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Editors

Nonlinear Economic Dynamics and Financial Modelling

Essays in Honour of Carl Chiarella

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Preface

The MDEF Workshop has been held at the University of Urbino since 2000. The 2014 Workshop is particularly dedicated to Carl Chiarella for his 70th birthday. As the second home (along with the University of Bielefeld, another second home), Carl visited Urbino in 1998 for the first time and the visit has become an almost annual event since then. In order to commemorate the occasion, a number of Carl's colleagues from around the world gladly agreed to contribute chapters to a special book dedicated to this event. The book is the outcome of this process. It contains the latest developments in nonlinear economic dynamics, financial market modeling, and quantitative finance, the three most active research areas Carl has been involved in.

This book is a collection of essays written by colleagues of Carl Chiarella in honour of his 70th birthday. Most of the authors have been collaborating with Carl in the past. We would first of all like to thank Laura Gardini and Gian Italo Bischi for stimulating discussion on the initiation of this special book and suggestion to dedicate it to Carl's 70th birthday in the 2014 MDEF Workshop. We would also like to express our gratitude to all contributors and in particular those who have collaborated with Carl, as well as to the referees involved in the review process. Finally, we would like to acknowledge the assistance of Kai Li who has worked on the book under much pressure.

Born in March 1944 in Sydney, Carl realized in his final high school years that he wanted to do something in life that would involve the use of mathematics, although that "something" would involve economics and finance was totally absent from his mind then. After completing his B.Sc. (Hons.) from the University of Sydney in 1965, Carl completed an M.Sc. at the University of Sydney in 1967, a Ph.D. at the University of New South Wales in 1969 in applied mathematics, and wrote a thesis on nuclear reactor physics. After spending two years at the University of Nancy as a postdoc Carl returned to Australia in 1971, and joined the School of Mathematical Science at University of Technology, Sydney (UTS). He has built his entire subsequent career at UTS since then (apart from a three year spell at the University of New South Wales from 1986 to 1989).

From his teenage years, Carl had an interest in the origins of the economic cycle. Despite heavy teaching load, Carl managed to pursue his long held interest in economics and completed a Master of Commerce in 1977 and a second Ph.D. in Economics at the University of New South Wales in 1988. His Ph.D. thesis was on the nonlinear viewpoint in economics. The thesis led to his first book, *The Elements of a Nonlinear Theory of Economic Dynamics*, published in the Springer-Verlag Lecture Note Series in 1990. In 1989, Carl was appointed as a Professor in the School of Finance and Economics at UTS, a position that he still occupies. Apart from his early work on nuclear reactor theory, Carl has made numerous scientific achievements and important contributions to the economics and finance area, in particular to nonlinear dynamic economic, financial market modeling, and option pricing.

As a mathematical economist, Carl has a strong research interest in modeling key economic adjustment processes as nonlinear dynamical systems. Carl's earlier work on chaotic economic dynamics in 1980s, in particular *The Cobweb Model: Its Instability and the Onset of Chaos* published in Economic Modeling in 1988 and *The Dynamics of Speculative Behaviour*, published in Annals of Operations Research in 1992, have been pioneering contributions in this area, which had profound influence on many researchers in this field. Carl has made a significant contribution to at least two areas of dynamic economic modeling. The first is on out-of-equilibrium models of macroeconomic dynamics. It develops a systematic approach to the disequilibrium tradition of macroeconomic dynamic analysis, leading to nine jointly authored books (with Peter Flaschel and others) on integrated Keynesian dynamic macroeconomic models, including three with Cambridge University Press and three with Springer-Verlag. The other is on financial market models with heterogeneous boundedly rational economic agents, showing that price movements of financial assets are the result of nonlinear dynamic feedback processes driven by the interaction of investors with heterogeneous beliefs and bounded rationality.

Through his many conferences and visits, the University of Urbino and the University of Bielefeld have become second home for Carl. Carl's visits to Urbino started in 1998 and have become regular since then. The attraction of Urbino for Carl is not only the glorious history, beautiful palaces, and churches, but also a group of brilliant researchers around Laura and Gian Italo in the theory of nonlinear dynamical systems. Through the Vienna Workshops on Economic Dynamics initiated by Gustav Feichtinger, Carl established his intensive research collaboration with the research groups around Peter Flaschel, Willi Semmler, and Volker Böhm in Bielefeld. Carl's collaborations with these groups belong to the highlights of his career.

As one of the main organizers of the annual Quantitative Methods in Finance conference at UTS since 1997, Carl has made a significant contribution to American option pricing, where he has mainly contributed to the development and numerical implementation of various solution methods. He has also been active in pricing interest rate derivative securities along two directions. The first is to implement on market data the various interest rate term structure and interest rate derivative pricing models that have been developed over the last two decades using nonlinear filtering and Bayesian updating methods. The second consists in finding improved computational procedures within the stochastic calculus framework of the term structure and option prices by allowing the volatility function of the Heath–Jarrow–Morton model to depend on the forward rate, and allowing for jump-processes in the underlying forward rate dynamics of this framework.

Carl has published more than 15 books and 200 papers, supervised more than 10 Ph.D. students, been involved in more than 30 research projects including the Australian Research Council (ARC) Discovery Grants. He was the Co-Editor of the *Journal of Economic Dynamics and Control* from 2004 to 2012 and has been Associate Editor of many leading finance and economics journals, including *Journal of Economic Behavior and Organization*, *Macroeconomic Dynamics*, *Computational Economics*, *Studies in Nonlinear Dynamics and Econometrics*, *European Journal of Finance*, *Quantitative Finance*, and *Asia-Pacific Financial Markets*. Of course, this is not a full list of Carl's numerous scientific achievements. The papers in this book deal with some of the many research topics Carl has addressed in many of his papers and books. They reflect the breadth of topics Carl has worked on during his career. We are grateful for the inspiration his work has given to all of us over so many years. Indeed his work inspires a new generation to further develop this exciting and challenging research agenda.

Bologna, April 2014 — Roberto Dieci
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Introduction

Roberto Dieci, Xue-Zhong He and Cars Hommes

The book opens with two brief articles summarising Carl's view on a broad range of research-related issues, mostly concerning the role of mathematical modelling in Economics and Finance. Both articles were originally published in Italian in the Springer journal "Lettera Matematica Pristem". The first article is the result of an interview given by Carl to Gian Italo Bischi—one of Carl's collaborators from the 'Urbino group'—during the annual meeting of AMASES (Italian Association for Mathematics Applied to Economics and Social Science) in Lecce, in September 2007. Besides some biographical details, the interview focuses on Carl's prominent research themes, on some issues related to the use of mathematical modelling in social sciences, on alternative approaches to economic modelling such as Econophysics and Agent-Based models, on Carl's direct experience (as a world traveller) of research organisation and funding in Italy, Europe, USA, Australia and so-called Asian emerging countries. The second chapter authored by Carl himself, is a brilliant discussion of the debates that have gone on amongst economists in the past century about what is the correct approach to modelling economic behaviour, of the future of Mathematical Economics and of the possible impact of the recent economic crisis on economic theorising.

The second part of the book containing six chapters deals with Nonlinear Economic Dynamics, the area of Economic Theory that mostly attracted Carl's interest

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since the beginning of his scientific career. The first two chapters stand in the tradition of the joint work of Carl with many co-authors in the field of disequilibrium macroeconomic dynamics, which typically deals with high-dimensional nonlinear dynamic models in continuous time. *Matthieu Charpe, Peter Flaschel, Christian R. Proaño and Willi Semmler* incorporate the basic elements of a firms' debt-finance model into a larger scale disequilibrium macroeconomic framework along the lines of Chiarella et al. (2005). In a fully interdependent macro-model they investigate both analytically and through numerical simulation the feedback impact of endogenously generated debt of firms on aggregate economic activity, on investors' confidence and on the stability of the economic system. The chapter by *Toichiro Asada* deals with the impact of macroeconomic stabilization policies under a Minsky-type "financial instability" hypothesis, again using the analytical framework of high-dimensional nonlinear Keynesian macrodynamic models. The chapter starts from a two-dimensional fixed price model without active stabilisation policy and considers, as extensions of this core version, a four-dimensional model of monetary stabilisation policy with flexible prices and a six-dimensional case with a monetary and fiscal policy mix. Besides providing a number of theoretical results concerning the stability of the steady state of the economy (depending on the fiscal and monetary parameters, and central bank's credibility), the chapter offers an economic interpretation of the main feedback mechanisms operating in the dynamic models. The next two chapters deal more closely with the bifurcations and the cyclical and complex dynamics that may emerge from traditional economic models when standard rationality and full-information assumptions are abandoned in favour of agents' bounded rationality and the use of simple rules of thumb in making decisions, which often results in nonlinearities. This is a research field in which Carl has made important contributions. The chapter by *Anna Agliari, Laura Gardini and Iryna Sushko* is inspired by Carl's early work (Chiarella 1990) on the issue of so-called dynamic instability of saddle-point type under perfect foresight, discussed within a continuous-time model of monetary dynamics (Sargent and Wallace 1973). A key feature of Carl's version of the monetary model was the assumption of a nonlinear S-shaped demand function, justified by realistic portfolio rules adopted by economic agents. Building on earlier work in collaboration with Carl (Agliari et al. 2004), the authors consider a discrete-time version of the perfect-foresight model with a similar (log) money demand function, linear within a 'normal' range of the expected inflation rate, and constant outside this range. As a result, the price dynamics of the physical good is described by a piecewise-linear one-dimensional map having two "kink points". Bounded cyclical orbits of any period and even chaotic dynamics may appear if the demand function is sufficiently sloped and price reacts slowly to excess money demand. The study is based on advanced and up-to-date analytical and numerical methods for piecewise linear models. *Akio Matsumoto and Ferenc Szidarovszky* build a dynamic monopoly model in which a bounded rational monopolist has partial and delayed information about the inverse demand function. In order to deal with this kind of uncertainty and to react smoothly to sudden market changes, the monopolist adopts a 'gradient' output decision rule based on average past data. After the benchmark case of fixed delay, the authors investigate the case of continuously distributed delays, under different weighting

