Relating the Resource Theories of Entanglement and Quantum Coherence

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Quantum coherence and quantum entanglement represent two fundamental features of nonclassical systems that can each be characterized within an operational resource theory. In this Letter, we unify the resource theories of entanglement and coherence by studying their combined behavior in the operational setting of local incoherent operations and classical communication (LIOCC). Specifically, we analyze the coherence and entanglement trade-offs in the tasks of state formation and resource distillation. For pure states we identify the minimum coherence-entanglement resources needed to generate a given state, and we introduce a new LIOCC monotone that completely characterizes a state's optimal rate of bipartite coherence distillation. This result allows us to precisely quantify the difference in operational powers between global incoherent operations, LIOCC, and local incoherent operations without classical communication. Finally, a bipartite mixed state is shown to have distillable entanglement if and only if entanglement can be distilled by LIOCC, and we strengthen the well-known Horodecki criterion for distillability.

The ability for quantum systems to exist in "superposition states" reveals the wavelike nature of matter and represents a strong departure from classical physics. Systems in such superposition states are often said to possess quantum coherence. There has currently been much interest in constructing a resource theory of quantum coherence [1–11], in part because of recent experimental and numerical findings that suggest quantum coherence alone can enhance or impact physical dynamics in biology [12–15], transport theory [2,16,17], and thermodynamics [18–20].

In a standard resource-theoretic treatment of quantum coherence, the free (or "incoherent") states are those that are diagonal in some fixed reference (or incoherent) basis. Different classes of allowed (or incoherent) operations have been proposed in the literature [1,3,9–11] (see also Refs. [21,22] for comparative studies of these approaches); however, an essential requirement is that the incoherent operations act invariantly on the set of diagonal density matrices. Incoherent operations can then be seen as one of the most basic generalizations of classical operations (i.e., stochastic maps) since their action on diagonal states can always be simulated by classical processing. Note also that most experimental setups will have a natural basis to work in, and arbitrary unitary time evolutions might be physically difficult to implement. In these settings, there are practical advantages to identifying "diagonal preserving" operations as being "free" relative to coherent-generating ones.

One does not need to look far to find an important connection between incoherent operations and quantum entanglement, the latter being one of the most important resources in quantum information processing [23]. Consider the task of entanglement generation. This procedure is usually modeled by bringing together two or more quantum systems initially in a product state $\rho \otimes \sigma$ and then applying an entangling joint operation. However, using only incoherent operations, this will not be possible unless either ρ or σ already possesses coherence. The reason is that when $\rho \otimes \sigma$ is an incoherent bipartite state, any incoherent operation acting on both systems will leave the joint state incoherent (and hence unentangled). On the other hand, if the joint state is $|+\rangle|0\rangle$, with $|\pm\rangle = \sqrt{1/2}(|0\rangle \pm |1\rangle)$, then an application of CNOT yields the entangled state $\sqrt{1/2}(|00\rangle + |11\rangle)$. This example reveals that coherence, or at least coherent-generating operations, is a prerequisite for producing entanglement. In fact, as Streltsov *et al.* have shown [24], every coherent state can be used for the generation of entanglement in a manner similar to this example.

