x}{\bar{\gamma}_{m,k}}\right), \quad x \geq 0 \tag{1}$$

and

$$F_{\gamma_{m,k}}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}_{m,k}}\right), \quad x \geq 0 \tag{2}$$

where $\bar{\gamma}_{m,k}$ is the average received SNR of MS k in group m.

B. Fair and Greedy Schedulings

The total scheduling process for multicast systems can be described as follows. The BS first sends a multicast message[1] to all multicast group MSs. Then, all active MSs synchronize with the BS and estimate their channel gains using the multicast message. The MSs feed back their CSI (e.g., the received SNR or alternative channel quality indicator (CQI)) to the BS. We define $\gamma_m = \min_{k=1,\ldots,K_m} \{\gamma_{m,k}\}$ as the received SNR of group m. The achievable capacity C_m for group m is subject to the minimum capacity link between the BS and the MSs in that group, i.e., $C_m = \log_2(1+\gamma_m)$. Based on different schedulings, the BS selects the group with the maximum scheduling metrics among all groups (i.e., $\max_{m=1,\ldots,M} \{\Gamma_m\}$) to schedule data transmission. The scheduler selects the multicast group with the largest value of

$$\Gamma_m = \begin{cases} \gamma_m/\bar{\gamma}_m, & \text{fair scheduling} \\ \gamma_m, & \text{greedy scheduling} \end{cases} \tag{3}$$

where $\bar{\gamma}_m$ is the short-term average received SNR of group m. It is worth noting that $\bar{\gamma}_m$ for multicast systems defined later is different from that in unicast systems [13]. In unicast systems, the short-term average received SNR is directed against each individual MS, but in multicast systems, it is related to the received SNR distributions of all MSs in a group.

Greedy scheduling is used to maximize system capacity by exploiting the multicast groups' diversity gain. However, some multicast groups that are closer to the BS in average than the others (or equivalently, some multicast groups have relatively higher average received SNRs.) will dominate the channel access with the greedy scheduling, thereby biasing against the groups with lower average received SNRs. To solve the unfairness issue, fair scheduling is taken into account to potentially achieve fairness among all multicast groups at the expense of system capacity loss.

[1]The multicast message carries system configuration information (e.g., the multicast groups' identities).

C. Fairness Measure

In a similar manner, we apply the average system fairness [20] to measure fairness in wireless multicast systems exploiting multicast groups' diversity. This fairness measure has extensively been used in [11], [12] to describe the fairness characteristics in wireless unicast systems exploiting multiuser diversity.

It is assumed that all groups are equally important and have the same quality of service requirements. The self-fairness of group m is defined as [12], [20]

$$\zeta_m = \frac{-\log(P_m)}{\log(M)}, \tag{4}$$

where P_m is the proportion of resources allocated to group m, or equivalently the probability of group m being selected, and $\log(M)$ is a normalization factor. The average system fairness is then defined as [12], [20]

$$\zeta = \sum_{m=1}^{M} P_m \zeta_m = -\sum_{m=1}^{M} P_m \frac{\log(P_m)}{\log(M)}. \tag{5}$$

Note that a system is strictly fair if $P_m = 1/M, \forall\ m \in \{1, 2, \ldots, M\}$, regardless of their average SNRs, and the average system fairness is 1.

The fairness measure is closely related to the distribution of each MS's CSI in each group. It is shown from (3) that if all MSs' channels follow i.i.d. fading, the fair scheduling is equivalent to greedy scheduling and the system maintains strict fairness. On the other hand, if all MSs' channels follow i.n.d. fading, the unfairness with greedy scheduling that is induced by different MSs with different cdfs will be serious compared with fair scheduling. It is therefore necessary to analyze the fairness and capacity performances over i.n.d. fading channels in multicast systems with fair and greedy schedulings.

D. Adaptive Transmission Scheme

We consider the constant-power variable-rate uncoded [21] and coded [22] M-QAM as two alternative adaptive discrete rate modulation schemes. In the schemes, the whole SNR range is split into $J+1$ regions, and the constellation size M_j is assigned to the jth region ($j = 0, 1, ..., J$). While maintaining a prescribed BER (BER_0), the corresponding switching thresholds γ_j^{th} are given as follows.

For uncoded M-QAM:

$$\gamma_j^{th} = \begin{cases} [\text{erfc}^{-1}(2\text{BER}_0)]^2, & j = 1 \\ K_0(M_j - 1), & j = 0, 2, ..., J \\ +\infty, & j = J+1 \end{cases} \tag{6}$$

and for coded M-QAM:

$$\gamma_j^{th} = \begin{cases} K_j M_j, & j = 1, 2, ..., J \\ +\infty, & j = J+1 \end{cases} \tag{7}$$

where $\text{erfc}^{-1}(\cdot)$ is the inverse complementary error function, and $K_j = -1/b_j \ln(\text{BER}_0/a_j)$, where a_j and b_j are given in Table I [22]. These parameters are obtained by curve fitting with the least squares method, which makes the approximated

BER curves in (8) very close to the simulated BER points for all codes. $M_j = 2^j$ for uncoded M-QAM, and $M_j = 2^{j+1}$ for coded M-QAM. The corresponding coded M-QAM BER over AWGN channel is approximated as [22]

$$\mathrm{BER}_j \simeq a_j \exp\left(-\frac{b_j \gamma}{M_j}\right). \tag{8}$$

For uncoded M-QAM, a similar BER approximation [10] can be obtained by replacing a_j, b_j and M_j in (8) with $a_0 = 1/5$, $b_0 = 3/2$ and $M_j - 1$, respectively.