functions of past data. Unlike the case of static rational monopoly, cyclic dynamics can emerge from a quite simple economic structure in this case. In particular, the parameter representing the length of the delay has a threshold value above which stability is lost via a Hopf bifurcation. The bifurcation boundary in the parameter space is investigated analytically and numerically, under different time averaging patterns. The last two chapters in this part are concerned with the joint impact of nonlinearity and stochastic factors in macroeconomic dynamics, which also represents a major topic among Carl's research interests in recent years. The chapter by *Simone Landini, Mauro Gallegati, Joseph E. Stiglitz, Xihao Li and Corrado Di Guilmi* develops an Agent-Based Model (ABM) of an economy with heterogeneous and interacting firms subject to financial constraints. It focuses on the macro effects of firms' learning and decision process, according to the notion of "social atom" (Buchanan 2007). In a nonlinear stochastic environment, the aggregate observables generated by the ABM are analysed by means of master equations and combinatorial master equations. The chapter is concerned with the dominance and survival of firms' behavioural rules, and the role played by "financial fragility" in a complex environment. It is found that financially fragile firms—the most active ones in learning—contribute more to growth and determine periods of expansion sustained by credit supply but, at the same time, their behaviour may compromise system stability. Besides providing insights into an alternative micro-foundation of macro-models, the chapter offers a new interpretation of system phase transitions. *Reiner Franke* starts from recent empirical evidence against the hypothesis of normal distribution of aggregate output, and reconsiders this issue for quarterly US output data using a number of statistical tests, among which is the "shape parameter" of the exponential power distribution, the two polar values of which constitute the normal and the Laplace distribution with its fatter tails. It turns out that evidence against normality of output growth rates is weaker than one might expect, once a structural break between the periods of Great Inflation (GI) and Great Moderation (GM) is properly taken into account. However, if the Laplace can be rejected in favour of normality in one subsample (GI), in the other subsample (GM) normality is rejected and the Laplace cannot be ruled out. The chapter provides new empirical results and methodological insights on the important issues of nonlinearity and non-normality of economic time series.

The third part of the book focusses on Financial Market Modelling, one of the main areas of Carl's work. The first two chapters present agent-based models of financial markets with limit order books, extending some of Carl's earlier work on this topic in Chiarella et al. (2009). *Giulia Iori and Polina Kovaleva* consider an ABM of an order-driven market in which agents hold heterogeneous beliefs and study the interrelations between pre-trade quote transparency and stylised properties of order-driven markets. Their ABM is able to replicate stylised facts such as negative skewness of stock returns and clustered volatility when book depth is visible to traders. Full quote transparency contributes to convergence in traders' actions, while partial transparency restrictions may lead to long-range dependencies. *Daniel Ladley and Paolo Pellizzari* study optimal trading strategies in order book-based continuous double auction markets. Their framework is still analytically tractable and optimal trading strategies can be identified using numerical techniques. They find that the optimal strategies are well approximated by linear strategies using only the best quotes. This study illustrates that, in complex markets, optimal behaviour may be well approximated by simple (linear) heuristics, a major theme in Carl's work. The following chapter by *Wai-Mun Chia, Mengling Li and Huanhuan Zheng,* studies regime switching models in foreign exchange markets and fits in a rapidly growing literature on estimating heterogeneous agent models (HAMs), an area where Carl has made major contributions, e.g. recently in Chiarella et al. (2012). Three different empirical models are compared, endogenous switching with fractions determined by relative performance, endogenous switching based on macroeconomic fundamentals and models with heterogeneous beliefs based on a Markov-switching process. In-sample and out-of-sample forecasting are compared across these different HAMs using monthly AUD/USD exchange rate data. The last two chapters in this part of the book are concerned with time series modelling and empirical analysis of financial markets. *Andreas Röthig and Andreea Röthig* study the time-varying cross-market trading activities of speculators in US currency futures markets, extending earlier work in Röthig and Chiarella (2011). They investigate linkages between speculative activities in different currency futures markets. The results show positive responses of total/long/short speculative activities in the GBP, CAD and JPY futures markets to an increase in total/long/short speculation in the CHF futures markets, indicating the presence of cross-market herding activities. These cross-market linkages between speculative activities are relatively stable over time from 1994–2013 and therefore do not suggest that changes in regulation or new market participants or trading strategies had a significant and lasting impact on cross-market speculative activities. The final chapter in this part of the book, by *Ramaprasad Bhar and A.G. Malliaris,* deals with the Stochastic Discount Factor (SDF) methodology as a general empirical framework for asset pricing. In particular, the authors suggest a multifactor model for the SDF taking both macroeconomic fundamental factors, such as the yield curve, the VIX index and a measure for trading liquidity, as well as behavioural factors into account to identify significant determinants of the daily equity premium. The chapter proposes to include momentum return as a behavioural factor in the SDF and offers a way to address this issue empirically. The chapter also shows how copula methods can be used in this context to overcome analytical complexities for software implementation in defining the dependence between asset returns and the SDF. These five chapters illustrate the broad contributions of Carl to Financial Market Modelling.

The fourth part of the book consisting of seven chapters focusses on Quantitative Finance, another main area of Carl's work. The first two chapters develop a new framework for risk management of interest-rate products, extending some of Carl's earlier work on this topic in Chiarella and Kwon (2003), Chiarella et al. (2007) and Chiarella et al. (2010). To develop a new methodology for risk management of interest-rate sensitive products, *Masaaki Kijima and Yukio Muromachi* present a risk evaluation model for interest-rate sensitive products within the no-arbitrage framework. They first consider a yield-curve model under the observed probability measure, based on the results of the principal component analysis (PCA), to generate future scenarios of interest rates, and then identify market prices of risk for the pricing of interest-rate derivatives under the risk-neutral measure at any future time. Thus risk measures such as Value-at-Risk (VaR) of portfolios with interest-rate sensitive products can be evaluated through simple Monte Carlo simulation. They also show, however, that some market models often used in practice are not consistent with the no-arbitrage paradigm. Motivated by a significant increase in the spread between LIBORs of different tenors as well as the spread between LIBOR and the discount curve during the financial crisis, *Laura Morino and Wolfgang J. Runggaldier* extend Carl's work (Chiarella et al. 2007, 2010) beyond a pure credit risk setting to a more general post-crisis multicurve set-up. While Carl's work follows an HJM-based approach, here the authors use a short rate modelling with a short rate spread and consider a two-curve model with one curve for discounting (OIS swap curve) and one for generating future cash flows (LIBOR for a give tenor). The clean-valuation approach of pricing FRAs and CAPs without counterparty risk exhibits an "adjustment factor" when passing from the one-curve to the two-curve setting. The bottom-up short rate modelling where the short rate and a short rate spread are driven by affine factors allows for correlation between short rate and short rate spread as well as to exploit the convenient affine structure methodology. The next two chapters contribute to price American call option and futures price volatility with the framework of stochastic volatility, two areas Carl has made a significant contribution, see for example, Chiarella and Kwon (2001), Chiarella et al. (2009, 2010, 2013) and Adolfsson et al. (2013). To price an American call option when the underlying dynamics follow the Heston's stochastic volatility and the Cox-Ingersoll-Ross (CIR) stochastic interest rate, *Boda Kang and Gunter H. Meyer* formulate the call as a free boundary PDE problem on a finite computational domain with appropriate boundary conditions. Comparing with finite difference approximation, they find that the time discrete method of lines is accurate and efficient in producing option prices, early exercise boundaries and option hedge ratios such as delta and gamma. Using a continuous time forward price model with stochastic volatility, *Les Clewlow, Boda Kang and Christina Sklibosios Nikitopoulos* introduce three distinct volatility structures to capture the impact of long-term, medium-term and short-term futures price volatility in commodity futures markets. They then use an extensive 21-year database of commodity futures prices to estimate the model for six key commodities: gold, crude oil, natural gas, soybean, sugar and corn. They identify the shape and the persistence of each volatility factor, their contribution to the total variance, the extent to which commodity futures volatility can be spanned and the nature of the return-volatility relation. In the next chapter, based on the structural relationships in the electricity market, *John Breslin, Les Clewlow and Chris Strickland* develop a general framework for the modelling of Australian electricity market risk. The framework is consistent with temperature and load mean forecasts, market forward price quotes, the dependence of load on temperature and the dependence of price on load. The model uses basic building blocks of an HJM form for which Carl has contributed important results. The model can be used not only for accurate evaluation of the market risk of an electricity generation and retail company, but also for the valuation of electricity market derivatives and assets. They demonstrate the application of the framework to the Australian National Electricity

