A string number line intervention to promote students' relative thinking and understanding of scale

Introduction

Situations involving scale, rate, ratio, and proportion all require proportional reasoning (Ben-Chaim, Keret, & Ilany, 2007). Proportional reasoners can use multiplicative and relative thinking (in contrast to additive or absolute thinking), have a sense of co-variation, and can recognise situations of comparison requiring multiplicative reasoning (Fielding-Wells, Dole, & Makar, 2014). Not only is proportional reasoning essential for students to succeed in many mathematical contexts, it is also necessary in many other subject areas including art, geography, history, and science (Akatugba & Wallace, 2009). Proportional reasoning is also recognised as one of the most commonly used forms of mental computation in our daily lives and yet a large number of adults cannot reason proportionally (Lamon, 2012). Despite its recognition as a key element of numeracy, research shows that more focus is needed on helping students and teachers to develop the various aspects of proportional reasoning and the foundational concepts that underpin it (Kastberg, D'Ambrosio, & Lynch-Davis, 2012).

This article describes research on using a string number line intervention to support children's development of relative thinking and understanding of linear scale. The intervention proved effective for developing students' understanding of concepts related to scale and relative thinking as well as their mathematical language. It also proved useful for teachers as a means of assessing and monitoring their students' understanding and progress.

Background

Scale and proportion are essential beyond mathematics. They have also been recognised as cross-cutting concepts fundamental to understanding and reasoning in science (National Research Council, 2012). The interpretation and use of linear scale requires an understanding of range, scale intervals, and the use of relative thinking, particularly in situations where the intervals are not indicated or where they are not in increments of 1. This suggests that students who have difficulties in this area may also encounter difficulties in reading and using scales (e.g., on measuring devices or graph axes); in interpreting or representing data using graphs; or in creating subject-specific representations, such as timescales in science or history. Research on scale has identified possible reasons behind students' difficulties in using and interpreting scale. These include difficulties identifying relative situations (Van de Walle, Karp, & Bay-Williams, 2010); the tendency to rely on additive or absolute thinking (not considering the value of a quantity in relation to other quantities) (Lamon, 1993; Misailidou & Williams, 2003); ignoring certain relevant data; and measurement-related difficulties (Lesh, Post, & Behr, 1988).

The number line has been identified as a core mathematical tool that is used to support children's development of relative thinking (e.g., scale, magnitude, relative positioning of whole numbers and fractions); understanding of whole numbers, sequencing, and number relationships; and their understanding of equivalence (e.g., fractions, decimals, and percentages) (Geary, Hoard, Nugent & Byrd-Craven, 2008; National Council of Mathematics Teachers, 2006). According to Geary et al. (2008), unless children develop a mathematically accurate representation of the number line, there are implications for their learning of mathematics in high school and even beyond. Booth and Siegler (2006) found that there is a correlation between children's understanding of the linear number line and mathematics achievement. Despite its central importance in the development of children's mathematical reasoning, scaling has been largely ignored in research about proportional reasoning and little is known about the ability to scale and how this ability changes as children develop (Boyer & Levine, 2012). What is known is that in primary school one of the first encounters children have with formal scales occurs when they learn about numbers using a number line and that being able to deal with scale and the reasoning associated with it requires relative thinking.

The string number line is an example of an empty number line. The empty number line has been found to be useful for developing children's number sense and their confidence and ability to use numbers flexibly and is typically used as a support for counting, addition, and subtraction, often with no requirement for the lengths or distances between numbers to indicate relative value (Bobis, 2009). In this article, the use of the string number line is different; there is a definite requirement for the distances between numbers to be relative to their values because this is essential to developing an understanding of linear scale. Being a physical representation, the string number line allows students to manipulate the numbers and the distances between them, which makes it useful for developing students' understanding of scale and allows teachers to focus on developing children's understanding of the relative positioning of different numbers. It also allows children to reposition numbers and to use relative thinking when rescaling is required.

Impetus for the Study

Our previous research involving 2 500 middle years students revealed that many students from Year 5 to Year 9 had difficulties in a range of applications of proportional reasoning, including scale (see Hilton, Hilton, Dole, & Goos, 2016). The data collected about students' understanding of linear scale showed that a very high proportion of students had difficulties in identifying a missing value on a number line with 10 intervals between two known numbers, which were 20 units apart. Only a small percentage of primary school students (6.6% Year 5, 10.4% Year 6, and 20.5% Year 7), used the correct relative thinking to answer the question. The most commonly used erroneous reasoning involved absolute thinking in which the students counted on from the first number, ignoring the last number thereby ignoring the scale of the number line.