III. Performance Analysis without AMC

In this section, the capacity and fairness performance of the multicast system with fair and greedy schedulings is firstly analyzed without considering adaptive transmission scheme. The BS first selects the MS with the minimum $\gamma_{m,k}$ in group m, whose received SNR is defined as the received SNR of group m (i.e., γ_m), and then picks the group with the largest Γ_m among all groups. In the first step, the cdf of γ_m, $F_{\gamma_m}(x)$ is given by

$$\begin{aligned} F_{\gamma_m}(x) =& \Pr\{\gamma_m \le x\} = 1 - \Pr\{\gamma_m > x\} \\ =& 1 - \prod_{k=1}^{K_m} \Pr\{\gamma_{m,k} > x\} \\ =& 1 - \prod_{k=1}^{K_m} \left(1 - F_{\gamma_{m,k}}(x)\right). \end{aligned} \tag{9}$$

Taking the derivative of (9) with respect to x, we can obtain the pdf of γ_m, $f_{\gamma_m}(x)$ as,

$$f_{\gamma_m}(x) = \sum_{j=1}^{K_m} f_{\gamma_{m,j}}(x) \prod_{k=1,k\neq j}^{K_m} \left(1 - F_{\gamma_{m,k}}(x)\right). \tag{10}$$

The short-term average received SNR of group m, $\bar{\gamma}_m$ can be obtained by

$$\bar{\gamma}_m = \int_0^{+\infty} x f_{\gamma_m}(x) dx. \tag{11}$$

Let $\star = \underset{m}{\operatorname{argmax}}\{\Gamma_m\}$, and $f_{\gamma_\star}(x)$ be the pdf of $\gamma_\star$. Therefore, the multicast system capacity C is given by

$$C = \int_0^{+\infty} \log_2(1+x) f_{\gamma_\star}(x) dx. \tag{12}$$

Substituting (1) and (2) into (9) and (10), we have $F_{\gamma_m}(x)$ and $f_{\gamma_m}(x)$ in Rayleigh fading channels as $F_{\gamma_m}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}_m}\right)$ and $f_{\gamma_m}(x) = \frac{1}{\bar{\gamma}_m}\exp\left(-\frac{x}{\bar{\gamma}_m}\right)$ respectively, where $\frac{1}{\bar{\gamma}_m} = \sum_{k=1}^{K_m} \frac{1}{\bar{\gamma}_{m,k}}$. Interestingly, it is shown that the $\{\gamma_m\}$ $\forall m = 1, \ldots, M$ also follow exponential distribution, and the reciprocal of its average received SNR is the sum of the reciprocal of all MSs' average received SNRs in group m.

A. Fair Scheduling

1) System Capacity

For fair scheduling, $\star = \underset{m}{\operatorname{argmax}}\{\gamma_m/\bar{\gamma}_m\}$, and $f_{\gamma_\star}(x)$ can be given by (See Appendix A)

$$\begin{aligned} f_{\gamma_\star}(x) =& \sum_{m=1}^{M} f_{\gamma_m}(x) \prod_{n=1,n\neq m}^{M} F_{\gamma_n}\left(\frac{\bar{\gamma}_n}{\bar{\gamma}_m}x\right) \\ =& \sum_{m=1}^{M} \frac{1}{\bar{\gamma}_m} \exp\left(-\frac{x}{\bar{\gamma}_m}\right)\left[1 - \exp\left(-\frac{x}{\bar{\gamma}_m}\right)\right]^{M-1}. \end{aligned} \tag{13}$$

By substituting (13) into (12) and using the binomial expansion, the system capacity C is derived as

$$\begin{aligned} C =& \log_2 e \sum_{m=1}^{M}\sum_{n=0}^{M-1} \binom{M-1}{n} \frac{(-1)^n}{\bar{\gamma}_m} \\ & \times \int_0^{+\infty} \ln(1+x) \exp\left(-\frac{(n+1)x}{\bar{\gamma}_m}\right) dx \\ =& \frac{\log_2 e}{M} \sum_{m=1}^{M}\sum_{n=1}^{M} \binom{M}{n} (-1)^n \exp\left(\frac{n}{\bar{\gamma}_m}\right) \mathrm{Ei}\left(-\frac{n}{\bar{\gamma}_m}\right), \end{aligned} \tag{14}$$

where the Eq. (4.337.2) in [23] is used as follows:

For $\mathrm{Re}\{\mu\} \ge 0$ and $|\arg\beta| < \pi$,

$$\int_0^{+\infty} e^{-\mu x} \ln(1+\beta x) = -\frac{1}{\mu} e^{\frac{\mu}{\beta}} \mathrm{Ei}\left(-\frac{\mu}{\beta}\right)$$

and $\mathrm{Ei}(\cdot)$ is the exponential integral function defined by $\mathrm{Ei}(x) = \int_{-\infty}^{x} e^t/t dt$.

2) System Fairness

The probability of group m being selected, P_m, can be shown (see Appendix B)

$$P_m = \int_0^{+\infty} f_{\gamma_m}(x) \prod_{n=1,n\neq m}^{M} F_{\gamma_n}\left(\frac{\bar{\gamma}_n}{\bar{\gamma}_m}x\right) dx. \tag{15}$$

Substituting $f_{\gamma_m}(x)$ and $F_{\gamma_m}(x)$ over Rayleigh fading channels into (15), we have $P_m = 1/M, \forall m = 1, \ldots, M$. Therefore, strict fairness (i.e., $\zeta = 1$) is achieved by fair scheduling, even though all MSs experience i.n.d. Rayleigh fadings.

B. Greedy Scheduling

1) System Capacity

For greedy scheduling, $\star = \underset{m}{\operatorname{argmax}}\{\gamma_m\}$. Following the similar derivation in Appendix A, $f_{\gamma_\star}(x)$ is given by

$$\begin{aligned} f_{\gamma_\star}(x) =& \sum_{m=1}^{M} f_{\gamma_m}(x) \prod_{n=1,n\neq m}^{M} F_{\gamma_n}(x) \\ =& \sum_{m=1}^{M} \frac{1}{\bar{\gamma}_m} \exp\left(-\frac{x}{\bar{\gamma}_m}\right) \prod_{n=1,n\neq m}^{M} \left[1 - \exp\left(-\frac{x}{\bar{\gamma}_n}\right)\right] \\ =& \sum_{m=1}^{M} \frac{1}{\bar{\gamma}_m} \sum_{\tau\in T_m^M} \mathrm{sign}(\tau) \exp\left(-\frac{x}{\bar{\gamma}_m} - \tau x\right), \end{aligned} \tag{16}$$