Notice that the transformation $|\pm\rangle|0\rangle \rightarrow \sqrt{1/2}(|00\rangle + |11\rangle)$ requires performing an entanglement-generating incoherent operation. To capture both coherence and entanglement in a common resource-theoretic framework, one must modify the scenario by adopting the "distant lab" perspective in which two or more parties share a quantum system but they are spatially separated from one another [23,25]. In this setting, entanglement cannot be generated between the parties and it becomes another resource in play. When the constraint of locality is added to the incoherent framework, the allowable operations for Alice and Bob are then local incoherent operations and classical communication (LIOCC). The hybrid coherence-entanglement theory described here is similar in spirit to previous work on the locality-restricted resource theories of purity [26–29] and asymmetry [30]. We do not point to a specific biological or thermodynamic process as motivation for our study of LIOCC—although, one could envision
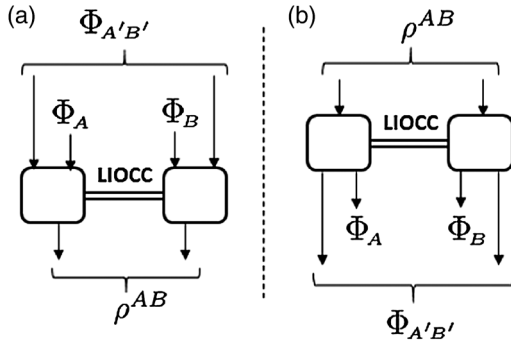
FIG. 1. (a) A LIOCC formation protocol asymptotically generates an arbitrary state ρ^{AB} from an initial supply of local coherent bits (Φ_A/Φ_B) and shared entanglement bits ($\Phi_{A'B'}$). (b) A LIOCC dilution protocol performs the reverse transformation.

potential physical applications in certain coherence-enhanced transport networks where the nodes interact through classical signaling. Rather, we promote LIOCC as the natural setting to explore the interplay between coherence and entanglement as resource primitives in quantum information theory. For example, how much local coherence and shared entanglement do Alice (A) and Bob (B) need to prepare a particular bipartite state ρ^{AB} using LIOCC [Fig. 1(a)]? Conversely, how much coherence and entanglement can be distilled from a given state ρ^{AB} using LIOCC [Fig. 1(b)]? The latter task can also be seen as a type of collaborative randomness distillation, where Alice and Bob work together to generate local sources of genuine randomness for each other [6].

Our main results are the following. (1) We completely characterize the achievable coherence-entanglement rate region for the task of asymptotically generating some pure state $|\Psi\rangle^{AB}$ (Theorem 1). (2) We introduce a new LIOCC monotone that combines both coherence and entanglement measures (Theorem 4), and we show it quantifies the optimal rate in which Alice and Bob can simultaneously distill local coherence from a pure state. (3) We identify an achievable rate region for the coherence-entanglement distillation of a pure state and show optimality at almost all corner points (Theorem 5). (4) In analogy to Refs. [26–29], we introduce and compute for pure states the nonlocal coherence deficit and the LIOCC coherence deficit [Eqs. (6) and (7)]. (5) We show that LIOCC operations alone are sufficient to decide whether entanglement can be distilled from a mixed state using general local operations and classical communication (LOCC).

Let us begin by briefly describing the theory of bipartite coherence in more detail. Assigned to both Alice and Bob's system is a particular basis called their incoherent basis. We denote Alice's incoherent basis by $\{|x\rangle^A\}_{x=0}^{d_A-1}$ and Bob's incoherent basis by $\{|y\rangle^B\}_{y=0}^{d_B-1}$ so that the incoherent basis for their joint system $\mathcal{H}^A \otimes \mathcal{H}^B$ is $\{|x\rangle^A|y\rangle^B\}_{x,y=0}^{d_A-1,d_B-1}$. Then, any bipartite state belongs to the set of incoherent states $\mathcal{I}$ if and only if (iff) it has the form

$$\sigma^{AB} = \sum_{xy} p_{xy}|x\rangle\langle x|^A \otimes |y\rangle\langle y|^B. \qquad (1)$$

Following the framework of Baumgratz et al. [3], a local incoherent operation for Alice is given by a complete set of Kraus operators $\{K_\alpha\}_\alpha$ such that $(K_\alpha \otimes \mathbb{I}^B)\rho^{AB}(K_\alpha \otimes \mathbb{I}^B)^\dagger/\mathrm{tr}[K_\alpha K_\alpha^\dagger \otimes \mathbb{I}^B \rho^{AB}] \in \mathcal{I}$ for all $\rho^{AB} \in I$. If ever she introduces a local ancilla system $\mathcal{H}^{A'}$, the incoherent basis for this additional system is labeled in the same way $\{|x\rangle^{A'}\}_{x=0}^{d_{A'}-1}$. Analogous statements characterize the notion of incoherent operations on Bob's system. In the LIOCC setting, Alice and Bob take turns performing local incoherent operations and sharing their measurement data over a classical communication channel.