While proportional reasoning and its foundational concepts and skills do not always develop naturally (Bangert-Drowns, Hurley & Wilkinson, 2004), our previous research suggested that targeted teaching can assist students to develop stronger proportional reasoning skills (Hilton et al., 2016). Our current study focuses deliberately on a small group of teachers and specific teaching interventions to help us to learn more about how these interventions can support teaching and learning of proportional reasoning. In its first year, eight Years 3-5 teachers participated in a series of one-day professional development workshops with classroom interventions between each. The study aims to design and trial interventions that are easily implemented by teachers using simple resources and to determine their impact on particular aspects of students' ability to reason proportionally. This article focuses on the impact of a structured intervention using a string number line on children's ability to solve problems involving linear scale, relative thinking, and re-scaling.

The String Number Line Intervention

The Eight Lessons

The teachers were given a script that contained a structured series of lesson prompts to be used on each of four days per week for two weeks (a total of 8 lessons). It was expected that each day's activities would require around 10 minutes, although the teachers initially spent 15-20 minutes per day. As they became more familiar with the activities, the language and the children's abilities, the time became closer to the anticipated 10 minutes per day. In addition to discussion prompts and suggested questions, the teachers used a glossary of terms to ensure that they used accurate and consistent mathematical language when conducting the intervention. The main reasons for providing the script and glossary for the teachers was (1) to help teachers foreground the mathematical language associated with scale; (2) to support those teachers who felt unsure about how they could promote their students' understanding of scale and rescaling on the number line; and (3) to provide an opportunity for teachers to enact learning from the workshops in their classes. Appendix 1 shows the main ideas associated with each of the lessons. Figure 1 shows a string number line as it was used in this study. The first photo illustrates an introductory step in Lesson 1. The second photo requires students to find the midpoint and place the missing number.

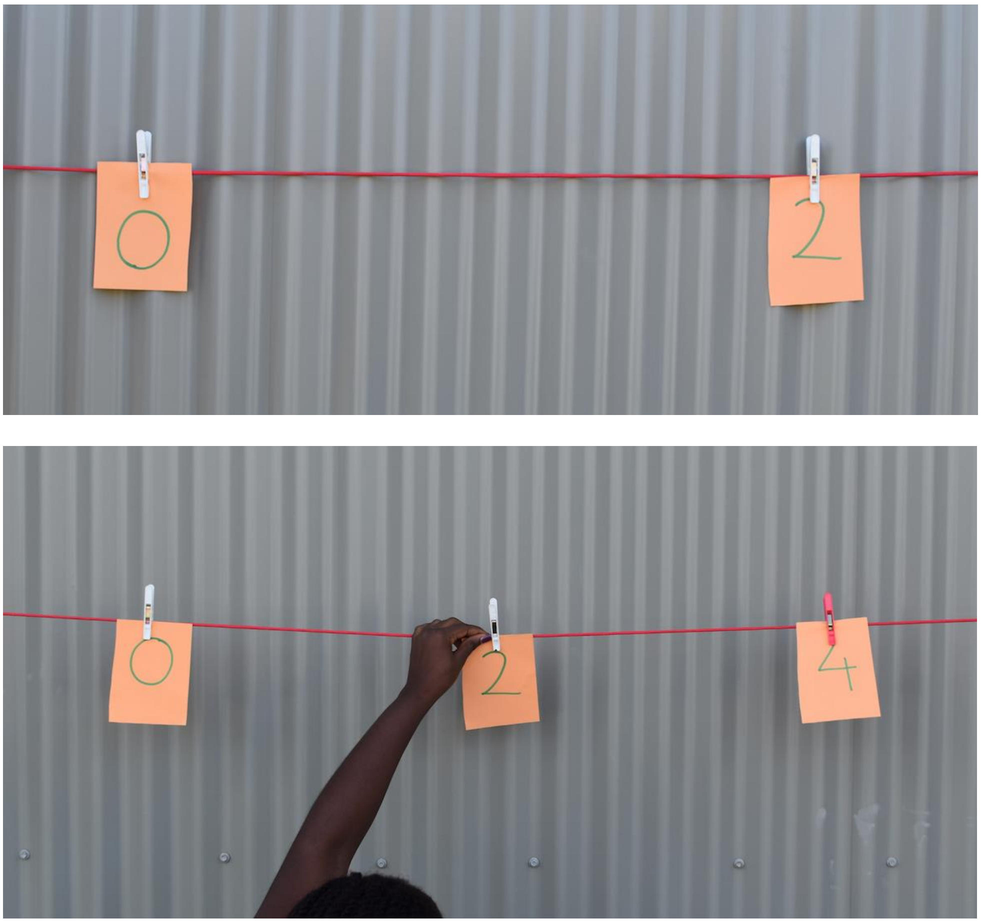

Figure 1. Rescaling and placing numbers on the string number line.