where by adopting the notations of [24], [25], the set T_m^M is obtained by expanding the product of $\prod_{n=1,n\neq m}^{M}\left[1-\exp\left(-\frac{x}{\bar{\gamma}_n}\right)\right]$, and then taking the natural logarithm of each term. Sign$(\cdot)$ corresponds to the sign of each term in the expansion. To understand the meaning of the above notations, an example ($M=4, m=2$) is given by

$$\begin{aligned}
&\prod_{n=1,n\neq 2}^{4}\left[1-\exp\left(-\frac{x}{\bar{\gamma}_n}\right)\right]\\
=&\left[1-\exp\left(-\frac{x}{\bar{\gamma}_1}\right)\right]\left[1-\exp\left(-\frac{x}{\bar{\gamma}_3}\right)\right]\left[1-\exp\left(-\frac{x}{\bar{\gamma}_4}\right)\right]\\
=&1-\exp\left(-\frac{x}{\bar{\gamma}_1}\right)-\exp\left(-\frac{x}{\bar{\gamma}_3}\right)-\exp\left(-\frac{x}{\bar{\gamma}_4}\right)\\
&+\exp\left(-\frac{x}{\bar{\gamma}_1}-\frac{x}{\bar{\gamma}_3}\right)+\exp\left(-\frac{x}{\bar{\gamma}_1}-\frac{x}{\bar{\gamma}_4}\right)\\
&+\exp\left(-\frac{x}{\bar{\gamma}_3}-\frac{x}{\bar{\gamma}_4}\right)-\exp\left(-\frac{x}{\bar{\gamma}_1}-\frac{x}{\bar{\gamma}_3}-\frac{x}{\bar{\gamma}_4}\right),
\end{aligned}\tag{17}$$

and

$$\begin{aligned}
T_2^4=\Bigg\{&0,\frac{1}{\bar{\gamma}_1},\frac{1}{\bar{\gamma}_3},\frac{1}{\bar{\gamma}_4},\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_3}\right),\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_4}\right),\\
&\left(\frac{1}{\bar{\gamma}_3}+\frac{1}{\bar{\gamma}_4}\right),\left(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_3}+\frac{1}{\bar{\gamma}_4}\right)\Bigg\}.
\end{aligned}$$

Taking the sign of each term in (17), we get sign$(\tau)=\{1,-1,-1,-1,1,1,1,-1\}$.

By substituting (16) into (12), the system capacity C is given by

$$\begin{aligned}
C=&-\log_2 e\sum_{m=1}^{M}\sum_{\tau\in T_m^M}\frac{\text{sign}(\tau)}{1+\bar{\gamma}_m\tau}\\
&\times\exp\left(\frac{1}{\bar{\gamma}_m}+\tau\right)\text{Ei}\left[-\left(\frac{1}{\bar{\gamma}_m}+\tau\right)\right].
\end{aligned}\tag{18}$$

2) System Fairness

Following the similar derivation in Appendix B, the probability of group m being selected for greedy scheduling is given by

$$\begin{aligned}
P_m=&\int_0^{+\infty}f_{\gamma_m}(x)\prod_{n=1,n\neq m}^{M}F_{\gamma_n}(x)\,dx\\
=&\frac{1}{\bar{\gamma}_m}\sum_{\tau\in T_m^M}\text{sign}(\tau)\int_0^{+\infty}\exp\left[-\left(\frac{1}{\bar{\gamma}_m}+\tau\right)\right]xdx\\
=&\sum_{\tau\in T_m^M}\frac{\text{sign}(\tau)}{1+\bar{\gamma}_m\tau}.
\end{aligned}\tag{19}$$

Therefore, the average system fairness ζ can be obtained by substituting (19) into (4) and (5). Especially, when $\bar{\gamma}_1=\bar{\gamma}_2=\ldots=\bar{\gamma}_M$, $P_m=1/M, \forall m=1,\ldots,M$ and strict fairness is achieved by greedy scheduling.

IV. Performance Analysis with AMC

In this section, we apply fair scheduling and greedy scheduling in an adaptive transmission based multicast system, and derive closed-form expressions for the system spectral efficiency, outage probability, system fairness and average BER performance. For multicast transmission, the BS adjusts the modulation and coding rate according to the instantaneous channel condition, and schedules data transmission to the group with the maximum scheduling metrics among all groups, only if this group's received SNR is greater than the switching threshold γ_1^{th}. Otherwise, the outage happens and no data transmission is scheduled.

A. System Spectral Efficiency

The system spectral efficiency in terms of bps/Hz is defined as [10]

$$\left\langle\frac{R}{W}\right\rangle=\sum_{j=1}^{J}p_j\log_2 M_j,\tag{20}$$

where $\log_2 M_j$ is the data rate in region j, and $p_j=\int_{\gamma_j^{th}}^{\gamma_{j+1}^{th}}f_{\gamma_\star}(x)dx$ is the probability of the selected group's received SNR falling in region j. For fair scheduling, substituting (13) into (20), the system spectral efficiency over Rayleigh fading channels is given by

$$\begin{aligned}
\left\langle\frac{R}{W}\right\rangle=&\frac{1}{M}\sum_{j=1}^{J}\log_2 M_j\left\{\sum_{m=1}^{M}\left[1-\exp\left(-\frac{\gamma_{j+1}^{th}}{\bar{\gamma}_m}\right)\right]^M\right.\\
&\left.-\left[1-\exp\left(-\frac{\gamma_j^{th}}{\bar{\gamma}_m}\right)\right]^M\right\}.
\end{aligned}$$

For greedy scheduling, substituting (16) into (20), the system spectral efficiency is given by

$$\begin{aligned}
\left\langle\frac{R}{W}\right\rangle=&\sum_{j=1}^{J}\log_2 M_j\left\{\prod_{m=1}^{M}\left[1-\exp\left(-\frac{\gamma_{j+1}^{th}}{\bar{\gamma}_m}\right)\right]\right.\\
&\left.-\prod_{m=1}^{M}\left[1-\exp\left(-\frac{\gamma_j^{th}}{\bar{\gamma}_m}\right)\right]\right\}.
\end{aligned}$$

