Large-Scale Continuous
2.5D Robotic Mapping

by

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Submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

at the
Centre for Autonomous Systems
Faculty of Engineering and Information Technology
University of Technology Sydney

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Declaration of Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signed: Luye Sun

Date: 22/02/2018
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Abstract

Autonomous robotic systems require building representations of the environment in order to accomplish their particular tasks. Creating rich, continuous probabilistic maps is essential for the robot to perceive the world. As the complexity of the task increases, robots need more sensors or more data to build maps. Noisy and incomplete data is common for the robotic sensors outputs; Gaussian Process (GP), a flexible and powerful statistical model, has become a popular method to cope with the incompleteness of sensory information, incorporate and handle uncertainties appropriately and allow a multi-resolution representation of space. GP regression has been applied in robotic mapping to predict spatial correlations and fill in gaps in unknown areas across the field. The key component of GP for robotic mapping is that it captures spatial correlations and thus increases the accuracy of the representation when fusing data. When multiple sources of data are available, spatial correlations also can be used in fusion to improve accuracy. For large datasets, however, exploiting correlations can become prohibitively expensive. One attractive strategy for reducing storage and computational cost is submapping, which works by dividing the environment into small regions. If no information is shared between maps, submaps are statistically independent.

This thesis investigates how to effectively and efficiently model the necessary spatial correlations that are required to build accurate large-scale maps. Three near-optimal probabilistic mapping frameworks that exploit global and local strategies such as submapping are proposed. SubGPBF
applies submapping techniques imposing the conditional independence between submaps. It develops a novel approach to propagate information forward and backwards, which allows spatial correlations to be transferred between submaps after fusing sensor data only within submaps.

**GMRF-BF** is a global mapping approach, which exploits the inherent structure of the recently proposed continuous Gaussian Markov Random Field (GMRF) and Bayesian fusion in information form, to model spatial correlations using a sparse information matrix. This leads to information-form Bayesian fusion that is linear in cost. To further increase computational efficiency, this thesis combines ideas from the previous two approaches (the GMRF model and the information-form submapping) to propose another new framework named **subGMRF-BF**. The forward and backward update algorithms formulated in information form are introduced to produce a highly efficient approach due to the conditional independence between submaps when assuming Gaussian distribution.

All three frameworks lend themselves to generate accurate 2.5D probabilistic maps at high resolution. They can handle varying noise from disparate sensor sources and incorporate spatial correlations in a statistically sound way. They are all efficient in memory requirements as there is no need to recover the full covariance matrix or information matrix.

The performance of the three frameworks was evaluated on one controlled terrain elevation dataset and a real water pipe thickness dataset. Five other methods, including one optimal global (fully correlated) method and one without spatial correlations, were used to benchmark the proposed methods. The experiments show that the accuracy, reliability and consistency are improved when the spatial correlations are correctly modelled and incorporated. The experiments also show that all three frameworks achieve storage and computational gain compared with the fully correlated benchmark approach, while subGMRF-BF outperforms all others.
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First and foremost I would like to express my sincere gratitude to my supervisors, Prof. Jaime Valls Miro and Dr Teresa Vidal-Calleja, for being enthusiastic in guiding my research, for the freedom to learn and explore exciting topics, and for their patience. I hope I continue to collaborate with you in the future.

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Contents

Declaration of Authorship iii
Abstract v
Acknowledgements vii
List of Figures xiii
List of Tables xv
List of Algorithms xvii
Acronyms xix
Nomenclature xxiv

1 Introduction 1
1.1 Motivation ........................................ 1
1.2 Research Problems and Scope ............................. 3
1.3 Main Contributions ................................... 5
1.4 Thesis Structure ..................................... 6
1.5 Publications ....................................... 8

2 Related Work 11
2.1 Localisation and Mapping ................................ 11
2.2 Probabilistic 2.5D Mapping with Statistical Tools .............. 13
2.2.1 Probabilistic Mapping ................................ 13
2.2.2 Gaussian Processes for Probabilistic Mapping .............. 15
2.2.3 Gaussian Markov Random Fields for Probabilistic Mapping .... 16
2.3 Probabilistic Fusion for 2.5D Mapping ........................ 18
2.4 Efficient Approximation Methods ........................... 20
2.4.1 Gaussian Processes Approximations ..................... 20
2.4.2 Bayesian Committee Machine .......................... 21
2.4.3 Submapping Techniques ............................. 23
2.4.4 Tree data structures ................................. 23
2.4.5 Markovian Approximation ............................. 24
## 3 Background Theories and Techniques