The Pre-Post Intervention Instrument

The teachers gave their students a pre-intervention instrument prior to the first lesson and this was repeated in the week following the intervention. A total of 204 students completed both instruments. The instrument consisted of 10 items that reflected the content of each of the 8 lessons. Each item showed a number line, similar in layout to a physical string number line and students were required to insert one or more missing numbers. Figure 2 shows a sample item.

Figure2. Sample item from the instrument.

Did the Students' Reasoning Improve?

The results for the pre- and post-intervention instruments were compared using paired-sample *t*-tests. There was a statistically significant difference between the scores for the pre-intervention and the post-intervention for all year levels. The teachers were also asked to annotate the script to record their reflections during the intervention and to note the ways in which students responded to the questions and activities. The scripts indicated a number of patterns in the thinking of the students. Early in the intervention, the teachers noted that

- some students used counting strategies to determine the value when one number was missing
- once the interval value was established, additive strategies were sometimes used to find missing consecutive numbers
- many children wanted to draw blank number lines to help them solve the problem, used trial and error, or asked to measure the distances
- some students could give the correct answer but weren’t able to articulate why

When solving the problems,

- the students found the items much easier when the starting number was zero
- if the starting number was not zero, students sometimes used erroneous logic (e.g., When solving

2 ☐ 8 ☐, some students found the difference between 2 and 8, i.e., 6 and halved it before realising that 3 is not the mid-point of 2 and 8)

- When the first number was missing, some students counted on and then counted back by the same amount
- pre-algebraic strategies were evident in some students’ reasoning (e.g., ‘I found that 7 + ☐ = 10, which was 3 so then I said 7 – 3 = 4’)

By the end of the intervention, the teachers noted that the children were better able to use the known values to determine the interval value and solve for the missing values. They also noted a shift from additive and absolute thinking to multiplicative or relative thinking and a distinct increase in the children’s use of the mathematical language.

Did the Teachers Find the String Number Line Intervention Useful?

The teachers perceived the string number line intervention as valuable and their responses reflected a number of different benefits for both teaching and learning. It was useful for diagnostic assessment because it allowed teachers to easily determine which students were using additive thinking or multiplicative thinking. The teachers felt that because the use of the string number line required physical manipulation the children developed an understanding more quickly and effectively. They noted that the structured nature of the activity supported their teaching by modelling a developmental sequence and providing model questions and explanations and felt that this approach prompted them to use different strategies, for example varying the starting number rather than always starting at zero. Teachers liked the short repetitious nature of the activities, which were used on a daily basis, for supporting students who were struggling with the concepts because of the. At the same time, the teachers noted that there was still the opportunity for challenging the more able students becasue some prompts elicited higher order thinking.

The intervention promoted the children's use of the mathematical language and began to use such terms as line, interval, and interval value. Teachers described this as empowering for the students because they were better able to articulate their reasoning. Some teachers used the opportunity to develop words cards or word walls to support the development and use of mathematical language. The teachers also felt that the intervention provided them with a personal learning opportunity, describing how their confidence and knowledge of scaling and re-scaling were enhanced and their increased fluency in the use of the mathematical terminology.

Conclusion

This study showed that the use of a structured intervention utilising a string number line and script to target particular concepts and mathematical language was effective for developing children's understanding of scale and their ability to use relative thinking. This study has provided some insight into the ways in which children reason when thinking about scale and the number line. An added benefit of the structured intervention was the improvement in the use of mathematical language by both teachers and students. This is a significant finding as it addresses an important issue identified by Staples and Truxaw (2012) that the use of mathematical language is a key aspect of developing conceptual understanding. Certainly, the teachers felt that the consistent use of language with their students supported its development and provided students with a means by which to articulate their thinking. While this small scale study only involved eight teachers, it provided an opportunity to investigate the effectiveness of an intervention that targeted the specific needs of those teachers and their students. The findings suggest that the string number line is an effective tool when accompanied by sequenced questions and activities that target children's development of scale and its underpinning concepts. The string number line is simple, quick to set up, and perhaps most importantly, it is both a physical and visual representation that allows children to position and reposition numbers while explaining their thinking. Further research is ongoing and will focus on investigating similar interventions for developing fractional understanding.