B. Outage Probability

The outage occurs when the selected group's received SNR is lower than the switching threshold γ_1^{th}. The outage probability O_p is defined as the probability of the selected group's received SNR falling below γ_1^{th}, and it is given by [26]

$$O_p=\int_0^{\gamma_1^{th}}f_{\gamma_\star}(x)dx.\tag{21}$$

Substituting (13) and (16) into (21), the outage probability O_p for fair and greedy schedulings over Rayleigh fading channels is given by

$$O_p=\frac{1}{M}\sum_{m=1}^{M}\left[1-\exp\left(-\frac{\gamma_1^{th}}{\bar{\gamma}_m}\right)\right]^M,$$

and

$$O_p=\prod_{m=1}^{M}\left[1-\exp\left(-\frac{\gamma_1^{th}}{\bar{\gamma}_m}\right)\right].$$

C. System Fairness

For fair scheduling in an adaptive transmission system, the necessary condition of one group being selected is its received SNR greater than the switching threshold γ_1^{th}. Therefore, the probability of group m being selected, P_m, is given by

$$\begin{aligned}P_m =& \int_{\gamma_1^{th}}^{+\infty} f_{\gamma_m}(x) \prod_{n=1,n\neq m}^{M} F_{\gamma_n}\left(\frac{\bar{\gamma}_n}{\bar{\gamma}_m}x\right) dx\\ =& \int_{\gamma_1^{th}}^{+\infty} \frac{1}{\bar{\gamma}_m} \exp\left(-\frac{x}{\bar{\gamma}_m}\right)\left[1-\exp\left(-\frac{x}{\bar{\gamma}_m}\right)\right]^{M-1} dx\\ =& \frac{1}{M}\left[1-\left(1-\exp\left(-\frac{\gamma_1^{th}}{\bar{\gamma}_m}\right)\right)^M\right]. \end{aligned} \tag{22}$$

It is easily shown from (22) that $P_m < 1/M$, and P_m increases with the increase of $\bar{\gamma}_m$ when M is fixed. As $M \to \infty$, $P_m \to 1/M$ and the average system fairness $\zeta \to 1$. Fair scheduling can not make an adaptive transmission based multicast system with strict fairness due to the outage.

For greedy scheduling, P_m is given by

$$\begin{aligned}P_m =& \int_{\gamma_1^{th}}^{+\infty} f_{\gamma_m}(x) \prod_{n=1,n\neq m}^{M} F_{\gamma_n}(x)\, dx\\ =& \frac{1}{\bar{\gamma}_m}\sum_{\tau\in T_m^M} \mathrm{sign}(\tau)\int_{\gamma_1^{th}}^{+\infty} \exp\left[-\left(\frac{1}{\bar{\gamma}_m}+\tau\right)\right] x dx\\ =& \sum_{\tau\in T_m^M} \frac{\mathrm{sign}(\tau)\exp\left[-\left(\frac{1}{\bar{\gamma}_m}+\tau\right)\gamma_1^{th}\right]}{1+\bar{\gamma}_m\tau}. \end{aligned} \tag{23}$$

Therefore, the average system fairness ζ can be obtained by substituting (23) into (4) and (5).

D. System Average BER

The system average BER ($\overline{\mathrm{BER}}$) in an adaptive transmission system is defined as the ratio of the average number of bits in error to the system spectral efficiency, and it is given by [10]

$$\overline{\mathrm{BER}} = \frac{\sum_{j=1}^{J} \overline{\mathrm{BER}}_j \log_2 M_j}{\sum_{j=1}^{J} p_j \log_2 M_j}, \tag{24}$$

where $\overline{\mathrm{BER}}_j$ is the average BER of the selected group's received SNR falling in region j, which is expressed as $\overline{\mathrm{BER}}_j = \int_{\gamma_j^{th}}^{\gamma_{j+1}^{th}} \mathrm{BER}_j f_{\gamma_\star}(x) dx$. By substituting (8) and (13) into $\overline{\mathrm{BER}}_j$, we have $\overline{\mathrm{BER}}_j$ for fair scheduling as

$$\begin{aligned}\overline{\mathrm{BER}}_j =& \sum_{m=1}^{M}\sum_{n=0}^{M-1}\binom{M-1}{n}\frac{(-1)^n a_j}{\bar{\gamma}_m}\\ &\times \int_{\gamma_j^{th}}^{\gamma_{j+1}^{th}} \exp\left[-\left(\frac{b_j}{M_j}+\frac{n+1}{\bar{\gamma}_m}\right)x\right] dx\\ =& \frac{a_j M_j}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\binom{M}{n}\frac{(-1)^n n}{\bar{\gamma}_m b_j + M_j n}\\ &\left\{\exp\left[-\left(\frac{b_j}{M_j}+\frac{n}{\bar{\gamma}_m}\right)\gamma_{j+1}^{th}\right]\right.\\ &\left.-\exp\left[-\left(\frac{b_j}{M_j}+\frac{n}{\bar{\gamma}_m}\right)\gamma_j^{th}\right]\right\}. \end{aligned} \tag{25}$$

For greedy scheduling, substituting (8) and (16) into $\overline{\mathrm{BER}}_j$, we have

$$\begin{aligned}\overline{\mathrm{BER}}_j =& \sum_{m=1}^{M}\sum_{\tau\in T_m^M}\frac{a_j \mathrm{sign}(\tau)}{\bar{\gamma}_m}\\ &\times \int_{\gamma_j^{th}}^{\gamma_{j+1}^{th}} \exp\left[-\left(\frac{b_j}{M_j}+\frac{1}{\bar{\gamma}_m}+\tau\right)x\right] dx\\ =& \sum_{m=1}^{M}\sum_{\tau\in T_m^M}\frac{a_j M_j \mathrm{sign}(\tau)}{\bar{\gamma}_m b_j + M_j(\bar{\gamma}_m\tau+1)}\\ &\left\{\exp\left[-\left(\frac{b_j}{M_j}+\frac{1}{\bar{\gamma}_m}+\tau\right)\gamma_j^{th}\right]\right.\\ &\left.-\exp\left[-\left(\frac{b_j}{M_j}+\frac{1}{\bar{\gamma}_m}+\tau\right)\gamma_{j+1}^{th}\right]\right\}. \end{aligned} \tag{26}$$

Substituting (25) and (26) into (24), the system $\overline{\mathrm{BER}}$ with fair and greedy schedulings can be easily obtained respectively.