The canonical resource states in the bipartite LIOCC framework are the maximally coherent bits (CoBits) [3] $\Phi_A := \sqrt{1/2}(|0\rangle^A + |1\rangle^A)$ and $|\Phi_B\rangle := \sqrt{1/2}(|0\rangle^B + |1\rangle^B)$ for Alice and Bob's systems, respectively, as well as the entangled state $|\Phi_{AB}\rangle := \sqrt{1/2}(|00\rangle + |11\rangle)$, which we will call the maximally coherent entangled bit (eCoBit). Notice that unlike entanglement theory, only those bipartite states related to $|\Phi_{AB}\rangle$ by an incoherent local unitary transformation can be regarded as equivalent to $|\Phi_{AB}\rangle$. For example, as we will see below, one eCoBit cannot be incoherently transformed into the state $\sqrt{1/2}(|0+\rangle + |1-\rangle)$, even asymptotically.

We now describe the primary tasks studied in this Letter, which can be seen as the resource-theoretic tasks recently analyzed by Winter and Yang in Ref. [7] but now with additional locality constraints. All of the detailed proofs can be found in the Supplemental Material [38], which also contains Refs. [39–53], and here we just present the results. Let us begin with the problem of asymptotic state formation shown in Fig. 1(a). A triple $(R_A, R_B, E^{\mathrm{co}})$ is an achievable coherence-entanglement formation triple for the state ρ^{AB} if for every $\epsilon > 0$ there exists a LIOCC operation $\mathcal{L}$ and integer n such that

$$\mathcal{L}(\Phi_A^{\otimes\lceil n(R_A+\epsilon)\rceil} \otimes \Phi_B^{\otimes\lceil n(R_B+\epsilon)\rceil} \otimes \Phi_{A'B'}^{\otimes\lceil n(E^{\mathrm{co}}+\epsilon)\rceil}) \overset{\epsilon}{\approx} \rho^{\otimes n}$$

Dual to the task of formation is resource distillation, as depicted in Fig. 1(b). A triple $(R_A, R_B, E^{\mathrm{co}})$ is an achievable coherence-entanglement distillation triple for ρ^{AB} if for every $\epsilon > 0$ there exists a LIOCC operation $\mathcal{L}$ and integer n such that

$$\mathcal{L}(\rho^{\otimes n}) \overset{\epsilon}{\approx} \Phi_A^{\otimes\lfloor n(R_A-\epsilon)\rfloor} \otimes \Phi_B^{\otimes\lfloor n(R_B-\epsilon)\rfloor} \otimes \Phi_{AB}^{\otimes\lfloor n(E^{\mathrm{co}}-\epsilon)\rfloor}.$$

As we are dealing with asymptotic transformations, we should expect the optimal rate triples to be given by entropic quantities. Recall that for a bipartite state ω^{AB}, the von Neumann entropy of, say, Alice's reduced state ω^A is given by $S(A)_\omega = -\mathrm{tr}[\omega^A \log \omega^A]$. The quantum mutual

information of ω^{AB} takes the form $I(A:B)_\omega := S(A)_\omega - S(A|B)_\omega$, where $S(A|B)_\omega := S(AB)_\omega - S(B)_\omega$. For a pure state $|\Psi\rangle^{AB}$, the entropy of entanglement $E(\Psi) := S(A)_\Psi = S(B)_\Psi$ is the unique measure of entanglement in the asymptotic regime [54], and it can be generalized to mixed states as the entanglement of formation $E_F(\rho)$ [55]. We will also be interested in these entropic quantities after sending our state ω^{AB} through the completely dephasing channel, $\Delta(\omega) := \sum_{xy} |xy\rangle\langle xy|\omega|xy\rangle\langle xy|$. It will be convenient to think of $\Delta(\omega)$ as encoding random variables XY having joint distribution $p(x,y) = \langle xy|\Delta(\omega)|xy\rangle$. For this reason, we follow standard convention and replace the labels $(A,B) \to (X,Y)$ when discussing a dephased state.