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Appendix 1. Main ideas associated with each lesson in the intervention

Lesson	Idea
1	Naming and locating the middle number between 0 and an even number
2	Manipulating the starting number, while keeping numbers even and maintaining the interval length
3	Determining the starting number or the end number having been given the mid-point
4	Determining the starting number or the end number when the middle number changes (re-scaling)
5	Determining the value of multiple numbers given the first two numbers (and thus the interval value)
6	Determining the value of two non-consecutive numbers (thereby determining the interval value)
7	Repeating lesson 6 but with more missing numbers or larger numbers (teacher judgment used here)
8	Rescaling - interval distance is changed so numbers need to be relocated

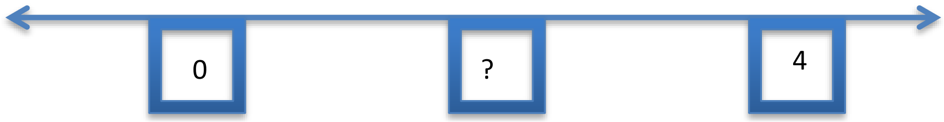
Note: All numbers were whole numbers, no fractional thinking was involved.

Appendix 2. Extract from the intervention: The first two lessons

Setting up the number line:
The best resources for the string number line are a thick string/cord and pegs that fit the string snugly. This allows the number cards to be held firmly and prevents them from spinning around on the string. The best approach is to secure the ends of the string so that students are not required to hold the ends, thereby allowing all students to participate in the activity.

DAY 1: Naming and locating the middle number between 0 and an even number. All examples keep the numbers whole and avoid fractional answers at this stage.

Example 1. Ask students to find the position of half way and then name that position. For these exercises the interval distance stays the same stays but the interval value may vary.

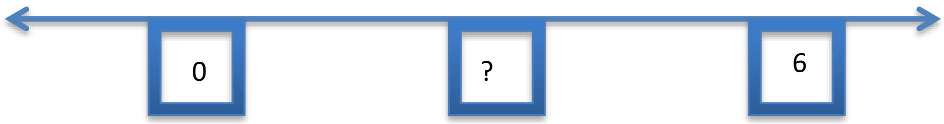


Q: How was the mid-point determined?

Q: How was the number value determined? NOTE: children who say 'halve the end number (4) are only correct when the starting number is 0.

Even at this early stage, there is an opportunity to identify whether children are using additive or multiplicative thinking. This is also an opportunity for teachers to use these terms with the children. For example, a child who determines the interval value by counting on is using additive thinking and trial and error (e.g., 0, 1, 2, no; 0, 2, 4, yes) but one who knows that the interval value is found by dividing (4 – 0) by 2 is using multiplicative thinking.

Example 2: Repeat as for Example 1 but use 0 and 6. Do not change the position of the number cards.



Q: What is the middle number now? How do you know?

Q: Why did the middle number change?

Q: Why did the end number increase by 2 but the middle number increased by only 1 from the previous example?

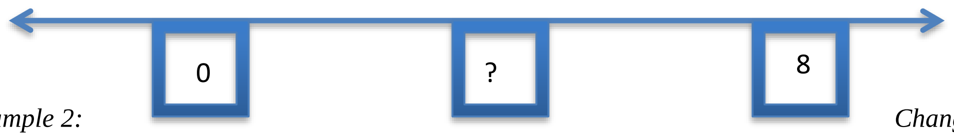
Q: Did the interval length change?

Q: Did the interval value change? What is it now?

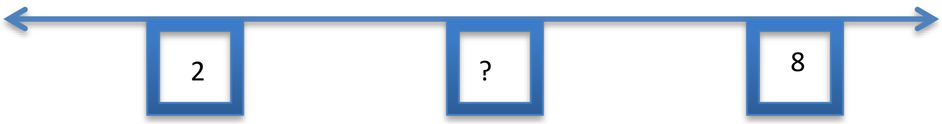
Q: Why did the interval value change but the interval length did not?

DAY 2: The purpose today is to manipulate the starting number. We are keeping numbers even and whole and the interval length does not change.

Example 1: Start with the number line below and repeat questions from Day 1 examples for how to find the middle number.



Example 2: Change the starting number to 2. Do not change the positions of the number cards.



Q: What is the middle number now? How do you know?

Q: Why did the middle number change?

Q: Why did the starting number increase by 2 but the middle number increase by 1?

Q: Did the interval distance change?

Q: What numeric value does the interval now represent?

Q: Why did the number that the interval represents change but the length of the interval did not?

Repeat the exercise with the starting number at 4 and 6. Repeat any questions you feel are needed to consolidate concepts.