V. General Framework for Cross-Layer Design

So far, to guarantee the reliability at the physical layer, the proposed scheduling schedules data transmission at the rate that the MS with the minimum received SNR in the selected group can support, leading to low system capacity and long delay. However, for practical multimedia multicast services, the system is able to tolerate an acceptable level of data loss to achieve high system capacity or low delay. Here we present a general framework on how to apply the proposed scheduling in a cross-layer design subject to the loss rate QoS constraint specified from upper-layer protocol. We consider a time division multiplexing system, where different time slot proportions are allocated to the MSs sorted in order of SNRs.

Without loss of generality, we consider that the approach to selecting the scheduled multicast group is the same as in the Section II. B (i.e., the BS selects the group with the maximum scheduling metrics among all groups to schedule data transmission.). However, the scheduling rate selection is different from the proposed one which selects the rate that the MS with the minimum received SNR in the selected group can achieve. The BS firstly picks up a received SNR from $K_\star$ MSs' received SNRs as the *dominating* SNR, denoted by $\gamma_{\star,dom}$, and begins transmission at the rate of $\log_2(1+\gamma_{\star,dom})$, where we have $K_\star$ optional $\gamma_{\star,dom}$. If $\gamma_{\star,k} \geq \gamma_{\star,dom}$, MS k

can receive the signal correctly without any rate loss. On the other hand, MS k can not decode the received signal correctly given $\gamma_{\star,k} < \gamma_{\star,dom}$, thus resulting in complete rate loss. Averaging over all time slots, a certain level of data loss that depends on the upper-layer protocol can be usually tolerable for multimedia services. Let q_{cons} denote the loss rate QoS constraint, and $q_{m,k}$ denote the loss rate of MS k in group m, which is defined as the ratio of the total amount of data that has to be dropped due to decoding incorrectly for MS k in group m to that of the data received by group m. Generally, $q_{m,k} \leq q_{cons}$ should be satisfied for any MS.

Let the subscript $\pi(k)$ denote the permutation of $\gamma_{m,k}, k = 1, 2, ..., K_m$ such that $\gamma_{m,\pi(k)}$ decreases as k increases from 1 to K_m. Further, let $\lambda_{m,k}$ denote time slot proportion allocated to MS k of group m, where $0 \leq \lambda_{m,k} \leq 1$ and $\sum_{k=1}^{K_m} \lambda_{m,k} = 1$. According to different $\lambda_{m,k}$s, we consider the following two schedulings.

A. Fixed scheduling

Let $\lambda_{m,\pi(k)} = 1$ over all time slots for a fixed k provided that group m is selected, i.e., the position of the dominating SNR is fixed as $\pi(k)$. In this case, we call it *fixed scheduling*. Specifically, if $\lambda_{m,\pi(K_m)} = 1$ for any m (i.e., $\gamma_{m,dom} = \gamma_{m,\pi(K_m)}$), the capacity of group m is determined by the minimum received SNR among all MSs in group m, thus guaranteeing no rate loss for any MS, which shows that the proposed normalized SNR-based fair scheduling in the Section II. B is a special case of fixed scheduling. On the other hand, If $\lambda_{m,\pi(1)} = 1$ for any m, the highest system capacity without loss rate constraint can be obtained. Particularly, when all MSs in group m experience i.i.d fading channels, $q_{m,k}$ for a fixed k under fixed scheduling equals $1 - k/K_m$. To satisfy the constraint of $q_{m,k} \leq q_{cons}$, let $k = \lceil K_m(1 - q_{cons}) \rceil$, where $\lceil (\cdot) \rceil$ is the smallest integer greater than or equal to $(\cdot)$.

B. Dynamic scheduling

$\lambda_{m,\pi(k)}$ may vary in accordance with different q_{cons} to maximize the system capacity, i.e., the position of the dominating SNR can be dynamically adjusted. In this case, we call it *dynamic scheduling*, which can be used by solving the following system capacity optimization problem,

$$\begin{aligned} &\underset{\lambda_{m,\pi(k)}}{\text{argmax}} \left\{ \sum_{m=1}^{M} C_m \right\} \\ \text{s.t.} \quad & q_{m,k} \leq q_{cons}, \\ & 0 \leq \lambda_{m,\pi(k)} \leq 1, \quad \sum_{k=1}^{K_m} \lambda_{m,\pi(k)} = 1, \\ & m = 1, 2, ..., M, k = 1, 2, ..., K_m, \end{aligned}$$

where $q_{m,k} = 1 - R_{m,k}/C_m$, and $R_{m,k}$ denotes the average data rate of MS k of group m.

Fixed scheduling provides a simple approach to guarantee the loss rate, although it can not flexibly control the loss rate. By adaptively controlling the position of dominating SNR, dynamic scheduling can efficiently maximize the system capacity, while at the same time, flexibly satisfy the rate loss constraint variation. The implementation of dynamic scheduling needs to solve more complicated optimization problem, leading to great complexity.

