

MODELLING OPTIMAL DECISIONS FOR
FINANCIAL PLANNING IN RETIREMENT USING
STOCHASTIC CONTROL THEORY



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Declaration of authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

In this thesis, we develop an expected utility model for retirement behaviour in the decumulation phase of Australian retirees with sequential family status subject to consumption, housing, investment, bequest, and a government-provided means-tested Age Pension. We account for mortality risk and risky investment assets, and we introduce a “health” proxy to capture the decreasing level of consumption for older retirees. The model is calibrated using the maximum likelihood method with empirical data on consumption and housing from the Australian Bureau of Statistic’s 2009-2010 ‘Household Expenditure Survey’ and ‘Survey of Income and Housing’. The calibrated model fits the characteristics of the data well to explain the behaviour of Australian retirees, and is then used to examine the optimal decisions given recent Age Pension policies and different family settings. Specifically, we examine optimal decisions for housing at retirement, and the optimal consumption and risky asset allocation depending on age and wealth for the Age Pension policies 2015-2017.

As the piecewise linearity in the Age Pension function requires the stochastic control problem to be solved numerically, we utilise the Least Squares Monte Carlo method to extend the problem with additional dimensions and control variables. This method is difficult to use with utility functions, as it can lead to a bad fit or bias from transforming variables. We suggest methods to account for this bias, and show that the Least Squares Monte Carlo is then accurate when applied to expected utility stochastic control problems. We then extend the optimal decisions to include annuitisation, as well as the option to scale housing in retirement or to access home equity through a reverse mortgage, and examine optimal decisions with respect to the Age Pension in retirement.

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