Market. In the following chapter, *Mark Craddock* presents a tractable solution to the Yakubovich parabolic PDE $u_t = x^2 u_{xx} + x u_x - x^2 u$, which arises in a number of financial mathematical problems such as Asian option pricing, the Hartman-Watson law and pricing zero coupon bonds in the Dothan model. After deriving the heat kernel for the PDE, Mark uses the Fourier sine transform to reduce the kernel to a simple form, which may be explicitly evaluated as a series of error functions. Some financial applications are then discussed. In the final chapter, by applying the approach of changing numeraire, *Gerald Cheang and Gim-Aik Teh* extend the European call option pricing formula in the literature to the case when both stock prices and interest rates are driven by jump-diffusion processes. The pricing model is an extended Merton jump-diffusion stock price model with a stochastic interest rate term structure that is an HJM-type model with jumps. The approach does not require Fourier transforms as used in the existing literature. It allows us to price the option when the bond price dynamics is also discontinuous. When the jump-sizes are fixed instead, then they get the special case of the HJM model with fixed jumps as in Chiarella and Nikitopoulos Sklibosios (2003).

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On the Volatility of Commodity Futures Prices

Les Clewlow, Boda Kang and Christina Sklibosios Nikitopoulos

1 Introduction

Commodity futures markets are playing a leading role in the current financial arena. Commercial participants with physical positions in commodities have traditionally been the main traders of commodity futures markets. However, a rapidly increasing number of financially motivated traders such as hedge funds, institutional investors and insurance companies have entered the markets and have been using commodities derivatives for portfolio diversification, and for hedging inflation and the weak U.S. dollar (Tokic (2011)). Commodity markets have experienced noteworthy price swings and significant volatility especially over the past decade. Consequently, the analysis and the management of this volatility is of paramount importance.

In this chapter, a forward price model within the Heath et al. (1992) framework for the entire term structure of futures prices is combined with a multi-factor stochastic volatility model. The proposed three-factor model aims to capture the impact of short-term, medium-term, as well as long-term futures price volatility by using exponential decaying and hump-shaped stochastic volatility factors. For these volatility specifications, the forward price model admits finite-dimensional realizations and is affine in the state space. Consequently, the model is estimated by using an extended dataset of futures prices for six major commodities traded on the CME, spanning 21 years from 1st January 1990 to 31st December 2010. Selecting the most liquid

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commodity futures markets, the following commodity futures prices are included in the study; gold, crude oil, natural gas, soybean, sugar and corn.

The model is well suited to identify the shape, the persistence and the intensity of each volatility factor. An exponential decaying volatility factor typically gauges the impact of short-term variations, subject to the rate of the decay. A hump-shaped volatility factor, in general, captures the impact of medium-term variations, subject to the location of the hump peak. The volatility structure of an infinite maturity forward price gauges the impact of long-term variation. The rate of mean reversion and the volatility for each of the stochastic volatility factors are also assessed and indicate the level of persistence and the intensity of each volatility factor. Furthermore, the model determines the extent to which commodity futures volatility can be spanned by futures contracts (i.e. hedged by futures contracts only), and the nature of the return-volatility relation in commodity futures markets.

Forward price models such as Miltersen and Schwartz (1998), Clewlow and Strickland (2000) and Miltersen (2003), as well as convenience yield models of Gibson and Schwartz (1990) and Korn (2005) have studied commodity futures markets but they are restricted to deterministic volatility. Stochastic volatility models have been proposed by Schwartz and Trolle (2009a) and Chiarella et al. (2013) and have analysed crude oil futures market volatility, under exponential decaying and hump-shaped volatility specifications, respectively. The contribution of this paper rests on using three distinct volatility structures and aiming to analyze their nature and assess their contribution.

The paper is organized as follows. Section 2 presents a Markovian affine forward price model with stochastic volatility. Section 3 describes and analyses the commodity futures price data used in the analysis and outlines the estimation method used to estimate the model. Section 4 presents and discusses the results. Section 5 concludes.

2 A Markovian Commodity Futures Price Model

A filtered probability space $(\Omega, \mathscr{A}_T, \mathscr{F}_0, P)$, $T \in (0, \infty)$ with $\mathscr{F}_0 = (\mathscr{A}_t)_{t\in[0,T]}$ is assumed, satisfying the usual conditions.[1] The uncertainty in the commodity futures market is modelled via a generic stochastic volatility process $\mathbf{V} = \{\mathbf{V_t}, t \in [0, T]\}$. Let us denote as $S(t, \mathbf{V_t})$ the spot commodity price at time $t \geq 0$, and $F(t, T, \mathbf{V_t})$ the commodity futures price at time t, for delivery at time T, (for all maturities $T \geq t$), thus by definition, $S(t, \mathbf{V_t}) = F(t, t, \mathbf{V_t})$, $t \in [0, T]$. Furthermore, no-arbitrage arguments in commodity futures markets imply that the futures price process is equal to the expected future commodity spot price under an equivalent risk-neutral probability measure Q (see Duffie (2001)), namely

$$F(t, T, \mathbf{V_t}) = \mathbf{E}^Q[S(T, \mathbf{V_T})|\mathscr{A}_t].$$

[1] The usual conditions satisfied by a filtered complete probability space are: (a) $\mathscr{F}_0$ contains all the P-null sets of $\mathscr{F}$ and (b) the filtration is right continuous. See Protter (2004) for technical details.

Consequently, the commodity futures price is a martingale under the risk-neutral measure and the commodity futures price process should follow a driftless stochastic differential equation under the risk-neutral measure. Accordingly, a three-factor model is proposed of the form

$$\frac{dF(t, T, \mathbf{V_t})}{F(t, T, \mathbf{V_t})} = \sum_{i=1}^{3} \sigma_i(t, T, \mathbf{V_t}) dW_i(t), \tag{1}$$

where, $W(t) = \{W_1(t), W_2(t), W_3(t)\}$ is a three-dimensional Wiener process. The $\mathscr{F}_0$-adapted futures price volatility processes $\sigma_i(t, T, \mathbf{V_t})$ have the functional forms, for all $T > t$,

$$\begin{aligned} \sigma_1(t, T, \mathbf{V_t}) &= \kappa_1 \sqrt{\mathbf{V_t^1}}, \\ \sigma_2(t, T, \mathbf{V_t}) &= \kappa_2 e^{-\eta_2(T-t)} \sqrt{\mathbf{V_t^2}}, \\ \sigma_3(t, T, \mathbf{V_t}) &= \kappa_3 (T-t) e^{-\eta_3(T-t)} \sqrt{\mathbf{V_t^3}}, \end{aligned} \tag{2}$$

with κ_i, $(i = 1, 2, 3)$ and η_i, $(i = 2, 3)$ constants. The volatility process $\mathbf{V_t} = \{\mathbf{V_t^1}, \mathbf{V_t^2}, \mathbf{V_t^3}\}$ is a three-dimensional Heston (1993) type process such that

$$d\mathbf{V_t^i} = \mu_i(\nu_i - \mathbf{V_t^i})dt + \varepsilon_i \sqrt{\mathbf{V_t^i}} dW_i^V(t), \tag{3}$$

where μ_i, ν_i, and ε_i are constants for $i = 1, 2, 3$, and $W^V(t) = \{W_1^V(t), W_2^V(t), W_3^V(t)\}$ is the three-dimensional Wiener process driving the stochastic volatility process $\mathbf{V}_t$, for all $t \in [0, T]$. The first volatility factor can be considered as the factor capturing the volatility of the futures price returns with infinite maturity, thus representing the long-term volatility in commodity futures markets. The second volatility factor predominantly gauges the volatility of the short-term futures price returns as it allows a volatility structure that decays exponentially as the time to maturity increases. The third volatility factor generates humps in the volatility structure, thus reveals principally the impact of the volatility of medium-term futures prices. The proposed volatility structure also allows each of these volatility factors to be driven by a different stochastic volatility processes, consequently the proposed model has the potential to capture the impact and the nature of market shocks to the entire volatility term structure (including short-term, medium-term and long-term) and determine their contribution to the total variance.