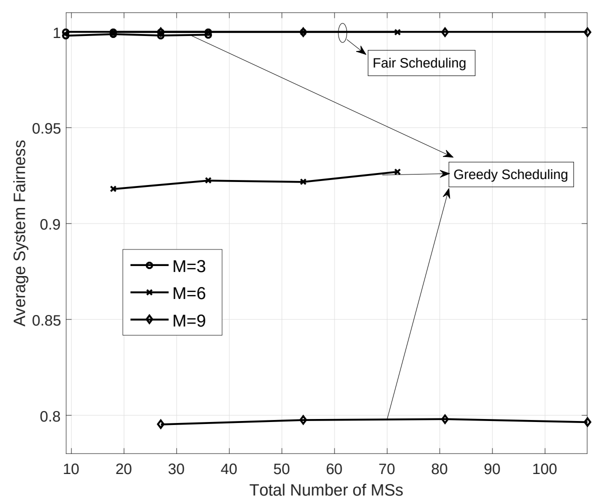

Fig. 1. Average system fairness ζ versus total number of MSs for M=3, 6 and 9, respectively.

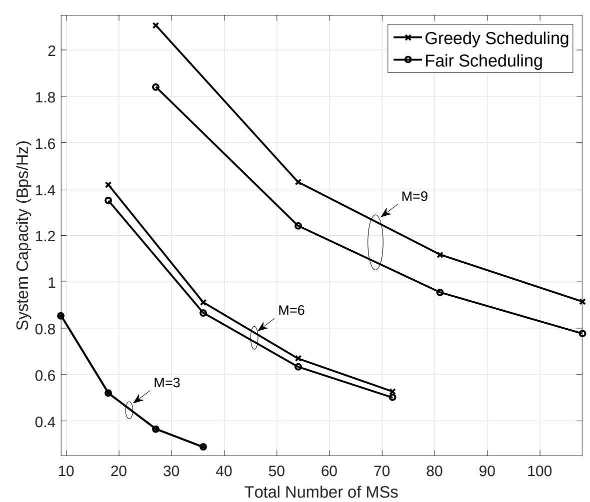

Fig. 2. System capacity C in bps/Hz versus total number of MSs for M=3, 6 and 9, respectively.

VI. Numerical Results

A. Numerical Results without AMC

Fig. 1 and 2 show the average system fairness and the system capacity in bps/Hz versus the total number of MSs with fair and greedy schedulings by changing the values of the number of multicast groups M, and the number of each group K_m under an i.n.d. Rayleigh fading channel environment. The setup in the figures is as follows. We adopt $M = 3, 6, 9$ and $K_m = 3, 6, 9, 12$, respectively, where the same value of K_m, $\forall m = 1, \ldots, M$ is used when M is fixed. When

$M = 3$, the average received SNR of each MS in all groups is generated from uniform distribution between 0dB and 3dB [i.e., uniform(0,3)]. When M increases from 3 to 6, the average received SNRs of the MSs in three new groups are generated from uniform(3,6). When M increases from 6 to 9, the average received SNRs of the MSs in three increased groups are generated from uniform(6,9). The same sets of random average received SNRs are used for Fig. 1 and 2 to demonstrate the tradeoff with fair and greedy schedulings between system capacity and average system fairness. As shown in Figs. 1 and 2, the numerical results with solid lines obtained using the derived expressions match the simulation results with markers.

As expected from Fig. 1, the fair scheduling achieves strict fairness since all groups have the same opportunities to access the channel, whereas the greedy scheduling demonstrates lower fairness as M increases since the groups with higher average received SNRs have greater opportunities to access the channel. When $M = 3$, fair scheduling and greedy scheduling have almost the same performance in terms of the system capacity and average system fairness, respectively. The reason is that all groups have the close average received SNRs due to the average received SNR of each MS with the same distribution. It is shown from Fig. 2 that the system capacity is degraded with the increase of K_m when M is fixed, because the achievable capacity for a multicast group is dominated by the MS with the lowest received SNR among all MSs. On the other hand, the capacity performance is improved with the increase of M when K_m is fixed, which is due to the multicast groups' diversity gain and the increased average received SNRs of the MSs in new groups. The fair scheduling achieves strict fairness at the cost of a small loss of system capacity compared with greedy scheduling.

B. Numerical Results with AMC

We present the numerical results with lines using the derived expressions in Figs. 3-6 in terms of system spectral efficiency, outage probability, system fairness and average BER for fair scheduling and greedy scheduling in an adaptive transmission based multicast system over Rayleigh fading channels with different average received SNR distributions, which are verified through Matlab simulation results with markers. We assume that there are eight MSs in each multicast group, and observe the performance variation as the number of groups M increases from 3 to 15 with step size 3. To compare the performance in the adaptive discrete rate modulation schemes, we generate the average received SNR of each MS following the distribution uniform(12,15) when $M = 3$. As the number of groups increases, the average received SNR distributions for three new groups follow uniform(15,18) when $M = 6$, uniform(18,21) when $M = 9$, uniform(21,24) when $M = 12$ and uniform(24,27) when $M = 15$, respectively.

In simulations with coded M-QAM adaptive transmission scheme, we use a_j, b_j and M_j as Table 1 in [22]. For convenience, part of this table is reproduced in this paper as Table 1. These parameters are actually applied in an AWGN channel, whereas for a frequency-selective fading channel, we can exploit efficient equalization and detection approaches to approach the performance of the corresponding system over an AWGN channel [27].

TABLE I
PARAMETERS a_j AND b_j VERSUS M_j

j	M_j	a_j	b_j
1	4	896.0704	10.7367
2	8	404.4353	6.8043
3	16	996.5492	8.7345
4	32	443.1272	8.2282
5	64	296.6007	7.9270

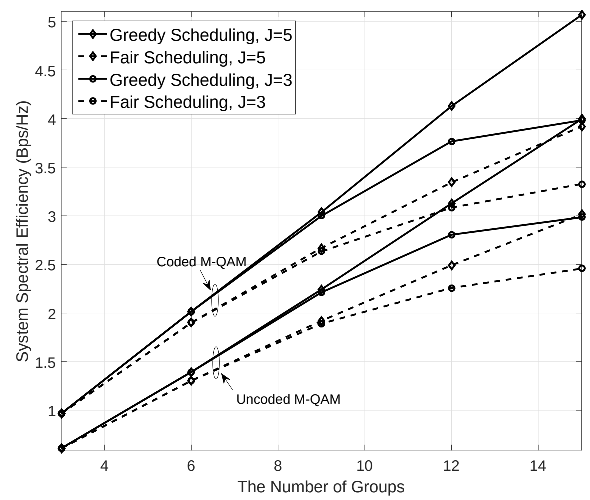

Fig. 3. System spectral efficiency $\left\langle \frac{R}{W} \right\rangle$ in bps/Hz versus the number of groups M ($\text{BER}_0 = 10^{-3}$).