2.5 Links to the Proposed Work ........................................... 24
2.6 Summary ........................................................................ 27

3 Background Theories and Techniques .................................. 29

3.1 Symbols and Notation ................................................. 29
3.2 Probability Theory Preliminary ...................................... 30
  3.2.1 Some Basic Concepts and Rules .............................. 30
  3.2.2 Multivariate Normal Distribution ............................ 32
  3.2.3 Independence and Conditional Independence ............... 33
  3.2.4 Probabilistic Graphical Models ............................... 34
  3.2.5 Linear Gaussian Systems ................................... 35
3.3 Gaussian Processes ...................................................... 36
  3.3.1 Definition ................................................... 37
  3.3.2 GP Regression and Model Selection ......................... 37
  3.3.3 Covariance Function ....................................... 41
  3.3.4 GP Software ........................................... 43
3.4 Gaussian Markov Random Fields and the SPDE approach ......... 43
  3.4.1 Definition ................................................... 43
  3.4.2 GMRF Regression and the SPDE approach ................. 45
  3.4.3 GMRF Software ....................................... 46
3.5 Bayesian Data Fusion for Linear Gaussian Systems ............... 47
  3.5.1 Covariance-form Bayesian Fusion .......................... 47
  3.5.2 Naïve Bayesian Fusion ................................... 48
  3.5.3 Information-form Bayesian Fusion ......................... 49
3.6 GPBF: GP and Bayesian Fusion for 2.5D Mapping .................. 50
  3.6.1 Problem Statement ....................................... 50
  3.6.2 Approach ........................................... 51
3.7 Summary ................................................................. 52

4 Gaussian Processes for Bayesian Fusion with Conditionally Independent 2.5D Submapping ...................................................... 53

4.1 Problem Statement and Approach Overview .......................... 54
4.2 SubGPBF: the Incremental CI Submapping Approach ............. 55
  4.2.1 The Graphical Model and CI Submaps ...................... 55
  4.2.2 GP for Prior Mapping .................................... 59
  4.2.3 Spatially Correlated Bayesian Fusion ....................... 59
  4.2.4 Forward Update ......................................... 60
  4.2.5 Backward Update ........................................ 63
4.3 Comparison .............................................................. 66
  4.3.1 Forward Update and Backward Update ..................... 66
  4.3.2 Forward Update and Augmentation ......................... 66
4.4 Variants and Applications ............................................. 66
  4.4.1 Adapting the Forward Update Algorithm to Prediction ...... 67
  4.4.2 Adapting SubGPBF to Sequential Mapping with One Data Source ...... 68
4.5 Summary .................................................. 68

5 Gaussian Markov Random Fields for Bayesian Fusion and 2.5D Mapping 71
  5.1 Problem Statement and Approach Overview 72
  5.2 GMRF-BF: the Information-form Global Mapping Approach 73
    5.2.1 GMRF-SPDE for Prior Mapping 73
    5.2.2 Information-form Bayesian Fusion with Correlations 74
    5.2.3 Map Recovery 75
  5.3 Summary .................................................. 76

6 Gaussian Markov Random Fields for Bayesian Fusion with Conditionally Independent 2.5D Submapping 77
  6.1 Problem Statement and Approach Overview 78
  6.2 SubGMRF-BF: the Incremental CI Submapping Approach 81
    6.2.1 The Graphical Model and Sparsity of Information Matrix 81
    6.2.2 GMRFs for Prior Mapping and Correlated Bayesian Fusion 81
    6.2.3 Information-form Forward Update 82
    6.2.4 Information-form Backward Update 84
    6.2.5 Map Recovery 86
  6.3 Summary .................................................. 86

7 Experimental Results 87
  7.1 Experimental Procedure 87
    7.1.1 Comparison Approaches 87
    7.1.2 Datasets and Sensor Information
      7.1.2.1 Terrain Dataset (Synthetic Noisy Data) 89
      7.1.2.2 Pipe Wall Thickness Dataset (Real Experimental Data) 89
    7.1.3 Evaluation 91
  7.2 Results 101
    7.2.1 Qualitative Evaluation 101
    7.2.2 Quantitative Evaluation 103
  7.3 Summary ................................................. 106

