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A Low Cost Interpolation Based Detection Algorithm for Medium-size Massive MIMO-OFDM Systems

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***Abstract*—The great potential of exploiting millimeter wave (mmwave) frequency spectrum for emerging fifth-generation (5G) wireless networks has motivated the study of massive multiple-input multiple-output (MIMO) for achieving high data rate. For medium-size massive MIMO with orthogonal frequency division multiplexing (OFDM) uplink systems, the minimum mean square error (MMSE) based soft-output detector is often used due to its better bit error rate (BER) performance compared to the matched filter detector. Although the multipath channel can be converted into a set of parallel flat-fading channels by using OFDM thus reducing the complexity of receiver design, the tone by tone (per subcarrier) detection methods based on the state-of-the-art low complexity MMSE still incur considerably high computational complexity since the number of tones is typically very large. To reduce the complexity, the interpolation-based matrix inversion algorithms for small-size MIMO-OFDM systems have been proposed, which compute the matrix inversion by interpolating separately the adjoint and determinant. In this paper, we find that the (regularized) Gram matrix inversions have strong correlation between different subcarriers. By exploiting this strong correlation, we propose a linear interpolation based MMSE detection algorithm that directly interpolates the inverted MMSE matrices for a small number of subcarriers to obtain matrix inversions for all other subcarriers, thereby significantly reducing the number of matrix inversion required. Extensive simulations show that with small BER performance loss compared to the exact MMSE detector, the proposed algorithm can reduce the complexity to the level of the matched filter algorithm.**

***Index Terms*—Detection, Interpolation, Mmwave, Medium-size Massive MIMO-OFDM, MMSE.**

I. Introduction

Due to the large bandwidth and small antennas physical size, millimeter wave (mmWave) beamforming [1], [2] has been regarded as an attractive solution for 5G wireless communication systems [3], [4]. Mmwave communications leverage potentially available spectrum from 30 GHz to 300 GHz to provide high data rates, thus being considered for many applications including personal area network (PAN) (e.g., WirelessHD [5]), wireless local area networks (WLAN) (e.g., IEEE 802.11ad [6]), vehicular communications associated with radar [7] and wearable networks [8]. In recent years, the massive multiple-input multiple-output (MIMO) which typically employs a magnitude of more antennas in receiver than that in transmitter has attracted a lot of attention [9], [10]. Mmwave's high carrier frequency facilitates packing a large number of antenna elements in a physically limited space, thus enabling massive MIMO to be feasible. In addition, due to the ability to cope with severe frequency-selective fading channel without complex equalization, orthogonal frequency division multiplexing (OFDM) has been considered to joint massive MIMO in mmWave systems [11], [12].

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When the number of antennas becomes extremely large and assuming channel matrix $\mathbf{H}$ has independent and identically distributed (i.i.d.) entries, the channel vectors become orthogonal to each other with the consequence that $\mathbf{H}^H\mathbf{H}$ converges to a scaled identity matrix, which enables simple detection algorithms such as a matched filter to achieve very good performance. However, for practical medium-size massive MIMO systems, the minimum mean square error (MMSE) with soft-output algorithms [13], [14] are often preferred due to its better bit error rate (BER) performance over matched filter and relatively low complexity [13]. When the orthogonal frequency division multiplexing (OFDM) is employed in such systems, the per-tone based MMSE detection incurs very high complexity. Specifically, although [13] can successfully work around the direct computation of the matrix inversion, the matrix-matrix multiplication for the Gram matrix with cubic complexity is still needed. The overall complexity is $\mathcal{O}(NN_t^2N_r)$, where N is the number of subcarriers, and N_t and N_r are the numbers of the transmit and receive antennas, respectively. Recently, [14] proposes an algorithm which can reduce the total complexity to $\mathcal{O}(NN_tN_r)$ level, but it is still much higher than the matched filter algorithm.