Our first main result completely characterizes the achievable rate region for the LIOCC formation of bipartite pure states.

Theorem: For a pure state $|\Psi\rangle^{AB}$ the following triples are achievable coherence-entanglement formation rates

$$(R_A, R_B, E^{co}) = (0, S(Y|X)_{\Delta(\Psi)}, S(X)_{\Delta(\Psi)}) \quad (2)$$

$$(R_A, R_B, E^{co}) = (S(X)_{\Delta(\Psi)}, S(Y|X)_{\Delta(\Psi)}, E(\Psi)) \quad (3)$$

$$(R_A, R_B, E^{co}) = (0, 0, S(XY)_{\Delta(\Psi)}) \quad (4)$$

as well as the points obtained by interchanging $A \leftrightarrow B$ in Eqs. (2)–(4). Moreover, these points are optimal in the sense that any achievable rate triple must satisfy (i) $E^{co} \geq E(\Psi)$, (ii) $R_A + R_B \geq S(XY)_{\Delta(\Psi)}$, and (iii) $R_B + E^{co} \geq S(XY)_{\Delta(\Psi)}$.

For a mixed state ρ^{AB}, a formation protocol can be constructed that achieves the average rates for any ensemble $\{p_k, |\varphi_k\rangle^{AB}\}$ such that $\rho = \sum_k p_k |\varphi_k\rangle\langle\varphi_k|$ [55]. For instance, one can consider an ensemble whose average bipartite coherence attains the coherence of formation C_F for ρ; i.e., it is an ensemble $\{p_k, |\varphi_k\rangle^{AB}\}$ for ρ that minimizes $\sum_k p_k S(XY)_{\Delta(\varphi_k)}$ [6,7]. Then, for a mixed state ρ, the coherence rate sum $R_A + R_B$ of Eq. (3) can attain the coherence of formation $C_F(\rho)$. In the global setting where Alice and Bob are allowed to perform joint operations across system AB, it has been shown that $C_F(\rho)$ quantifies the optimal coherence consumption rate for generating ρ using global incoherent operations [7]. Our result then intuitively says that in the restricted LIOCC setting, the same coherence rate is sufficient to generate ρ; however, they now need additional entanglement at a rate $\sum_k p_k E(\varphi_k)$, where the ensemble $\{p_k, |\varphi_k\rangle^{AB}\}$ minimizes the average coherence of ρ.

The proof of Theorem 1 uses two lemmas that may be of independent interest. The first generalizes a result presented in Ref. [3], and the second is an incoherent version of Nielsen's majorization theorem [56].

Lemma: An arbitrary $d \times d$ unitary operator U can be performed on a system using incoherent operations and $\lceil \log d \rceil$ CoBits.

Lemma: Suppose $|\psi\rangle^{AB}$ and $|\phi\rangle^{AB}$ have reduced density matrices that are diagonal in the incoherent bases for both

parties and both states. Then $|\psi\rangle \to |\phi\rangle$ by LIOCC iff the squared Schmidt coefficients of $|\phi\rangle$ majorize those of $|\psi\rangle$.

Next, we introduce a new LIOCC monotone and provide its operational interpretation. To do so, we recall the recently studied task of assisted coherence distillation, which involves one party helping another distill as much coherence as possible through general quantum operations performed on the helper side and incoherent operations performed on the distillation side [57]. For a given state ρ^{AB}, the optimal asymptotic rate of coherence distillation on Bob's side when Alice helps is denoted by $C_a^{A|B}(\rho^{AB})$. When the roles are switched, the optimal asymptotic rate is denoted by $C_a^{B|A}(\rho^{AB})$. It was shown in Ref. [57] that $C_a^{A|B}(\rho^{AB}) = S(Y)_{\Delta(\Psi)}$ and $C_a^{B|A}(\rho^{AB}) = S(X)_{\Delta(\Psi)}$. With these quantities in hand, we define for a bipartite pure state $|\Psi\rangle^{AB}$ the function

$$\begin{aligned} C_L(\Psi) &:= C_a^{A|B}(\Psi) + C_a^{B|A}(\Psi) - E(\Psi) \\ &= S(X)_{\Delta(\Psi)} + S(Y)_{\Delta(\Psi)} - E(\Psi). \end{aligned} \quad (5)$$