Fig. 3 shows system spectral efficiency $\left\langle \frac{R}{W} \right\rangle$ in bps/Hz versus the number of groups M for uncoded and coded M-QAM adaptive transmission schemes with $\text{BER}_0 = 10^{-3}$. As

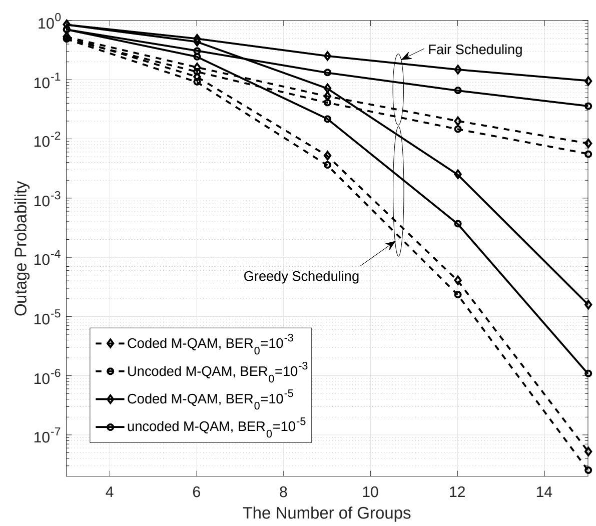

Fig. 4. Outage probability O_p versus the number of groups M ($J = 3$).

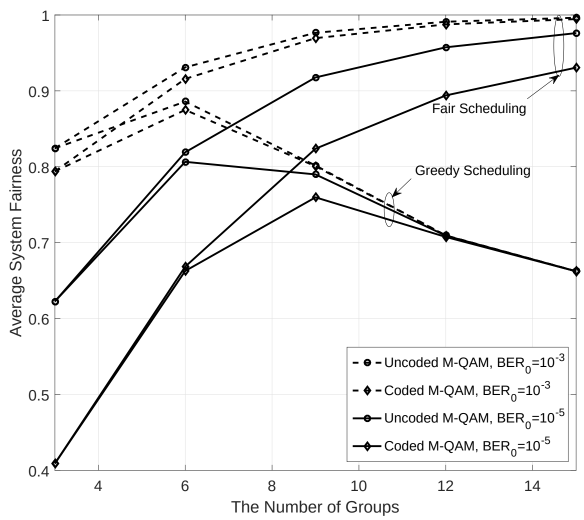

Fig. 5. Average system fairness ζ versus the number of groups M ($J = 3$).

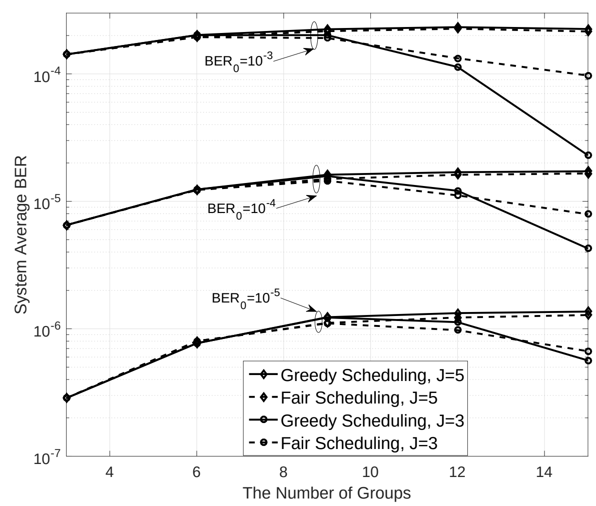

Fig. 6. System average BER versus the number of groups M (Uncoded M-QAM).

the number of groups increases and accordingly the average received SNR increases with the above defined uniform distribution, there is a system spectral efficiency improvement. Meanwhile, the performance in $J = 5$ outperforms that of $J = 3$ with the increase of M due to the adoption of a larger constellation size. In addition, the coded M-QAM exploits higher system spectral efficiency than the uncoded M-QAM due to the coding gain, and the greedy scheduling performs better than that of fair scheduling by exploiting the multicast groups diversity gain as expected.

Fig. 4 shows outage probability O_p versus the number of groups M for uncoded and coded M-QAM adaptive transmission schemes with $J = 3$. As shown in Fig. 4, the outage probability of greedy scheduling drops more quickly than that of fair scheduling as the number of groups increases. Particularly, when the number of groups is large, the outage (i.e., no transmission) happens less likely. As the prescribed BER_0 increases, the outage probability decreases, which can be verified by (6) and (7). The season is that the outage probability is an increasing function of γ_1^{th} shown in (21), while γ_1^{th} is a decreasing function of BER_0.

Fig. 5 shows average system fairness ζ versus the number of groups M for uncoded and coded M-QAM adaptive transmission schemes with $J = 3$. As shown in Fig. 5, the average system fairness of fair scheduling is far better than that of greedy scheduling at the expense of system spectral efficiency loss as the number of groups increases. The average system fairness ζ approaches to 1 as M increase to a large number as expected. However, the average system fairness of greedy scheduling becomes worse when M is larger than six, because the average received SNR differences between the groups with low average SNR and the groups with high average SNR become greater as M increases. The average system fairness increases with the increase of the prescribed BER_0, as the average system fairness is an increasing function of BER_0 as shown in (6), (7), (22) and (23).

Fig. 6 shows system average BER versus the number of groups M for uncoded M-QAM adaptive transmission schemes. For different schedulings with $J = 3$ and $J = 5$, the system average BER is always lower than the prescribed BER_0, which is due to the fact that the instantaneous BER is always below the prescribed BER_0 according to the switching thresholds. The system average BER of fair scheduling is very close to that of greedy scheduling with $J = 5$, because the system average BER is determined by the ratio of the average number of bits in error to the system spectral efficiency in (24). Although the greedy scheduling has higher system spectral efficiency than fair scheduling, its average BER is also higher for higher order modulation which is more frequently used.