8 Conclusion 107
  8.1 Final Remarks 107
  8.2 Limitations of the Research 110
  8.3 Future Work ............................................. 111

A Bayesian Fusion for Linear Gaussian Systems 113

B Proof of the Forward Update Algorithm 117

C Proof of the Forward Update Algorithm: An Alternative Way 121

D Proof of the Backward Update Algorithm 123
E  The Explicit Link between GMRF-SPDEs and Matérn GPs 127
F  Proof of the Information-Form Forward Update Algorithm 131
G  Proof of the Information-Form Backward Update Algorithm 137
H  Application of SubGPBF to Mapping with One Large Data Set 143

Bibliography 149
List of Figures

1.1 2D view of pipe wall thickness maps .............................................. 2
1.2 Overall thesis structure ................................................................. 7
3.1 Schematic representation of a Bayes network showing three sets of nodes .... 35
3.2 Flowchart of the GPBF framework ......................................................... 51
4.1 Synthetic terrain datasets ................................................................. 54
4.2 Flowchart of the proposed subGPBF framework ...................................... 56
4.3 2D demonstration of subGPBF ........................................................... 57
4.4 Schematic representation of all CI submaps ........................................... 57
5.1 Flowchart of the proposed GMRF-BF framework ................................... 72
6.1 Flowchart of the proposed subGMRF-BF framework ................................ 79
6.2 Schematic representation of the entries that are calculated during submapping 80
7.1 The synthetic terrain datasets and groundtruth ...................................... 90
7.2 Pipes’ remaining wall thickness maps in 2D view .................................. 91
7.3 Results of GPBF on elevation map ....................................................... 92
7.4 Results of SparseGPBF on elevation map .............................................. 92
7.5 Results of GMRF-BF on elevation map ............................................... 93
7.6 Results of GPBF-BCM on elevation map ............................................. 93
7.7 Results of CCIS on elevation map ....................................................... 94
7.8 Results of SubGPBF on elevation map ............................................... 94
7.9 Results of ICIS on elevation map ....................................................... 95
7.10 Results of SubGMRF-BF on elevation map .......................................... 95
7.11 Results of NaïveGPBF on elevation map ............................................. 96
7.12 Results of GPBF on pipe wall thickness map ....................................... 96
7.13 Results of SparseGPBF on pipe wall thickness map ................................ 97
7.14 Results of GMRF-BF on pipe wall thickness map .................................. 97
7.15 Results of GPBF-BCM on pipe wall thickness map ............................... 98
7.16 Results of CCIS on pipe wall thickness map ........................................ 98
7.17 Results of SubGPBF on pipe wall thickness map ................................... 99
7.18 Results of ICIS on pipe wall thickness map ......................................... 99
7.19 Results of SubGMRF-BF on pipe wall thickness map ............................. 100
7.20 Results of NaïveGPBF on pipe wall thickness map ................................ 100
List of Tables

3.1 Examples of some covariance functions $k(x, x')$. ................... 41
7.1 Computational complexity of all compared methods .................. 102
7.2 Computational time of terrain data (in seconds) ..................... 103
7.3 Computational time of pipes’ wall thickness data (in seconds) ...... 104
7.4 RMSE± std of terrain data in the 5-run Monte-Carlo simulation (in metre) . . . . . . 104
7.5 RMSE of pipe wall thickness data (in mm) .............................. 105
# List of Algorithms

1. **SubGPBF** ......................................... 58
2. **Forward update** ..................................... 62
3. **Backward update** .................................... 65
4. **SubGPBF for one dataset** ............................. 69
5. **SubGMRF-BF** ........................................ 80
6. **Information-form forward update** ..................... 83
7. **Information-form backward update** ................... 85
Acronyms

1D One-Dimensional. xvii, 112

2.5D Two-and-a-half Dimensional. xvii, 1, 13–16, 18, 25, 38, 50, 53, 75–78, 86, 107, 128