Its extension to mixed states can be defined by a convex roof optimization [52]: $C_{\mathcal{L}}(\rho^{AB}) = \inf_{\{p_k, |\varphi_k\rangle^{AB}\}} \sum_k p_k C_{\mathcal{L}}(\varphi_k^{AB})$ for which $\rho^{AB} = \sum_k p_k |\varphi_k\rangle\langle\varphi_k|$.

Theorem: The function C_L is a LIOCC monotone. We note that this is the first monotone of its kind since it behaves monotonically under LIOCC, but not general LOCC or even local quantum incoherent operations and classical communication (LQICC), the latter being an operational class in which only one of the parties is required to perform incoherent operations (as opposed to LIOCC where both parties must perform incoherent operations) [57].

Using the monotonicity of C_L, we are able to derive tight upper bounds on the coherence distillation rates.

Theorem: For a pure state $|\Psi\rangle^{AB}$ the following triples are achievable coherence-entanglement distillation rates

$$(R_A, R_B, E^{co}) = (S(X)_{\Delta(\Psi)} - E(\Psi), S(Y)_{\Delta(\Psi)}, 0), \quad (6)$$

$$(R_A, R_B, E^{co}) = (0, S(Y|X)_{\Delta(\Psi)}, I(X:Y)_{\Delta(\Psi)}), \quad (7)$$

as well as the points obtained by interchanging $A \leftrightarrow B$ in Eqs. (6) and (7). Moreover, these points are optimal in the sense that any achievable rate triple must satisfy (i) $R_A + R_B \leq C_{\mathcal{L}}(\Psi)$ and (ii) $R_B + E^{co} \leq S(Y)_{\Delta(\Psi)}$. This theorem endows $C_{\mathcal{L}}$ with the operational meaning of quantifying how much local coherence can be simultaneously distilled from a pure state. For a state $|\Psi\rangle$ the maximum that Alice can help Bob distill coherence is $C_a^{A|B}$ while the maximum that Bob can help Alice is $C_a^{B|A}$. Evidently, they cannot both simultaneously help each other at these optimal rates. Instead, they are bounded away from simultaneous optimality at a rate equaling their shared entanglement.

It is still unknown the precise range of achievable distillation triples $(R_A, R_B, E_{\max}^{co})$, where $E_{\max}^{co}$ is the maximum eCoBit distillation rate. While we are able to prove that $E_{\max}^{co}$ is the regularized version of $I(X:Y)_{\Delta(\Psi)}$ optimized over all LIOCC protocols, we have no single-letter expression for this rate nor do we know the achievable local coherence rates for optimal protocols.

A natural question is whether $E_{\max}^{co}(\Psi) = E(\Psi)$. While this question remains open, we can show that $E(\Psi)$ is achievable if the Schmidt basis of the final state need not be incoherent. More precisely, we say a number R is an achievable LIOCC entanglement distillation rate if for every $\epsilon > 0$ there exists a LIOCC protocol $\mathcal{L}$ acting on n copies of Ψ such that $\mathcal{L}(\Psi^{\otimes n}) \overset{\epsilon}{\approx} \Lambda_d$, where Λ_d is a $d \otimes d$ maximally entangled pure state (i.e., $\Lambda^A = \Lambda^B = \mathbb{I}/d$) with $\frac{1}{n}\log d > R - \epsilon$. The largest achievable distillation rate will be denoted by $E_D^{\text{LIOCC}}(\Psi)$.