In addition, the following correlation structure of innovations between volatility and futures price returns is assumed

$$\mathbf{E}^Q[dW_i(t) \cdot dW_j^V(t)] = \rho_i dt, \text{ for } i = j; \text{ and } 0, \text{ for } i \neq j, \tag{4}$$

where ρ_i are constants for $i = 1, 2, 3$. The correlation structure of innovations between volatility and futures prices determines the extent to which the volatility risk can be hedged (spanned) by futures contracts. When the Wiener processes $W_i(t)$ and $W_i^V(t)$ are uncorrelated then the volatility risk is unhedgeable by futures contracts. When the Wiener processes $W_i(t)$ and $W_i^V(t)$ are correlated, then the volatility risk can be partially hedged by futures contracts. Consequently, futures derivatives that are sensitive to volatility, such as options on futures, cannot be completely hedged by using only futures contracts. Furthermore, this modelling framework allows us to assess the dynamic relationship between futures price returns and volatility changes. A negative (positive) correlation implies a negative (positive) relation between price returns and their volatility, a well-known empirical phenomenon termed as asymmetric (inverted asymmetric) volatility. Yet again, the proposed model can capture the volatility reaction of each of the volatility factors, as these factors are modelled as separate identities.

For modelling convenience, we express the system of Eqs. (1) and (3) in terms of independent Wiener process, such that

$$\frac{dF(t,T,\mathbf{V_t})}{F(t,T,\mathbf{V_t})} = \sum_{i=1}^{3} \sigma_i(t,T,\mathbf{V_t})dW_i^1(t), \tag{5}$$

$$d\mathbf{V_t^i} = \mu_i(\nu_i - \mathbf{V_t^i})dt + \varepsilon_i\sqrt{\mathbf{V_t^i}}\left(\rho_i dW_i^1(t) + \sqrt{1-\rho_i^2}dW_i^2(t)\right), \tag{6}$$

where $W^1(t) = W(t)$ and $W^2(t)$ are three-dimensional independent Wiener processes. Accordingly, the volatility factors $\mathbf{V_t^i}$ with $\rho_i = 0$ carries a risk that cannot be spanned by futures contracts alone and when $\rho_i < 0$ ($\rho_i > 0$) then the volatility factor $\mathbf{V_t^i}$ has an asymmetric (inverted asymmetric) reaction.

It is well known that for general volatility specifications, the forward price model for pricing the commodity futures (5) is Markovian in an infinite dimensional state space. However, the volatility specifications (2) produce finite dimensional realisations of the forward price model, see Chiarella and Kwon (2001) and Björk et al. (2004).

Theorem 1 *Under the volatility specifications of* (2), $\ln F(t,T,\mathbf{V_t})$ *is affine in nine state variables, as described below:*

$$\ln F(t,T,\mathbf{V_t}) = \ln F(0,T,V_0) + \sum_{n=1}^{4}\beta_n(T-t)\phi_n(t) - \frac{1}{2}\sum_{j=1}^{5}\gamma_j(T-t)\chi_j(t), \tag{7}$$

where

$$\beta_1(T-t) = \kappa_1, \quad \gamma_1(T-t) = \kappa_1^2,$$
$$\beta_2(T-t) = \kappa_2 e^{-\eta_2(T-t)}, \quad \gamma_2(T-t) = \kappa_2^2 e^{-2\eta_2(T-t)}$$

$$\beta_3(T-t) = \kappa_3(T-t)e^{-\eta_3(T-t)}, \quad \beta_4(T-t) = \kappa_3 e^{-\eta_3(T-t)},$$
$$\gamma_3(T-t) = \beta_3(T-t)^2, \quad \gamma_4(T-t) = 2\beta_3(T-t)\beta_4(T-t), \gamma_5 = \beta_4(T-t)^2.$$

The state variables $\phi_n(t)$, $n = 1,\dots,4$ and $\chi_j(t)$, $j = 1,\dots,5$ satisfy the stochastic differential equations

$$\begin{aligned}
d\phi_1(t) &= \sqrt{\mathbf{V_t^1}}dW_1(t),\\
d\phi_2(t) &= -\eta_2\phi_2(t)dt + \sqrt{\mathbf{V_t^2}}dW_2(t),\\
d\phi_3(t) &= -\eta_3\phi_3(t)dt + \sqrt{\mathbf{V_t^3}}dW_3(t),\\
d\phi_4(t) &= (-\eta_3\phi_4(t) + \phi_3(t))dt,\\
d\chi_1(t) &= \mathbf{V_t^1}dt,\\
d\chi_2(t) &= \left(-2\eta_2\chi_2(t) + \mathbf{V_t^2}\right)dt,\\
d\chi_3(t) &= (-2\eta_3\chi_3(t) + \mathbf{V_t^3})dt,\\
d\chi_4(t) &= (-2\eta_3\chi_4(t) + \chi_3(t))dt,\\
d\chi_5(t) &= (-2\eta_3\chi_5(t) + 2\chi_4(t))dt,
\end{aligned} \tag{8}$$

subject to $\phi_n(0) = \chi_j(0) = 0$, for all n and j. The associated stochastic volatility process $\mathbf{V_t} = \{\mathbf{V_t^1}, \mathbf{V_t^2}, \mathbf{V_t^3},\}$ follows the dynamics (6).

Proof The technical details are summarized in the Appendix. □

The proposed model admits finite dimensional realizations within the affine class of Duffie and Kan (1996) and it is consistent, by construction, with the currently observed futures price curve; consequently, it is time-inhomogeneous. However for estimation purposes, it is necessary to reduce the model to a time-homogeneous one as presented in Sect. 3.2.

In addition, the market price of volatility risk is modelled with "complete" affine specifications (see Doran and Ronn (2008) and Dai and Singleton (2000)) and more specifically as

$$\begin{aligned}
dW_i^{\mathbf{P}}(t) &= dW_i(t) + \lambda_i\sqrt{\mathbf{V_t}^i}dt,\\
dW_i^{\mathbf{P}V}(t) &= dW_i^V(t) + \lambda_i^V\sqrt{\mathbf{V_t}^i}dt,
\end{aligned} \tag{9}$$

for $i = 1, 2, 3$, where $W_i^{\mathbf{P}}(t)$ and $W_i^{\mathbf{P}V}(t)$ are Wiener processes under the physical measure $\mathbf{P}$. Next, the proposed model is estimated by fitting the model to commodity futures prices of six key commodities: gold, crude oil, natural gas, soybean, sugar and corn.

3 Data and the Estimation Method

3.1 Data

An extended dataset of six commodity futures prices provided by CME is used that spans 21 years from 1st January 1990 to 31st December 2010. The selected commodities are the most actively traded commodities with crude oil leading the ladder, following by gold, soybeans, natural gas, sugar and corn.[2] Even though historical data for some of the commodities go back to 1970s, for consistency purposes, our analysis is concentrated in the past 21 years for all commodities. Furthermore, the selected commodities can be regarded as representative commodities within the metal (including gold), energy (including crude oil and natural gas) and agricultural (including soybeans, sugar and corn) commodities products.

Over this period, all commodity markets experienced extreme price movements and volatility due to noteworthy financial market and macroeconomic events such as the oil price crisis in 1990, the financial crisis in 2008, the crude oil bubble in 2008 and the ongoing food crisis initiated in 2008. Throughout the sample period, the number of available futures contracts for all commodities with positive open interest per day has increased significantly as well as the maximum maturity of futures contracts with positive open interest. For example, for crude oil, the open interest per day has increased from 17 on the 1st of January 1990 to 67 on the 31st of December 2010 and the maximum maturity of crude oil futures contracts with positive open interest has increased from 499 (calendar) days to 3,128 days.

