UNIVERSITY OF TECHNOLOGY, SYDNEY

Large-Scale Continuous 2.5D Robotic Mapping

by

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Submitted in partial fulfillment of the requirements for the degree of
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at the
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Faculty of Engineering and Information Technology
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Declaration of Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

Autonomous robotic systems require building representations of the environment in order to accomplish their particular tasks. Creating rich, continuous probabilistic maps is essential for the robot to perceive the world. As the complexity of the task increases, robots need more sensors or more data to build maps. Noisy and incomplete data is common for the robotic sensors outputs; Gaussian Process (GP), a flexible and powerful statistical model, has become a popular method to cope with the incompleteness of sensory information, incorporate and handle uncertainties appropriately and allow a multi-resolution representation of space. GP regression has been applied in robotic mapping to predict spatial correlations and fill in gaps in unknown areas across the field. The key component of GP for robotic mapping is that it captures spatial correlations and thus increases the accuracy of the representation when fusing data. When multiple sources of data are available, spatial correlations also can be used in fusion to improve accuracy. For large datasets, however, exploiting correlations can become prohibitively expensive. One attractive strategy for reducing storage and computational cost is submapping, which works by dividing the environment into small regions. If no information is shared between maps, submaps are statistically independent.

This thesis investigates how to effectively and efficiently model the necessary spatial correlations that are required to build accurate large-scale maps. Three near-optimal probabilistic mapping frameworks that exploit global and local strategies such as submapping are proposed. **SubGPBF** applies submapping techniques imposing the conditional independence between submaps. It develops a novel approach to propagate information forward and backwards, which allows spatial correlations to be transferred between submaps after fusing sensor data only within submaps. **GMRF-BF** is a global mapping approach, which exploits the inherent structure of the recently proposed continuous Gaussian Markov Random Field (GMRF) and Bayesian fusion in information form, to model spatial correlations using a sparse information matrix. This leads to information-form Bayesian fusion that is linear in cost. To further increase computational efficiency, this thesis combines ideas from the previous two approaches (the GMRF model and the information-form submapping) to propose another new framework named **subGMRF-BF**. The forward and backward update algorithms formulated in information form are introduced to produce a highly efficient approach due to the conditional independence between submaps when assuming Gaussian distribution.

All three frameworks lend themselves to generate accurate 2.5D probabilistic maps at high resolution. They can handle varying noise from disparate sensor sources and incorporate spatial correlations in a statistically sound way. They are all efficient in memory requirements as there is no need to recover the full covariance matrix or information matrix.

The performance of the three frameworks was evaluated on one controlled terrain elevation dataset and a real water pipe thickness dataset. Five other methods, including one optimal global (fully correlated) method and one without spatial correlations, were used to benchmark the proposed methods. The experiments show that the accuracy, reliability and consistency are improved when the spatial correlations are correctly modelled and incorporated. The experiments also show that all three frameworks achieve storage and computational gain compared with the fully correlated benchmark approach, while subGMRF-BF outperforms all others.

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List of Algorithms

Acronyms

1D	One-Dimensional. xvii, 112
2.5D	Two-and-a-half Dimensional. xvii, 1, 13–16, 18, 25, 38, 50, 53, 75–78, 86, 107, 128
2D	Two-Dimensional. xvii, 2, 15, 16, 24, 50, 54, 74, 75
3D	Three-Dimensional. xvii, 1, 16, 22, 25, 74, 89, 112
ARD	Automatic Relevance Determination. xvii
ASV	Autonomous Surface Vehicle. xvii
AUC	the Area Under the receiver operating characteristic Curve. xvii
BCM	Bayesian Committee Machine. xvii, 6, 21, 22, 26, 87, 101
CAS	Centre for Autonomous Systems. vii, xvii
CCIS	Covariance-form Conditionally Independent Submapping. xvii, 8
CDED	Canadian Digital Elevation Data. xvii, 89
CI	Conditionally Independent. xvii, 4–8, 25, 26, 34, 53, 55, 58, 59, 62–64, 66, 68, 69, 77, 78, 80, 81, 83, 84, 86, 96, 105, 107, 108

CI property	Conditional Independence property. xvii, 4–6, 25, 30, 44, 58, 59, 61, 64, 69, 77, 78, 82, 83, 85, 86, 108–110, 117, 123, 131, 137, 145, 146
CML	Concurrent Mapping and Localisation. xvii
COM	Continuous Occupancy Map. xvii
DAG	Directed Acyclic Graph. xvii, 34
DEM	Digital Elevation Model. xvii
DGM	Directed Graphical Model. xvii, 34, 55
EK	Expected Kernel. xvii
EKF	Extended Kalman Filter. xvii, 12, 13, 48, 66
ESM	Expected Sub-Map. xvii
GM	Graphical Model. xvii, 34, 44, 55
GMRF	Gaussian Markov Random Field. vi, x, xvii, 4–8, 15–17, 24, 25, 27, 43–47, 52, 72–74, 76, 77, 80–82, 86, 93, 94, 101, 105, 108–110
GMRF-BF	Gaussian Markov Random Fields for Bayesian Fusion. xvii, 3–6, 8, 25, 26, 71, 72, 75–78, 88, 96, 99, 101–103, 105, 106, 108–111
GMRF-SPDE	Gaussian Markov Random Field via the Stochastic Partial Differential Equation. xvii, 17, 43, 45, 46, 71–73, 91, 93, 109, 110
GP	Gaussian Process. v, x, xvii, 2–7, 15, 16, 18–22, 24–27, 29, 36–43, 50–55, 59, 60, 63, 68, 69, 71, 73, 76, 88, 91, 93, 101, 105, 107, 109–112, 143, 144
GPBF	Gaussian Processes for Bayesian Fusion. xvii, 7, 24–26, 29, 50–52, 72, 76, 77, 87, 88, 96, 101–103, 105, 107, 111, 112
GPIS	Gaussian Process Implicit Surfaces. xvii, 112