Theorem: $E_D^{\text{LIOCC}}(\Psi) = E(\Psi)$.

It is interesting to compare the coherence distillation rates using incoherent operations under different types of locality constraints. In Refs. [26–29], similar comparisons were made in terms of purity (or work-information) extraction. Let C_D^{Global}, C_D^{LIOCC}, and C_D^{LIO} denote the optimal rate sum $R_A + R_B$ of local coherence distillation using global incoherent operations, LIOCC, and local incoherent operations (with no classical communication), respectively. In complete analogy to Refs. [26–29], we define the nonlocal coherence deficit of a bipartite state ρ^{AB} as $\delta(\rho^{AB}) = C_D^{\text{Global}}(\rho^{AB}) - C_D^{\text{LIOCC}}(\rho^{AB})$ and the LIOCC coherence deficit as $\delta_c(\rho^{AB}) = C_D^{\text{LIOCC}}(\rho^{AB}) - C_D^{\text{LIO}}(\rho^{AB})$. Intuitively, the quantity $\delta(\rho^{AB})$ quantifies the coherence in a state that can only be accessed using nonlocal incoherent operations. Likewise, $\delta_c(\rho^{AB})$ gives the coherence in ρ^{AB} that requires classical communication to be obtained. The results of Winter and Yang imply that $C_D^{\text{Global}}(\Psi) = S(XY)_{\Delta(\Psi)}$ and $C_D^{\text{LIO}}(\Psi) = S(X)_{\Delta(\Psi)} + S(Y)_{\Delta(\Psi)} - 2E(\Psi)$ for a bipartite pure state $|\Psi\rangle^{AB}$ [58]. Combined with Theorem 5, we can compute the two coherence deficits for pure states:

$$\delta(\Psi) = E(\Psi) - I(X:Y)_{\Delta(\Psi)}, \qquad (8)$$

$$\delta_c(\Psi) = E(\Psi). \qquad (9)$$

It is curious that the entanglement $E(\Psi)$ quantifies the coherence gain unlocked by classical communication. But note that a similar phenomenon exists in the resource theory of purity. Namely, the quantum deficit $\bar{\delta}(\Psi)$ and classical deficit $\bar{\delta}_c(\Psi)$ measure the analogous differences in local purity distillation by so-called "closed operations," and they are given by $\bar{\delta}(\Psi) = \bar{\delta}_c(\Psi) = E(\Psi)$ [26,27]. For the task of distilling CoBits, every protocol using incoherent operations can be seen as one using closed operations by accounting for all ancilla systems at the start of the protocol [59]. However, closed operations allow for arbitrary unitary rotations, which are forbidden in coherence theory. The term $I(X:Y)_{\Delta(\Psi)}$ in

$\delta(\Psi)$ identifies precisely the basis dependence in coherence theory and shows how this decreases $\delta(\Psi)$ relative to $\bar{\delta}(\Psi)$. On the other hand, there is evidently no basis dependency in the LIOCC classical deficit $\delta_c(\Psi)$ and it is equivalent to $\bar{\delta}_c(\Psi)$.

Although our distillation results so far have only applied to pure states, we can deduce a very general result concerning the distillability of mixed states.

Theorem: A mixed state ρ^{AB} has (LOCC) distillable entanglement if and only if entanglement can be distilled using LIOCC. The proof of this theorem is actually quite simple and uses the fact that an arbitrary quantum operation can be simulated using incoherent operations and CoBits (Lemma 2). In Ref. [57] it was shown how local coherence can always be distilled for both Alice and Bob from multiple copies of every entangled states using LIOCC. Hence, for a sufficiently large number of any distillable entangled state ρ^{AB}, Alice and Bob first distill sufficient local coherence using LIOCC, and then they simulate the LOCC protocol which distills entanglement.