As the number of available futures contracts per day is relatively large, for the estimation analysis, the most liquid futures contracts are used, with liquidity measured by the open interest. As contracts close to expiry have very low liquidity, the contracts selected for the study here all have more than 14 days to expiry. Each commodity has different available delivery months and their liquidity is concentrated on different contracts. Thus, the available contract months and the liquidity for each commodity are investigated, and the following selection per commodity has been made. For crude oil, the first seven monthly contracts are used, near to the trade date, followed by the three contracts which have either March, June, September or December expiration months and then we include the next five December contracts. Therefore, the

[2] Information on liquidity was collected by http://www.barchart.com, as well as by computing the average open interest available on the data. For example, on Sept 9, 2013 volumes were; 261.394 for Oct 13 crude oil, 151,589 for Dec 13 gold, 112,208 for Nov 13 soybeans, 100,027 for Oct 13 natural gas, 90,026 for Oct 13 sugar and 89,000 for Oct 13 corn.

number of crude oil futures contracts used on a daily basis ranges between 8 and 15. As natural gas has continuous monthly contracts, a maximum of 15 monthly contracts is selected (near to the trade date). For gold, the first three monthly contracts are included, followed by the four contracts with expiration months of February, April, June, August, October and December and finally four semi-annual contracts with maturities of June or December. Consequently, the number of gold futures contracts used on a daily basis varies between 8 and 11. For soybeans, the available contract months are January, March, May, July, August, September and November, therefore the first 15 contracts are used (near to the trade date), with these maturity months. The available and more liquid contract months for sugar are March, May, July and October, thus the first ten contracts are used, near to the trade date, for these maturities. The corn futures more liquid maturities are March, May, July, September and December thus all available maturities are included, with a maximum number of contracts per day being 15. In accordance with the above selection, the longest maturity, in terms of months, we have chosen for each commodity is 31 for gold, 126 for crude oil, 102 for natural gas, 60 for soybeans, 54 for sugar and 61 for corn.

It is worth noticing that for all commodities of interest the futures price surfaces have changed significantly throughout the sample period with extreme price variation over the last decade, as depicted in Fig. 1. This notable variation is also apparent from the descriptive statistics presented in Tables 1 and 2 that display the statistical features of the selected commodity futures price returns for 1 month and 12 month futures contracts for the commodities of interest.

3.1.1 Number of Stochastic Factors

The number of driving stochastic factors affecting the evolution of the futures curve is investigated by performing a principal component analysis (PCA) of the futures price returns. Table 3 displays varying levels of contributions for different commodities and Fig. 2 depicts the eigenvalues and associated volatility functions. Three factors potentially can explain between 93 % (for natural gas) to 99 % (for crude oil) of the total variation in futures returns. The proposed three-factor volatility structure (2) aims to capture the impact of shocks in the entire term structure (from short-term to long-term), and it could be represented by the three factors revealed by the PCA, even though their nature is distinctively determined by the model.

3.1.2 The Discount Function

The discount function $P(t, T)$ is obtained by fitting a Nelson and Siegel (1987) curve each trading day to LIBOR and swap data consisting of 1, 3, 6, 9 and 12 month LIBOR rates and the 2 year swap rate, similar to Schwartz and Trolle (2009a).

Let $f(t, T)$ denote the time$-t$ instantaneous forward interest rate to time T. Nelson and Siegel (1987) parameterize the forward interest rate curve as

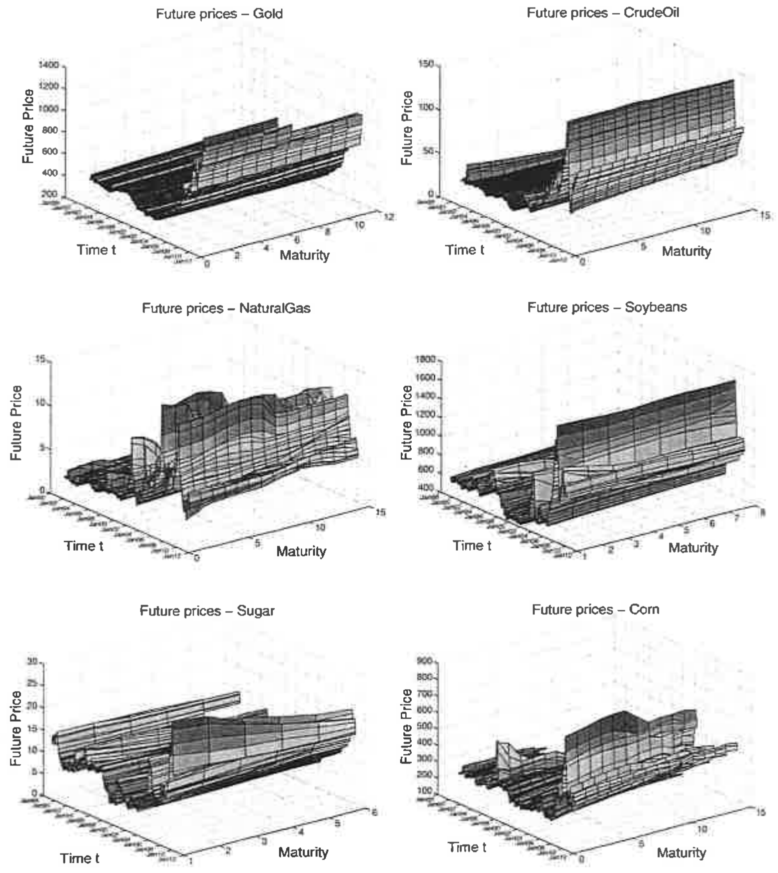

Fig. 1 Commodity futures prices. The figure plots the prices of selected commodity futures contracts from January 2, 1990 to December 31, 2010

$$f(t,T) = \alpha_0 + \alpha_1 e^{-\theta(T-t)} + \alpha_2 \theta (T-t) e^{-\theta(T-t)} \tag{10}$$

from which LIBOR and swap rates can be priced. This also yields for zero-coupon bond prices the expression

$$P(t,T) = \exp\left\{-\alpha_0(T-t) - \frac{1}{\theta}(\alpha_1+\alpha_2)\left(1-e^{-\theta(T-t)}\right) + \alpha_2(T-t)\right\}. \tag{11}$$

Table 1 Descriptive statistics—gold, crude oil and natural gas

Maturity	Gold		Crude oil		Natural gas	
	1 M	12 M	1 M	12 M	1 M	12 M
Mean	0.000220	0.000200	0.000291	0.000306	0.000185	0.000179
St. Dev.	0.011972	0.010799	0.026004	0.017870	0.037743	0.022392
Kurtosis	39.837774	28.133742	20.764654	8.156276	16.008901	19.721668
Skewness	−0.124339	−0.162282	−0.893782	−0.298393	0.301666	−0.069086

The table displays the descriptive statistics for daily log returns of futures prices between January 2, 1990 and December 31, 2010 for gold, crude oil and natural gas

Table 2 Descriptive statistics—soybeans, sugar and corn

Maturity	Soybeans		Sugar		Corn	
	1 M	12 M	1 M	12 M	1 M	12 M
Mean	0.000090	0.000084	0.000061	0.000225	0.000077	0.000096
St. Dev.	0.015746	0.013324	0.022504	0.014234	0.016618	0.012512
Kurtosis	12.025013	7.275205	12.437959	7.495627	21.100643	7.647190
Skewness	−0.932488	−0.341833	−0.345857	−0.243873	−0.847868	−0.147756

The table displays the descriptive statistics for daily log returns of futures prices between January 2, 1990 and December 31, 2010 for soybeans, sugar and corn

Table 3 Accumulated percentage of factor contribution

Commodity	One factor	Two factors	Three factors	Four factors
Gold	0.9735	0.9810	0.9881	0.9926
Crude oil	0.9503	0.9825	0.9919	0.9957
Natural gas	0.7636	0.8625	0.9308	0.9666
Soybeans	0.8973	0.9413	0.9639	0.9798
Sugar	0.8788	0.9560	0.9832	0.9956
Corn	0.8764	0.9236	0.9487	0.9712

The table displays the accumulated percentage of PCA factor contribution towards each commodity futures return variation. We found that three factors are able to explain most of the variations of the futures returns for the commodities of interest

On each observation date, the parameters $\alpha_0, \alpha_1, \alpha_2$ and θ are recalibrated, by minimizing the mean squared percentage differences between the model implied forward rates (as described in (10)) and the observed LIBOR and swap curve consisting of the 1, 3, 6, 9 and 12 month LIBOR rates and the 2 year swap rate on that date.