In OFDM systems, as the frequency domain channel coefficients between the adjacent subcarriers are typically highly correlated, interpolation method is often employed to compute channel coefficients or matrix inversions to reduce the complexity. For matrix inversion by interpolation, [15] and [16] exploited the fact that the adjoint matrix and the determinant can be represented as a polynomial matrix respectively, and thus they could be separately interpolated to compute the matrix inversion. The scheme in [15] required a large number of base interpolation points for matrix inversion computation. To reduce the number of base points, [17] proposed a Gaussian approximation based phase shifted interpolation method for matrix inversion, which approximately estimates the required number of base points for each real-time implementation thereby reducing the complexity further. [18] proposed a Banachiewicz formula based matrix inversion algorithm with low complexity. But all these interpolation based algorithms were designed for small-size MIMO and cannot be easily extended to massive MIMO-OFDM applications. Recently, [19] proposed a discrete Fourier transform (DFT) interpolation-based low complexity zero-forcing matrix computation technique for massive MIMO-OFDM systems, which achieves a good tradeoff between sum rate performance and complexity. However, its BER performance applied to soft output detector was not evaluated.

In this paper, from the asymptotic property of Gram matrix for massive MIMO, we conjecture that there should be strong correlation between the (regularized) Gram matrix inversions of adjacent subcarriers for medium-size massive MIMO-OFDM, which is then verified by simulations. To reduce the number of matrix inversion required, we propose a linear interpolation based MMSE detection algorithm, which exploits the strong correlation of the inversion of Gram matrices required by a zero forcing detector or regularized Gram matrices required by a MMSE detector between adjacent subcarriers in massive MIMO-OFDM systems. Extensive simulations show that, with the same level of complexity as the matched filter algorithm, the proposed algorithm only incurs small BER performance loss compared to the exact MMSE detector.

The remainder of this paper is organized as follows. Section II describes the massive MIMO-OFDM system model and the soft output MMSE detection algorithm. Then in Section III, the strong correlation of the matrix inversion for Massive MIMO detection is evaluated and a linear interpolation algorithm is proposed to compute the matrix inversion with low complexity. Simulation results are shown in Section IV and conclusion is given in Section V.

The notations used in this paper are as follows. Lower and upper case letters denote scalars. Bold lower and upper case letters represent column vectors and matrices, respectively. $\mathbb{E}[\cdot]$ is the expectation operation. The superscriptions "T", "H" and "-1" denote the transpose, conjugate transpose and inverse, respectively. $\mathbf{I}_N$ denotes the N-dimensional identity matrix.

II. System Model and Soft-output MMSE Detector

Consider a coded uplink massive MIMO-OFDM system with N_r receive antennas, N_t transmit antennas and N subcarriers. For subcarrier $n = 1, \ldots, N$, the received signal at the base station can be modelled as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n, \tag{1}$$

where $\mathbf{y}_n$ denotes a length-N_r observation vector on subcarrier n, $\mathbf{H}_n$ denotes an $N_r \times N_t$ MIMO system channel matrix of subcarrier n, $\mathbf{w}_n$ denotes a length-N_r circularly symmetric additive white Gaussian noise (AWGN) vector on subcarrier n with zero means and covariance of $\sigma^2 \mathbf{I}_{N_r}$, and $\mathbf{x}_n = [x_{n,1}, x_{n,2}, \ldots, x_{n,N_t}]^T$ is the data symbol vector transmitted on subcarrier n, which is mapped from an interleaved code sequence $\mathbf{c}$, i.e., each $x_{n,i} \in \mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_{2^Q}\}(|\mathcal{A}| = 2^Q)$ corresponds to a length-Q subsequence of $\mathbf{c}$ denoted by $\mathbf{c}_i = [c_{i,1}, c_{i,2}, \ldots, c_{i,Q}]^T$.