2D Two-Dimensional. xvii, 2, 15, 16, 24, 50, 54, 74, 75

3D Three-Dimensional. xvii, 1, 16, 22, 25, 74, 89, 112

ARD Automatic Relevance Determination. xvii

ASV Autonomous Surface Vehicle. xvii

AUC the Area Under the receiver operating characteristic Curve. xvii

BCM Bayesian Committee Machine. xvii, 6, 21, 22, 26, 87, 101

CAS Centre for Autonomous Systems. vii, xvii

CCIS Covariance-form Conditionally Independent Submapping. xvii, 8

CDED Canadian Digital Elevation Data. xvii, 89

CI Conditionally Independent. xvii, 4–8, 25, 26, 34, 53, 55, 58, 59, 62–64, 66, 68, 69, 77, 78, 80, 81, 83, 84, 86, 96, 105, 107, 108
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
<th>Page References</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI property</td>
<td>Conditional Independence property.</td>
<td>xvii, 4–6, 25, 30, 44, 58, 59, 61, 64, 69, 77, 78, 82, 83, 85, 86, 108–110, 117, 123, 131, 137, 145, 146</td>
</tr>
<tr>
<td>CML</td>
<td>Concurrent Mapping and Localisation.</td>
<td>xvii</td>
</tr>
<tr>
<td>COM</td>
<td>Continuous Occupancy Map.</td>
<td>xvii</td>
</tr>
<tr>
<td>DAG</td>
<td>Directed Acyclic Graph.</td>
<td>xvii, 34</td>
</tr>
<tr>
<td>DEM</td>
<td>Digital Elevation Model.</td>
<td>xvii</td>
</tr>
<tr>
<td>DGM</td>
<td>Directed Graphical Model.</td>
<td>xvii, 34, 55</td>
</tr>
<tr>
<td>EK</td>
<td>Expected Kernel.</td>
<td>xvii</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter.</td>
<td>xvii, 12, 13, 48, 66</td>
</tr>
<tr>
<td>ESM</td>
<td>Expected Sub-Map.</td>
<td>xvii</td>
</tr>
<tr>
<td>GM</td>
<td>Graphical Model.</td>
<td>xvii, 34, 44, 55</td>
</tr>
<tr>
<td>GMRF</td>
<td>Gaussian Markov Random Field.</td>
<td>vi, x, xvii, 4–8, 15–17, 24, 25, 27, 43–47, 52, 72–74, 76, 77, 80–82, 86, 93, 94, 101, 105, 108–110</td>
</tr>
<tr>
<td>GMRF-SPDE</td>
<td>Gaussian Markov Random Field via the Stochastic Partial Differential Equation.</td>
<td>xvii, 17, 43, 45, 46, 71–73, 91, 93, 109, 110</td>
</tr>
<tr>
<td>GPBF</td>
<td>Gaussian Processes for Bayesian Fusion.</td>
<td>xvii, 7, 24–26, 29, 50–52, 72, 76, 77, 87, 88, 96, 101–103, 105, 107, 111, 112</td>
</tr>
<tr>
<td>GPIS</td>
<td>Gaussian Process Implicit Surfaces.</td>
<td>xvii, 112</td>
</tr>
</tbody>
</table>
GPOM  Gaussian Processes Occupancy Map. xvii, 16
GRF  Gaussian Random Field. xvii, 16, 17, 43, 45, 46, 71, 127
GT  Groundtruth. xvii, 90, 91

HMM  Hidden Markov Model. xvii

i-backwardUpdate  Information-form Backward Update. xvii
i-correlateFusion  Information-form correlated Bayesian fusion. xvii
i-forwardUpdate  Information-form Forward Update. xvii
I-GPOM  Incremental Gaussian Processes Occupancy Map. xvii
ICIS  Information-form Conditionally Independent Submapping. xvii, 8
iff  if and only if. xvii, 33, 34, 38, 44
iid  independent and identically distributed. xvii, 38, 67, 90