3.2 Estimation Method

The estimation approach is quasi-maximum likelihood in combination with the Kalman filter. The model is cast into a state-space form, which consists of the system equations and the observation equations. For estimation purposes, a time-homogeneous version of the model (7) is considered, by assuming for all T,

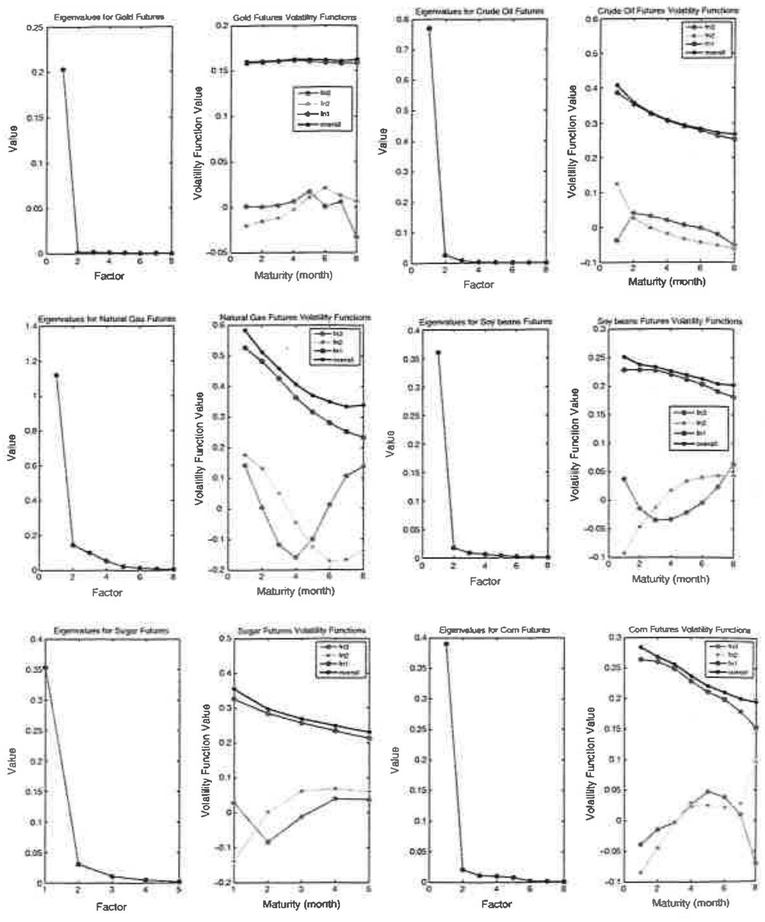

Fig. 2 PCA results (Eigenvalues and volatility functions)

$F(0, T) = f_o$, where f_o is a constant representing the long-term futures price (at infinite maturity). This constant is an additional parameter that is also to be estimated.

The system equations describe the evolution of the underlying state variables. In our case, the state vector is $X_t = \{X_t^m, m = 1, 2, \ldots, 12\}$, where X_t consists of the 12 state variables; $\mathrm{x}_j(t), j = 1, \ldots, 5, \phi_n(t), n = 1, \ldots, 4$, and $\mathbf{V}_t^i, i = 1, 2, 3$. The continuous time dynamics (under the physical probability measure) of these state variables are defined by Eqs. (6), (8) and (9). The corresponding discrete evolution is

$$X_{t+1} = \Phi_0 + \Phi_X X_t + w_{t+1}, \quad w_{t+1} \sim iid\, N(0, Q_t), \tag{12}$$

where Φ_0, Φ_X and Q_t can be computed in closed form.

From Eq. (7), log futures prices are linear functions of the state variables, thus the observation equation based on (7) can describe the relationship between observed futures prices and the state variables as

$$z_t = h(X_t) + u_t, \quad u_t \sim iid\, N(0, \Omega). \tag{13}$$

The loglikehood function is maximised by using the constrained optimization routine "e04jy" in the NAG library. Several different initial hypothetical parameter values are considered, first on monthly data, then on weekly data and finally on daily data, aiming to obtain global optima.

4 Empirical Results

4.1 Parameter Estimation

Table 4 presents the parameter estimates of a three-factor stochastic volatility model. Figures 3 and 4 depict the estimated deterministic part $\chi_i(t)$ of each volatility factor σ_i ($\sigma_i(t, T, \mathbf{V}_t^i) = \chi_i(t, T)\sqrt{\mathbf{V}_t^i}$) and the estimated time-series of volatility state factor $\mathbf{V}_t^i$.[3] Recall that the first volatility factor portrays the long-term volatility factor, while the second volatility factor is exponential decaying (dies out as the time to maturity increases) and is associated with short-term market uncertainty. The third volatility factor is hump-shaped, (volatility increases with time to maturity to a peak level, then decreases for longer times to maturity) and describes the medium-term volatility.

4.2 Gold Volatility

According to the parameter estimates for the gold futures market, see Table 4, only the first volatility factor is significant in magnitude and in contribution. For the other two factors, κ_i and ν_t^V are very close to zero, thus both the deterministic element and the stochastic element of these volatility factors are very close to zero. Table 5 shows that the long-term volatility factor contributes 99.98 % of the total variance, a result that is also consistent with PCA. The associated volatility state factor $\mathbf{V}_t^1$ is not persistent and reverts relatively quickly to the mean level (evidenced by the high value of μ_i) while its volatility is large, see also the top panels of Fig. 3. Thus,

[3] In absolute terms, $\mathbf{V}_t^i$ is the variance process and $\sqrt{\mathbf{V}_t^i}$ is the volatility process.

Table 4 Parameter Estimates

	f_o	Factor	κ_i	η_i	μ_i	ν_i	ϵ_i	ρ_i	λ_i	λ_i^V
Metals										
Gold										
	6.78	1	−0.4324	0.0000	−1.9822	0.0100	4.0110	0.8996	−3.0000	−2.1974
		2	−0.0109	4.9979	−1.6438	0.0305	1.3018	−0.9127	2.9762	−2.9740
		3	0.0308	4.8705	4.9083	0.0404	1.8998	0.9490	2.8698	−2.5760
Energy										
Crude oil										
	4.50	1	0.0364	0.0000	3.4978	0.1015	0.0100	0.5411	3.0000	−0.6957
		2	0.3735	0.3914	−2.0000	0.2624	5.0000	−0.8337	−3.0000	−1.1797
		3	−0.0804	1.6611	5.0000	5.0000	0.0619	0.9500	−3.0000	−1.6399
Natural gas										
	1.49	1	−0.2904	0.0000	−2.0000	0.9012	5.0000	0.3116	−3.0000	−1.1331
		2	0.6463	4.2571	5.0000	0.1914	5.0000	−0.9500	−3.0000	−2.8280
		3	2.4848	5.0000	−2.0000	5.0000	5.0000	−0.5886	−3.0000	−0.9792
Agricultural										
Soybeans										
	6.30	1	0.5595	0.0000	5.0000	0.0356	0.9964	0.7338	1.9856	1.4988
		2	0.0448	4.9300	4.8135	0.0130	0.9412	−0.0797	0.2685	−2.4983
		3	0.4287	2.8152	0.1185	4.8349	0.0100	−0.2922	0.2036	−0.8500
Sugar										
	1.94	1	0.2228	0.0000	−2.0000	0.5134	5.0000	−0.9155	−3.0000	−3.0000
		2	0.0000	−1.6488	−2.0000	0.1219	1.7671	−0.9221	−2.9525	−2.9494
		3	1.0191	3.3670	0.0828	5.0000	5.0000	−0.9500	0.0642	−0.0199
Corn										
	6.47	1	−0.4188	0.0000	−0.0762	0.1527	5.0000	0.9500	3.0000	−2.5545
		2	−0.0143	3.8723	−1.8526	0.1028	4.6859	−0.8463	0.1100	−0.8652
		3	0.9395	1.3926	1.3400	0.0100	0.4731	0.9500	−3.0000	−2.7719

The table displays the quasi maximum-likelihood estimates for the three-factor model specifications. F is the homogenous futures price at time 0, namely $F(0, t) = f_o, \forall t$

the long-term volatility is the dominant volatility factor in the gold futures market. Furthermore, the innovations of this volatility factor has a correlation of 0.9 with the innovations of the gold futures price returns, implying that gold futures price volatility can be mostly hedged by gold futures contracts. Moreover, the positive sign of the correlation coefficient confirms the well-documented positive gold return-volatility relation that in the spot gold market has predominantly been explained by the safe haven effect, see Baur (2012).