The task of a soft-output detector is to compute the soft value indicating reliability in the shape of log-likelihood ratio (LLR) for each code bit. We apply the low-complexity linear MMSE detector [18] to firstly compute the Gram matrix $\mathbf{G}_n = \mathbf{H}_n^H \mathbf{H}_n$ and the inverse of the regularized Gram matrix $\mathbf{V}_n = (\mathbf{G}_n + \sigma^2 \mathbf{I}_{N_t})^{-1}$ for each subcarrier. Then, we compute the MMSE filter outputs as

$$z_{n,i} = \mu_{n,i}^{-1} \mathbf{a}_{n,i}^H \mathbf{H}_n^H \mathbf{y}_n, \tag{2}$$

where $\mathbf{a}_{n,i}^H$ is the ith row of $\mathbf{V}_n$, $\mu_{n,i} = \mathbf{a}_{n,i}^H \mathbf{g}_{n,i}$ and $\mathbf{g}_{n,j}$ denotes the jth column of $\mathbf{G}_n$. Finally, the LLR $L(c_{i,q})$ for each subcarrier, $i = 1, \ldots, N_t$ and $q = 1, \ldots, Q$ is given by [20]

$$L(c_{i,q}) = \rho_{n,i} \left(\min_{a \in \mathcal{A}_q^0} |z_{n,i} - a|^2 - \min_{a \in \mathcal{A}_q^1} |z_{n,i} - a|^2 \right), \tag{3}$$

where $a \in \mathcal{A}_q^0 (\mathcal{A}_q^1)$ represents constellations whose q-th bit is $0(1)$, and $\rho_{n,i} = \frac{\mu_{n,i}}{1 - \mu_{n,i}}$. The above detection algorithm requires computational complexity of $\mathcal{O}(N_t^2 N_r)$ for calculating $\mathbf{G}_n$ and $\mathcal{O}(N_t^3)$ for calculating $\mathbf{V}_n$.

III. MMSE Detection Based on Interpolation

A. Correlation of Matrix Inversion for Massive MIMO-OFDM

In the following, we define the correlation of matrix $\mathbf{G}_n^{-1}$ between adjacent subcarriers, whose performance is very close to the correlation performance of $\mathbf{V}_n$ by extensive simulations, and then validate its strong correlation with the increase of the ratio $\rho = N_r / N_t$ in a massive MIMO-OFDM system by simulations.

The channel coefficient correlation between subcarrier n and subcarrier $n + d$ (assuming modulo N addition) is defined as [21]

$$C_{\mathbf{h}}(d) = \frac{\mathbb{E}\left[|\mathbf{h}^H(n+d)\mathbf{h}(n)|^2\right]}{\mathbb{E}\left[||\mathbf{h}^H(n+d)||^2\right] \mathbb{E}\left[||\mathbf{h}(n)||^2\right]}, \tag{4}$$

where $\mathbf{h}(n) = vec(\mathbf{H}_n)$ is the vector obtained by stacking the columns of $\mathbf{H}_n$ ($n \in [1, N]$) one on top of another, and $||(\cdot)||$ is the 2-norm of $(\cdot)$.

Similarly, the correlation between $\mathbf{G}_n^{-1}$ and $\mathbf{G}_{n+d}^{-1}$ can be defined as

$$C_{\mathbf{g}}(d) = \frac{\mathbb{E}[|\mathbf{g}^H(n+d)\mathbf{g}(n)|^2]}{\mathbb{E}\left[||\mathbf{g}^H(n+d)||^2\right] \mathbb{E}\left[||\mathbf{g}(n)||^2\right]}, \tag{5}$$

where $\mathbf{g}(n) = vec(\mathbf{G}_n^{-1})$.

Fig. 1 shows the correlations $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ versus subcarrier distance d with different sized MIMO ($N_t = 20$ and $N = 64$). It is shown that as the distance d increases, the correlations of $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ decrease accordingly as expected. On the other hand, the correlation $C_{\mathbf{g}}(d)$ between adjacent tones drops much slower than $C_{\mathbf{h}}(d)$ does. With