VII. Conclusion and Future Work

In this paper, we propose a normalized SNR-based fair scheduling for multiple multicast groups in multicast systems. Compared with greedy scheduling, we analyze the system capacity and fairness performances of the multicast system with fair scheduling over i.n.d. fading channels, and present closed-form system capacity and fairness expressions over i.n.d. Rayleigh fading channels. By employing the adaptive transmission in multicast systems with fair and greedy schedulings, we investigate the performances based on both uncoded and coded M-QAM schemes in terms of the system spectral efficiency, outage probability, system fairness and average BER. Numerical and simulation results show that the proposed fair scheduling improves the fairness between the groups with different average SNR distributions at the expense of slight capacity loss compared with greedy scheduling.

Our focus in this paper has been on the case that strict rate loss constraint is required. How to develop efficient low-complexity scheduling algorithms in a cross-layer design subject to flexible QoS constraints with the proposed scheduling will be interesting future research topics. In addition, taking advantage of dynamic single frequency networks, the Multicast-Broadcast Single-Frequency Network (MBSFN) transmission mode can greatly improve the spectral efficiency, where groups of adjacent BSs send the same signal simultaneously over the same frequency. The work in [28] has considered joint beamforming among cooperating BSs to improve the received SNR of the MS with the worst channel condition in a multicast group. It exploits the macrodiversity and effectively eliminates the inter-cell interference to dramatically increase the system capacity, and the fairness has not been considered yet. As shown in Section VI, our proposed scheduling outperforms the existing scheme in terms of fairness for multicast systems with single BS, but the work is also possible to be extended to the MBSFN with multiple cooperating BSs subject to fairness constraint.

Appendix A
Derivation of (13)

The cdf of $\gamma_\star$, $F_{\gamma_\star}(x)$, is given by

$$\begin{aligned} F_{\gamma_\star}(x) &= \Pr\{\gamma_\star \le x\} \\ &= \Pr\{\gamma_1 \le x, \gamma_2 \le x, ..., \gamma_M \le x\} \\ &= \Pr\{\Gamma_1\bar{\gamma}_1 \le x, \Gamma_2\bar{\gamma}_2 \le x, ..., \Gamma_M\bar{\gamma}_M \le x\} \\ &= \prod_{m=1}^{M} F_{\Gamma_m}\left(\frac{x}{\bar{\gamma}_m}\right). \end{aligned} \tag{27}$$

By taking the derivative of (27) with respect to x, we can obtain the pdf of $f_{\gamma_\star}(x)$ as follows,

$$f_{\gamma_\star}(x) = \sum_{m=1}^{M} \frac{1}{\bar{\gamma}_m} f_{\Gamma_m}\left(\frac{x}{\bar{\gamma}_m}\right) \prod_{n=1, n\neq m}^{M} F_{\Gamma_n}\left(\frac{x}{\bar{\gamma}_m}\right), \tag{28}$$

where $f_{\Gamma_m}(x)$ and $F_{\Gamma_m}(x)$ are the pdf and cdf of Γ_m (i.e., $\gamma_m/\bar{\gamma}_m$), respectively. By applying the Jacobian transformation, we have

$$f_{\Gamma_m}(x) = \bar{\gamma}_m f_{\gamma_m}(\bar{\gamma}_m x), \tag{29}$$

and

$$F_{\Gamma_m}(x) = F_{\gamma_m}(\bar{\gamma}_m x). \tag{30}$$

Therefore, (13) can be obtained by substituting (29) and (30) into (28).

Appendix B
Derivation of (15)

For fair scheduling, the probability of group m being selected is the probability of $\Gamma_m > \Gamma_n$ (i.e., $\gamma_m/\bar{\gamma}_m > \gamma_n/\bar{\gamma}_n), \forall n \in \{1, 2, \ldots, M\}$ and $n \neq m$. Therefore, P_m is derived as

$$\begin{aligned} P_m &= \Pr\left\{\frac{\gamma_1}{\bar{\gamma}_1} < \frac{\gamma_m}{\bar{\gamma}_m}, \frac{\gamma_2}{\bar{\gamma}_2} < \frac{\gamma_m}{\bar{\gamma}_m}, ..., \frac{\gamma_M}{\bar{\gamma}_M} < \frac{\gamma_m}{\bar{\gamma}_m}\right\} \\ &= \prod_{n=1, n\neq m}^{M} \Pr\left\{\gamma_n < \frac{\bar{\gamma}_n}{\bar{\gamma}_m}\gamma_m\right\} \\ &= \prod_{n=1, n\neq m}^{M} \int_0^{+\infty} f_{\gamma_m}(x) \int_0^{\frac{\bar{\gamma}_n}{\bar{\gamma}_m}x} f_{\gamma_n}(y)dydx \\ &= \int_0^{+\infty} f_{\gamma_m}(x) \prod_{n=1, n\neq m}^{M} F_{\gamma_n}\left(\frac{\bar{\gamma}_n}{\bar{\gamma}_m}x\right)dx. \end{aligned} \tag{31}$$

By substituting $f_{\gamma_m}(x)$ and $F_{\gamma_m}(x)$ over i.n.d. Rayleigh fading channels into (31), P_m is given by

$$\begin{aligned} P_m &= \int_0^{+\infty} \frac{1}{\bar{\gamma}_m} \exp\left(-\frac{x}{\bar{\gamma}_m}\right)\left[1 - \exp\left(-\frac{x}{\bar{\gamma}_m}\right)\right]^{M-1} dx \\ &= \sum_{m=1}^{M} \binom{M}{m-1} \frac{(-1)^{m-1}}{m} \\ &= \frac{1}{M}. \end{aligned}$$

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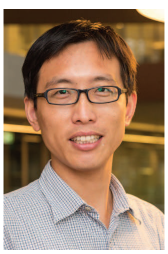

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