As shown in Ref. [60], a state ρ has distillable entanglement iff for some k there exists rank 2 operators A and B such that the (unnormalized) state $A \otimes B \rho^{\otimes k} A \otimes B$ is entangled. By Theorem 5 and following the same argumentation of Ref. [60], we can further require that the A and B are incoherent operators; that is, they have the form $A = |0\rangle\langle\alpha_0| + |1\rangle\langle\alpha_1|$ and $B = |0\rangle\langle\beta_0| + |1\rangle\langle\beta_1|$, where $\Delta(\alpha_0) := \Delta(|\alpha_0\rangle\langle\alpha_0|)$ is orthogonal to $\Delta(\alpha_1) := \Delta(|\alpha_1\rangle\langle\alpha_1|)$, and likewise for $\Delta(\beta_0) := \Delta(|\beta_0\rangle\langle\beta_0|)$ for $\Delta(\beta_1) := \Delta(|\beta_1\rangle\langle\beta_1|)$. We are thus able to add an additional condition to the distinguishability criterion of Ref. [60].

Corollary: A bipartite state ρ has distillable entanglement iff for any pair of orthonormal local bases $\mathcal{B}_A = \{|x\rangle^A\}$ and $\mathcal{B}_B = \{|y\rangle^B\}$ there exists some k and projectors $P_A = |\alpha_0\rangle\langle\alpha_0| + |\alpha_1\rangle\langle\alpha_1|$ and $P_B = |\beta_0\rangle\langle\beta_0| + |\beta_1\rangle\langle\beta_1|$ such that: 1. $(P_A \otimes P_B)\rho^{\otimes k}(P_A \otimes P_B)$ is entangled, and 2. $\text{tr}[\Delta_A(\alpha_0)\Delta_A(\alpha_1)] = \text{tr}[\Delta_B(\beta_0)\Delta_B(\beta_1)] = 0$, where Δ_Z is the completely dephasing map in the basis $\mathcal{B}_Z^{\otimes k}$.

Conclusion.—In this Letter, we have investigated the relationship between entanglement and coherence in the framework of local incoherent operations and classical communication. The findings of this study suggest that indeed entanglement and coherence are closely linked resources. For instance, Theorem 5 shows that the entanglement of a state plays a crucial role in limiting the amount of coherence that can be distilled from a state, a result highly reminiscent of the complementarity between local and nonlocal information studied in Ref. [28]. In a similar spirit, Theorem 7 shows that entanglement distillability can be studied through the lens of coherence theory. This latter result seems somewhat remarkable since despite coherence being a basis-dependent resource, its resource-theoretic analysis can be used to draw conclusions about entanglement, a basis-independent resource. Future work will be conducted to see whether the strengthened distillability

criterion of Corollary 8 can be useful in the long-standing search for nonpositive partial transpose (NPT) bound entanglement.

Finally, we would like to comment on the particular type of incoherent operations studied in this Letter. As noted in the introduction, there have been various proposals for the free class of operations in a resource theory of coherence. This Letter has adopted the incoherent operations (IO) of Baumgratz *et al.* [3], where each Kraus operator in a measurement just needs to be incoherence preserving. While the class IO has drawbacks in terms of formulating a full physically consistent resource theory of coherence [11,21], it nevertheless seems unlikely that the results of this Letter would remain true if other operational classes were considered. For example, the strictly incoherent operations proposed by Yadin *et al.* are unable to convert one eCoBit into a CoBit [11]. Thus, we believe that the interesting connections demonstrated here between IO coherence theory and entanglement make a positive case for why the class IO is relevant in quantum information theory, independent of any other motivation. In fact, one could even put coherence aside and view LIOCC as just being a simplified subset of LOCC. As we have shown here, nontrivial conclusions about entanglement can indeed be drawn by studying LOCC from "the inside." This approach is somewhat dual to the standard practice of studying LOCC using more general separable operations (SEP), the chain of inclusions being LIOCC $\subset$ LOCC $\subset$ SEP. Interesting future work would be to consider more general connections between coherence nongenerating and entanglement non-generating operations.

Note added—Recently, we learned of work by Streltsov and co-authors who have also initiated a study into local incoherent operations and classical communication [61].

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