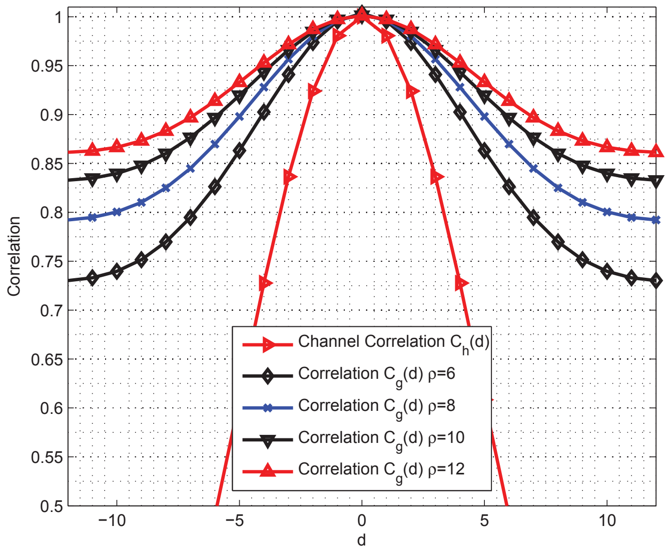

Fig. 1. Comparison between $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ with different ρ versus subcarrier distance d, where $N = 64$ and $N_t = 20$.

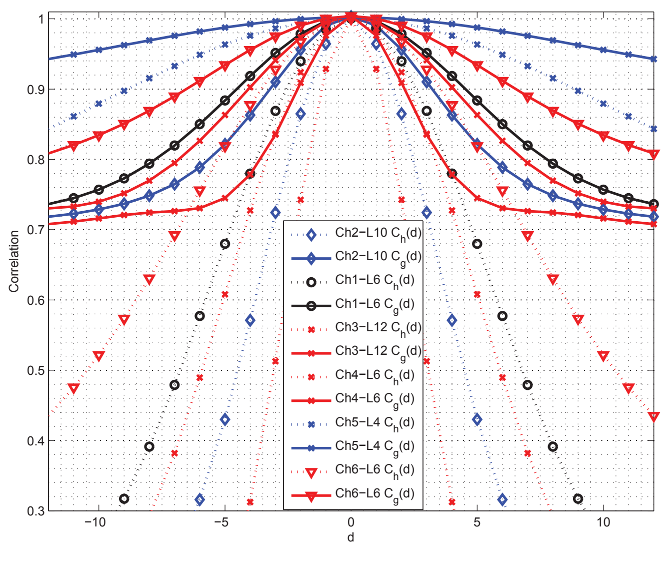

Fig. 2. Comparison between $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ under different channel models versus subcarrier distance d with $N = 64$, $N_t = 20$ and $\rho = 8$.

the increase of ρ, the dropping rate of $C_{\mathbf{g}}(d)$ (according to the increased d) becomes small, i.e., $\mathbf{G}_n^{-1}$ with larger ρ has stronger correlation .

Fig. 2 shows the $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ under different channel models listed in Table I. It is clear that for the channels with small L (the number of resolvable paths), both $C_{\mathbf{h}}(d)$ and $C_{\mathbf{g}}(d)$ are large. For channels with the same number of taps (e.g., channel No. 2, No.3 and No.4), the channel with larger maximum time delay spread has smaller correlation (e.g., channel No.4).

TABLE I
SIMULATED CHANNEL MODELS [22]

No.	Channel Model	Delay in ns
1	COST207_TU6alt Alt typical urban 6-tap	0 200 500 1600 2300 5000
2	COST259_RAx Rural ara, 10-tap 3GPP_TR_25.943	0 42 101 129 149 245 312 410 469 528
3	COST207_TU12 Typical urban, 12-tap	0 200 400 600 800 1200 1400 1800 2400 3000 3200 5000
4	COST207_HT THilly terrain, 6-tap	0 200 400 600 15000 17200
5	ITU_Pedestrian_A ITU Pedestria A, 4-tap	0 110 190 410
6	ITU_Vehicular_A ITU Vehicular A, 6-tap	0 310 710 1090 1730 2510

B. Direct Interpolation Based Matrix Inversion

We select the subcarriers with index of $S = \{1, 1 + D, ..., 1+KD, N\}$ as the base subcarriers, where $K = \lfloor \frac{N-1}{D} \rfloor$ is the nearest integer less than or equal to $\frac{N-1}{D}$, and then compute $\mathbf{V}_n (n \in S)$.