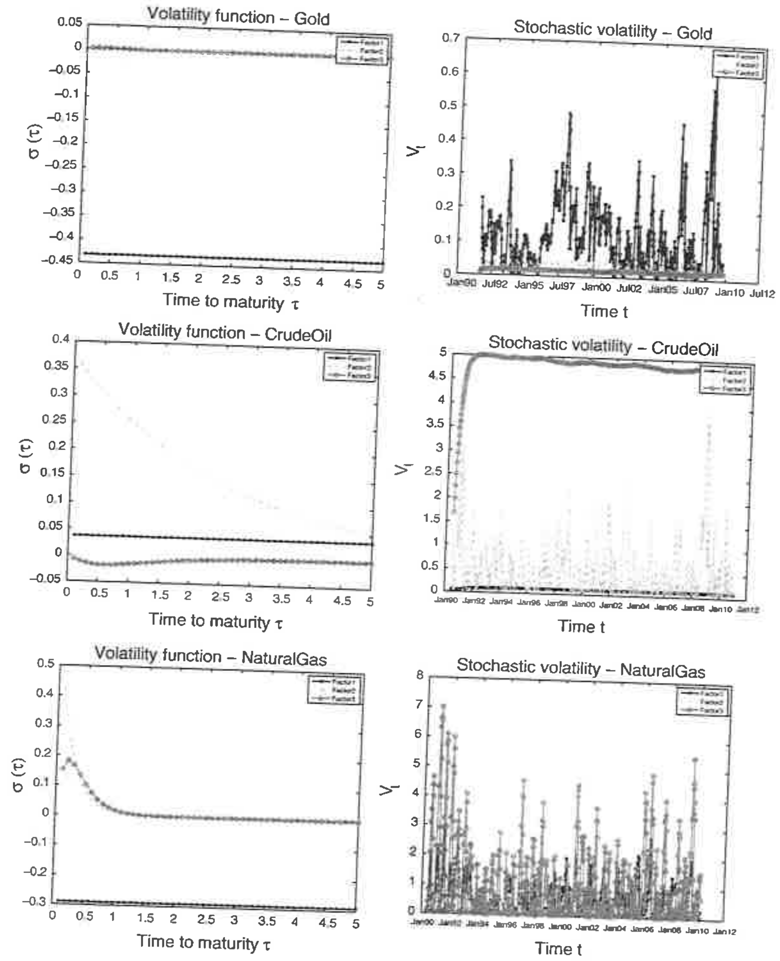

Fig. 3 Estimated volatility factors for metals and energy futures prices

4.3 Crude Oil Volatility

Table 4 demonstrates that all three volatility factors are important in the crude oil futures market. More specifically, from Table 5, the exponential decaying and hump-shaped volatility factors account for the majority of the volatility, with contributions of 65.43 and 27.84 % to the total variance, respectively. The exponential decaying

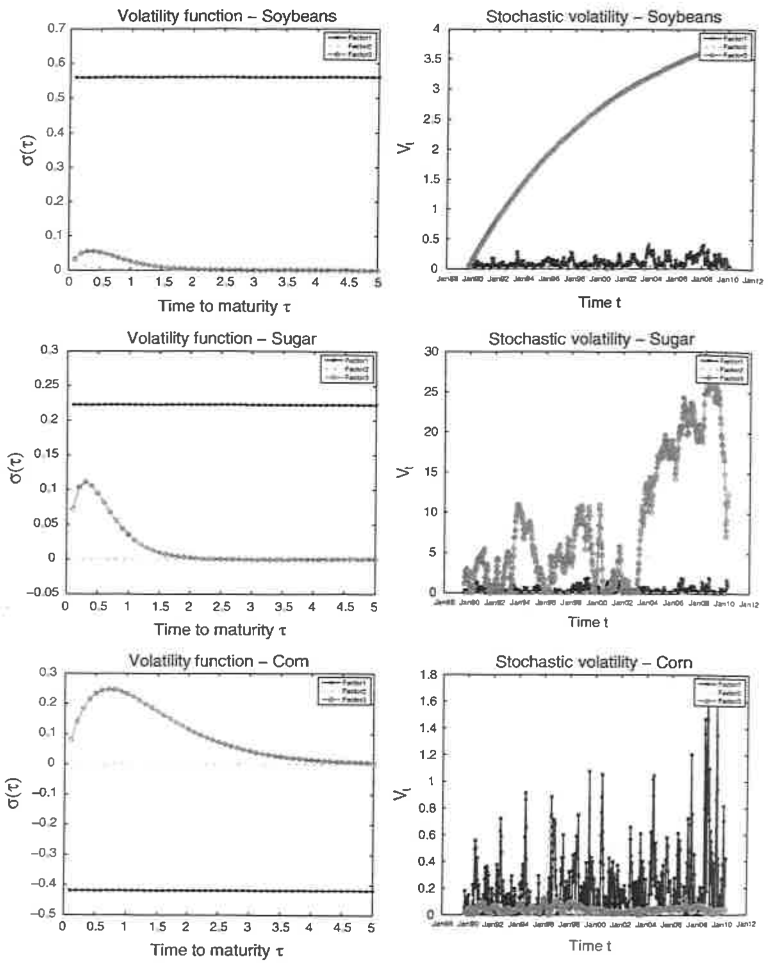

Fig. 4 Estimated volatility factors for agricultural futures prices

volatility state factor $\mathbf{V}_t^2$ is more persistent (reverts slower to the mean) and far more volatile compared to the hump-shaped state factor $\mathbf{V}_t^3$, see also the middle panels of Fig. 3. Therefore, the short-term volatility is the more influential volatility factor, followed by the medium-term and the long-term volatility factor. The correlation coefficient between volatility and futures price returns are −0.8337 and 0.95, respectively. These correlations imply that crude oil futures contracts can hedge more

Table 5 Contribution and shape of volatility factors

	Gold (%)		Crude Oil (%)		Natural Gas (%)	
σ_1	99.98	Long-term	6.73	Long-term	92.65	Long-term
σ_2	0.01	Exp	65.43	Exp	1.36	Exp
σ_3	0.01	Hump	27.84	Hump	5.99	Hump
	Soybean		Sugar		Corn	
σ_1	79.54	Long-term	70.68	Long-term	93.34	Long-term
σ_2	0.00	Exp	0.00	Exp	0.01	Exp
σ_3	20.36	Hump	29.32	Hump	6.65	Hump

The table reports the contribution of each volatility factor to the total variance for the three-factor model. The three volatility structures imposed by the model are long-term volatility (long-term), exponential decaying volatility (exp), and hump-shaped volatility (hump)

efficiently medium-term crude oil futures volatility rather than short-term crude oil futures volatility. The opposite signs of the correlation coefficients in these two dominant volatility factors verify the mixed return-volatility relation, a positive return-volatility relation as explained by the inventory effect, see Ng and Pirrong (1994) and a negative return-volatility relation as explained by the volatility feedback effect, see Salisu and Fasanya (2013).

4.4 Natural Gas Volatility

The parameter estimates for natural gas, see Table 4, demonstrate that the long-term volatility factor dominates in the natural gas futures market. More specifically, from Table 5, the first volatility factor contributes 92.65 % to the total variance. The second most contributing factor is the hump-shaped volatility factor with 6 % to the total variance. The leading volatility state factor $\mathbf{V}_t^1$ is not persistent (reverts quickly to the mean) and very volatile, see also the bottom panels of Fig. 3. Therefore, the long-term volatility is the more influential volatility factor most likely due to the impact of the seasonality associated with this commodity futures market. The correlation coefficient between this volatility factor and futures price returns is 0.3116 implying that natural gas futures contracts alone cannot hedge the natural gas futures volatility. The positive sign of the correlation coefficient of the dominant volatility factor can be explained by the inventory effect, see Ng and Pirrong (1994). Note that the other two factors exhibit a negative return-volatility relation as it can be explained by the volatility feedback effect, see Salisu and Fasanya (2013) but their contribution is marginal.

4.5 Soybean Volatility

From Table 5, two driving volatility factors are present, with the long-term volatility factor contributing 79.54 % of total variance and the hump-shaped volatility factor contributing 20.36 % of the total variance. The hump-shaped state factor $\mathbf{V}_t^3$ is highly persistent (reverts very slowly to the mean) and very volatile compared to the long-term volatility state factor $\mathbf{V}_t^1$ which is not as persistent and less volatile, see the top panels of Fig. 4. The correlation coefficients are 0.7338 for the long-term volatility factor and −0.2922 for the hump-shaped. Thus, soybean futures contracts alone cannot hedge medium-term volatility, yet they can hedge more efficiently the long-term volatility. Moreover, the more contributing long-term volatility factor involves a positive return-volatility relation, while the hump-shaped volatility factor entails a negative return-volatility relation.

4.6 Sugar Volatility

The analysis reveals two main driving volatility factors. The most contributing (with 70.68 % of total variance) is the long-term volatility factor and the second contributing volatility factor (with 29.32 % of the total variance) is the hump-shaped volatility factor, see Table 5. The long-term volatility state factor $\mathbf{V}_t^1$ is not as persistent (reverts quickly to the mean) but equally volatile compared to the hump-shaped state factor $\mathbf{V}_t^3$, see also the middle panels of Fig. 4. The correlation coefficient between the associated volatility factors and futures price returns are both negative and close to −1, thus sugar futures contracts can hedge most of the sugar futures volatility, and a negative return-volatility relation is implied.