INLA  Integrated Nested Laplace Approximation. xvii, 45

KLD  Kullback-Leibler Divergence. xvii

MAP  Maximum a Posteriori. xvii, 12, 48, 51, 59, 72, 74
MCMC  Markov chain Monte Carlo. xvii, 45
MDP  Markov Decision Process. xvii
MRF  Markov Random Field. xvii, 16, 17, 24
MSE  Mean Squared Error. xvii, 39
MVN  Multivariate normal. xvii, 4–6, 32, 37, 38, 44, 45, 52, 55, 60, 61, 67, 82, 101, 108, 110, 111, 125
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDT</td>
<td>Non Destructive Testing. xvii</td>
</tr>
<tr>
<td>NLML</td>
<td>Negative Log of the Marginal Likelihood. xvii</td>
</tr>
<tr>
<td>OGM</td>
<td>Occupancy Grid Map. xvii</td>
</tr>
<tr>
<td>PD</td>
<td>Positive Definite. xvii</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability Density Function. xvii, 30, 32, 44, 54, 60, 61, 81, 84, 123, 144</td>
</tr>
<tr>
<td>POMDP</td>
<td>Partially Observable Markov Decision Process. xvii</td>
</tr>
<tr>
<td>PSD</td>
<td>Positive Semi-Definite. xvii</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Squared Error. xvii, 105</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic. xvii</td>
</tr>
<tr>
<td>ROS</td>
<td>Robot Operating System. xvii</td>
</tr>
<tr>
<td>SE</td>
<td>Squared Exponential. xvii, 41, 42</td>
</tr>
<tr>
<td>SEIF</td>
<td>Sparse Extended Information Filter. xvii, 4</td>
</tr>
<tr>
<td>SLAM</td>
<td>Simultaneous Localisation And Mapping. xvii, 4, 6, 11–13, 25, 27, 48, 58, 66</td>
</tr>
<tr>
<td>SPDE</td>
<td>Stochastic Partial Differential Equation. xvii, 17, 45, 47, 76, 81, 127</td>
</tr>
<tr>
<td>subGMRF-BF</td>
<td>Gaussian Markov Random Fields for Bayesian Fusion with Conditionally Independent 2.5D Submapping. xvii, 3, 6, 8, 25–27, 75, 78, 86, 96, 101, 102, 105, 106, 108–112</td>
</tr>
<tr>
<td>subGPBF</td>
<td>Gaussian Processes for Bayesian Fusion with Conditionally Independent 2.5D Submapping. xvii, 3, 6–8, 25, 53–55, 65–69, 73, 75–78, 86, 96, 101, 102, 105, 106, 108–112</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>UGM</td>
<td>Undirected Graphical Model. xvii, 55</td>
</tr>
<tr>
<td>w.r.t.</td>
<td>with respect to. xvii, 40, 44</td>
</tr>
<tr>
<td>WGP</td>
<td>Warped Gaussian Process. xvii</td>
</tr>
<tr>
<td>WGPOM</td>
<td>Warped Gaussian Processes Occupancy Map. xvii</td>
</tr>
</tbody>
</table>
Nomenclature

\( \approx \) approximately equal

\( \mathcal{O}(\cdot) \) big O notation, which characterizes the growth rates of functions; for functions \( f \) and \( g \) on \( \mathbb{R} \), we write \( f(n) = \mathcal{O}(g(n)) \) if the ratio \( f(n)/g(n) \) remains bounded as \( n \to \infty \)

\( \theta \) vector of hyperparameters

\( \mu_b^a \) the mean estimate \( \mu \) of variable \( b \) given observation \( a \)

\( \xi_{s_1} \) state vector variables of submap \( s_1 \)

\( x \sim p(x) \) \( x \) is distributed according to distribution \( p(x) \)

\( y_{-ij} \) the set of variables \( y = \) except \( y_i, y_j \)

\( \perp \) independent

\( \text{Cov}[\cdot] \) covariance of random variables/vectors

\( \mathbb{E}[\cdot] \) expected value of a random variable

\( \iff \) if and only if

\( \mathbb{R} \) the real numbers

\( x \) a column vector

\( N_c(\eta, Q) \) equals to \( N(\mu, \Sigma) \), where \( Q = \Sigma^{-1}, \eta = Q \mu \)

\( \text{tr}(A) \) trace of matrix \( A \)
$\alpha$ proportional to

$\sim$ distributed according to

$\top$ transpose operation

$\triangleq$ an equality which acts as a definition

$\mathbb{V}[\cdot]$ variance of a random variable

$\{x_i\}_{i=1}^n$ a set that equals to $\{x_i,\ldots,x_n\}$

$I_n$ an identity matrix of dimension $n \times n$

$k(\cdot, \cdot)$ covariance function of Gaussian processes

$s_1^+$ the updated estimate of $s_1^-$

$s_1^-$ the prior estimate of submap $s_1$

$s_1^{++}$ the updated estimate of $s_1^+$

$X$ a matrix, or a matrix of training inputs

$x$ a scalar

$X^*$ a matrix of test inputs

$X^\top$ the transpose of matrix $X$

$X^{-1}$ the inverse of matrix $X$

$x_i$ the $i$-th element of $x$

iff if and only if