For non-base subcarriers with indices inside the range $(1, 1 + KD)$, the matrix inversion can be directly computed by linear interpolation as

$$\mathbf{V}_{kD+1+d} = (1 - \frac{d}{D})\mathbf{V}_{kD+1} + \frac{d}{D}\mathbf{V}_{(k+1)D+1}, \tag{6}$$

where $d \in [1, D - 1]$ and $k \in [0, K - 1]$.

For the subcarriers with indices between $1 + KD$ and N, the matrix inversion can be computed by linear interpolation as

$$\mathbf{V}_{KD+1+d} = (1 - \frac{d}{D_1})\mathbf{V}_{KD+1} + \frac{d}{D_1}\mathbf{V}_N, \tag{7}$$

where $d \in [1, D_1 - 1]$ and $D_1 = N - KD - 1$.

C. Computational Complexity Comparison

We focus on the number of complex value multiplications needed and only count quadratic or beyond terms. Note that the symmetric property of matrix $\mathbf{G}_n$ and $\mathbf{V}_n$ can reduce the complexity by a half. In Massive MIMO detection, matched filter based detection algorithm is assumed to be the algorithm with the lowest complexity although it suffers big performance loss for medium-size Massive MIMO. Therefore, we use the complexity with matched filter algorithm as the lower bound for comparison, which has the complexity of $\mathcal{O}(N_r N_t)$ for every subcarrier.

Table II is the summary of complexity comparison between the exact MMSE, the matched filter, the algorithms in [14], [15] and the proposed algorithm. In the table, the term $N_t^2 N_r$ corresponds to the computing of Gram matrix $\mathbf{G}_n$ and I is the number of terms in the Neumann Series expansion in [14]. The proposed algorithm has great computation saving compared to the exact implementation, while it has comparable complexity to the matched filter. For example, when $N_t = 16$, $N_r = 128$, $N = 256$ and $D = 16$, up to 84% and 77% of computation saving can be obtained by the proposed algorithm compared to the exact implementation and the algorithm in [14] with

$I = 5$, respectively. Compared to the matched filter algorithm, the complexity of the proposed algorithm is only 3.4 times higher.

TABLE II
COMPUTATIONAL COMPLEXITY COMPARISON

Algorithm	Number of Multiplications
Exact MMSE [20]	$N(N_t^3 + N_t^2 N_r + 4N_t N_r)$
Matched Filter	$N(N_t N_r)$
Low Complexity [14]	$N[(5 + 2I)N_t N_r]$
Interpolation-Based Matrix Inversion [15]	$2(N_t - 1)(L - 1)(N_t^3 + N_t^2 N_r + 4N_t N_r)$ $+[N - 2(N_t - 1)(L - 1)]N_t^2 C_{IP}$
Proposed one	$(K + 2)(N_t^3 + N_t^2 N_r + 4N_t N_r)$ $+[N - (K + 2)](N_t^2 + 2N_t N_r)$

For a large number of N_t, the algorithm II-A in [15] still requires a large amount of adjoint matrix and determinant computations. Especially when $N \simeq 2(N_t - 1)(L - 1)$, the complexity reduction can be negligible compared to the exact implementations in [20]. C_{IP} is an equivalent number of full multiplications per non-base subcarrier for interpolation, and often is a small number. Therefore, the complexity induced by interpolation is far lower than that of the matrix inversion. In general, the order required by matrix inversion computations of the proposed algorithm is far less than the algorithm II-A in [15] for massive MIMO-OFDM systems.