4.7 Corn Volatility

The parameter estimates for corn from Table 4 demonstrate that the long-term volatility factor dominates in the corn futures market. More specifically, from Table 5, the long-term volatility factor contributes 93.34 % to the total variance. The second contributing factor is the hump-shaped volatility factor with 6.65 % to the total variance. The leading volatility state factor $\mathbf{V}_t^1$ is highly persistent (reverts slowly to the mean) and very volatile, see also the bottom panels of Fig. 4. Therefore, the long-term volatility is the more influential volatility factor in the corn commodity futures market. The correlation coefficient between the associated volatility factors and futures price returns are positive and high, thus corn futures contracts can hedge the corn futures volatility. The positive sign of the correlation coefficient of the two dominant volatility factors implies a positive return-volatility relation, see Ng and Pirrong (1994) and relates to the inventory effect.

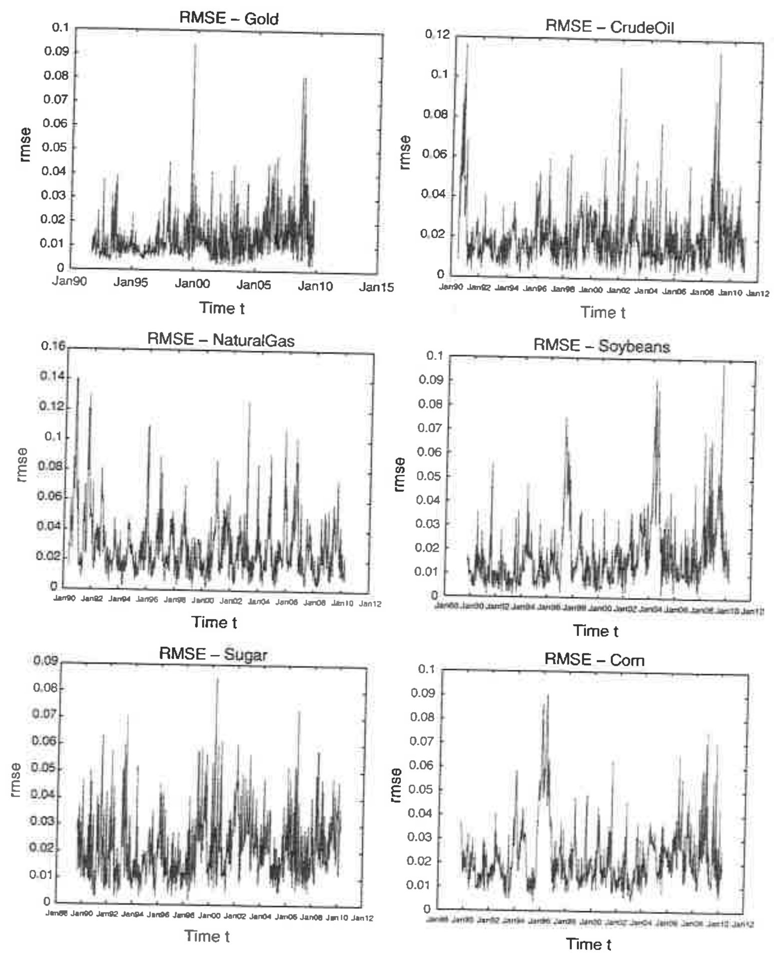

Fig. 5 Model goodness of fit

4.8 Model Fit

For illustrative purposes, the RMSEs of the percentage differences between actual futures prices and fitted futures prices to the proposed three-factor model are depicted in Fig. 5. For all commodities, the overall goodness of fit is satisfactory with most RMSE ranging between 2 and 4 % and occasionally reaching 10 % (only for natural gas the maximum RMSEs reached occasionally 14 %). As natural gas futures prices are subject to strong seasonal effects, alternate model specifications—e.g. higher dimensions of volatility factors, seasonal dynamics or regime switching volatility schemes—would have the potential to better capture the natural gas futures price dynamics. This also applies for the agricultural commodities, where the RMSEs are relatively low because of persistent seasonality effects. For crude oil and gold, the model fit is reasonable, with exceptional cases being associated with some major socio-economic effects such as the Gulf war and Global Financial Crisis (GFC).

5 Conclusion

In this paper, a tractable forward price model with stochastic volatility is proposed and an empirical study is carried out to analyze the volatility of the most liquid commodity futures markets, including the gold, crude oil, natural gas, soybeans, sugar and corn market. The model allows distinct volatility structures, including exponential decaying, hump-shaped and infinite maturity and can potentially gauge the impact of short-term, medium-term and long-term variation.

The study shows that for most of the commodities futures markets, at least two of the volatility structures are present (with varying levels of persistence and intensity). The long-term volatility is the dominant stochastic volatility factor for most commodities, except crude oil where exponential and hump-shaped volatility factors are contributing more. The extent to which the volatility can be hedged by futures contracts varies across commodities, with futures contracts being least capable to hedge volatility in the crude oil futures market, natural gas futures market and soybeans futures market.

Overall the proposed model provides a relatively good fit for these six commodities, even though the distinctive characteristics of each market is not properly accounted for. A comparative investigation between models with different specifications and their fit performance will better reflect on the quality of the model fit, a study that has been left for further research.

Under the proposed model, option prices can be obtained quasi-analytically, consequently the model can also be estimated by fitting to commodity futures options. Since options are volatility sensitive derivative instruments, this estimation study has the potential to provide constructive and insightful findings about the volatility in commodity futures markets, as well as volatility hedging effectiveness.

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Appendix: Proof of Theorem 1

We consider the process $X(t, T) = \ln F(t, T, \mathbf{V_t})$, where the forward price dynamics are given by (1) with the volatility specifications (2). Then an application of the Ito's formula derives

$$F(t, T, \mathbf{V_t}) = F(0, T) \exp\left[\sum_{i=1}^{3}\int_0^t \sigma_i(s, T, \mathbf{V_s})dW_i(s) - \frac{1}{2}\sum_{i=1}^{3}\int_0^t \sigma_i^2(s, T, \mathbf{V_s})ds\right]. \tag{14}$$

By introducing the state variables

$$\phi_1(t) = \int_0^t \sqrt{\mathbf{V_s^1}}dW_1(s), \quad \chi_1(t) = \int_0^t \mathbf{V_s^1}ds$$

$$\phi_2(t) = \int_0^t e^{-\eta_2(t-s)}\sqrt{\mathbf{V_s^2}}dW_2(s), \quad \chi_2(t) = \int_0^t e^{-2\eta_2(t-s)}\mathbf{V_s^2}ds$$

$$\phi_3(t) = \int_0^t e^{-\eta_3(t-s)}\sqrt{\mathbf{V_s^3}}dW_3(s), \quad \phi_4(t) = \int_0^t (t-s)e^{-\eta_3(t-s)}\sqrt{\mathbf{V_s^3}}dW_3(s)$$

$$\chi_3(t) = \int_0^t e^{-2\eta_3(t-s)}\mathbf{V_s^3}ds, \quad \chi_4(t) = \int_0^t (t-s)e^{-2\eta_3(t-s)}\mathbf{V_s^3}ds,$$

$$\chi_5(t) = \int_0^t (t-s)^2e^{-2\eta_3(t-s)}\mathbf{V_s^3}ds.$$

and performing some basic manipulations, Eq. (14) can be expressed as (7).

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A Multi-factor Structural Model for Australian Electricity Market Risk

John Breslin, Les Clewlow and Chris Strickland

1 Introduction

In this paper, we develop a general framework for the modelling of Australian electricity market risk based on the structural relationships in the market. The model framework is designed to be consistent with temperature and load mean forecasts, market forward price quotes, the dependence of load on temperature, and the dependence of price on load. The primary use of the model is for the accurate evaluation of the market risk of an electricity generation and retail company but it can also be used for the valuation of electricity market derivatives and assets. We demonstrate the application of our framework to the Australian National Electricity Market (NEM) by estimating the model using recent historical data from the NEM and then simulating the market using the estimated model.

Historically, the majority of published work on modelling electricity prices has taken the traditional finance approach of applying stochastic processes directly to the spot price (see for example Clewlow and Strickland (2000); Weron et al. (2004); Cartea and Figueroa (2005); Geman and Roncoroni (2006); Benth et al. (2007), (2008); Barndorff-Nielsen et al. (2010); Klüppelberg et al. (2010); Kholodnyi (2011); Veraart and Veraart (2013)). However, this approach has some fundamental disadvantages. Observation suggests that the spot price of electricity is linked to other key market variables, such as the temperature and electricity demand, and with a non-linear relationship which cannot be accurately captured by simple correlations. Furthermore, the dynamics of electricity spot prices are difficult to capture with a

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