IV. BER PERFORMANCE

In this section, we present BER simulation results of the proposed algorithm and the existing ones. The simulation settings are as follows: A rate-1/2, regular (3,6) low-density parity-check (LDPC) code with codeword length of 10000 bits is employed as the channel code and the maximum number of iterations of the decoder is 25. The signal modulation of 4-QAM with Gray mapping is used. For each signal-to-noise (SNR) value, we simulate at least 1000 codewords. In the simulations, there are clipping in soft-output part of the detector (LLR is constrained to range $[-50, 50]$). The MIMO size is $N_t \times N_r = 16 \times 128$, and the number of subcarriers N is set to 256. The channel No.1 ($L = 6$) in Table I is used. Within an OFDM symbol, the distance D of adjacent base subcarriers required for channel estimation with small estimation errors is generally less than $\frac{N}{L}$ [23]. Therefore, we select 17 base subcarriers with D = 16 for performance comparison.

Fig. 3 shows the BER performance comparison between the exact MMSE detection [20], the matched filter with exact $\mathbf{H}_n$, the algorithm II-A in [15], the algorithm proposed in [19] and our proposed algorithm with D = 16. It is obvious that the proposed algorithm outperforms the matched filter algorithm by up to 0.4 dB at the BER of 10^{-3}. Although $\mathbf{V}_n$ and $\mathbf{H}_n$ are directly computed by linear interpolation with proposed algorithm, its BER performance is nearly the same as the exact MMSE detection, which requires matrix inversions based on all subcarriers. $D = 32$. The BER performance of the algorithm II-A is also very close to the exact one, but it has higher computational complexity with equivalently $2(N_t - 1)(L - 1) = 150$ subcarriers' matrix inversions than

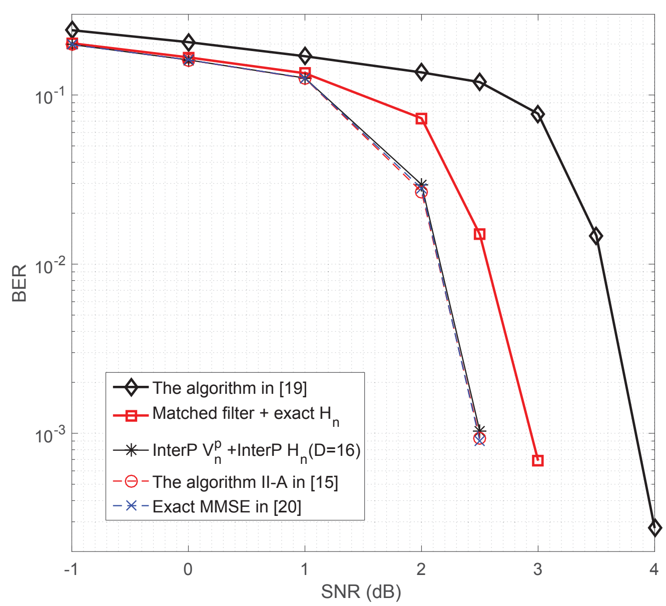

Fig. 3. BER performance comparison for exact MMSE, matched filter, the algorithm II-A in [15], the algorithm in [19] and the proposed interpolated algorithm.

the proposed one with 17 matrix inversions. For the algorithm II-A, the adjoint matrices and determinants of Gram matrix for the required base subcarriers are assumed to be obtained perfectly in simulations, and then linear interpolation with $C_{IP} = 1$ is used to compute the adjoint matrices and determinants (i.e., inverse matrices) of Gram matrix for all non-base subcarriers. The algorithm proposed in [19] has the worst performance among all algorithms, since the DFT interpolated based matrix (i.e., $\mathbf{G}_n^{-1}\mathbf{H}_n^H$) that is different from Gram matrix has weak correlation between different subcarriers.

V. CONCLUSION

In this paper, analysis and simulation show that the matrix inversions of the Gram matrix or the regularized Gram matrix are strongly correlated between adjacent subcarriers in a massive MIMO-OFDM system. Then, with the computed matrix inversions for base subcarriers, we propose a low complexity linear interpolation algorithm to directly compute the matrix inversions for non-base subcarriers. Simulation results show that the proposed algorithm leads to marginal performance loss but with considerable complexity saving.

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