GPOM	Gaussian Processes Occupancy Map. xvii, 16
GRF	Gaussian Random Field. xvii, 16, 17, 43, 45, 46, 71, 127
GT	Groundtruth. xvii, 90, 91
HMM	Hidden Markov Model. xvii
i-backwardUpdate	Information-form Backward Update. xvii
i-correlateFusion	Information-form correlated Bayesian fusion. xvii
i-forwardUpdate	Information-form Forward Update. xvii
I-GPOM	Incremental Gaussian Processes Occupancy Map. xvii
ICIS	Information-form Conditionally Independent Submapping. xvii, 8
iff	if and only if. xvii, 33, 34, 38, 44
iid	independent and identically distributed. xvii, 38, 67, 90
INLA	Integrated Nested Laplace Approximation. xvii, 45
KLD	Kullback-Leibler Divergence. xvii
MAP	Maximum a Posteriori. xvii, 12, 48, 51, 59, 72, 74
MCMC	Markov chain Monte Carlo. xvii, 45
MDP	Markov Decision Process. xvii
MRF	Markov Random Field. xvii, 16, 17, 24
MSE	Mean Squared Error. xvii, 39
MVN	Multivariate normal. xvii, 4–6, 32, 37, 38, 44, 45, 52, 55, 60, 61, 67, 82, 101, 108, 110, 111, 125

NDT	Non Destructive Testing. xvii
NLML	Negative Log of the Marginal Likelihood. xvii
OGM	Occupancy Grid Map. xvii
PD	Positive Definite. xvii
pdf	Probability Density Function. xvii, 30, 32, 44, 54, 60, 61, 81, 84, 123, 144
POMDP	Partially Observable Markov Decision Process. xvii
PSD	Positive Semi-Definite. xvii
RMSE	Root Mean Squared Error. xvii, 105
ROC	Receiver Operating Characteristic. xvii
ROS	Robot Operating System. xvii
SE	Squared Exponential. xvii, 41, 42
SEIF	Sparse Extended Information Filter. xvii, 4
SLAM	Simultaneous Localisation And Mapping. xvii, 4, 6, 11–13, 25, 27, 48, 58, 66
SPDE	Stochastic Partial Differential Equation. xvii, 17, 45, 47, 76, 81, 127
subGMRF-BF	Gaussian Markov Random Fields for Bayesian Fusion with Conditionally Independent 2.5D Submapping. xvii, 3, 6, 8, 25–27, 75, 78, 86, 96, 101, 102, 105, 106, 108–112
subGPBF	Gaussian Processes for Bayesian Fusion with Conditionally Independent 2.5D Submapping. xvii, 3, 6–8, 25, 53–55, 65–69, 73, 75–78, 86, 96, 101, 102, 105, 106, 108–112

UGM	Undirected Graphical Model. xvii, 55
w.r.t.	with respect to. xvii, 40, 44
WGP	Warped Gaussian Process. xvii
WGPOM	Warped Gaussian Processes Occupancy Map. xvii

Nomenclature

$\approx$	approximately equal
$\mathcal{O}(\cdot)$	big O notation, which characterizes the growth rates of functions; for functions f and g on $\mathbb{R}$, we write $f(n) = \mathcal{O}(g(n))$ if the ratio $f(n)/g(n)$ remains bounded as $n \to \infty$
$\boldsymbol{\theta}$	vector of hyperparameters
$\boldsymbol{\mu}_b^a$	the mean estimate $\boldsymbol{\mu}$ of variable b given observation a
$\boldsymbol{\xi}_{s_1}$	state vector variables of submap s_1
$\boldsymbol{x} \sim p(\boldsymbol{x})$	$\boldsymbol{x}$ is distributed according to distribution $p(\boldsymbol{x})$
$\boldsymbol{y}_{-ij}$	the set of variables $\boldsymbol{y}$ = except y_i, y_j
$\perp$	independent
$\mathbb{Cov}[\cdot]$	covariance of random variables/vectors
$\mathbb{E}[\cdot]$	expected value of a random variable
$\iff$	if and only if
$\mathbb{R}$	the real numbers
$\mathbf{x}$	a column vector
$\mathcal{N}_c(\boldsymbol{\eta}, Q)$	equals to $\mathcal{N}(\boldsymbol{\mu}, \Sigma)$, where $Q = \Sigma^{-1}$, $\eta = Q\boldsymbol{\mu}$
$\mathrm{tr}(A)$	trace of matrix A

$\propto$	proportional to
$\sim$	distributed according to
$\top$	transpose operation
$\triangleq$	an equality which acts as a definition
$\mathbb{V}[\cdot]$	variance of a random variable
$\{x_i\}_{i=1}^{n}$	a set that equals to $\{x_i, \ldots, x_n\}$
I_n	an identity matrix of dimension $n \times n$
$k(\cdot, \cdot)$	covariance function of Gaussian processes
s_1^{+}	the updated estimate of s_1^{-}
s_1^{-}	the prior estimate of submap s_1
s_1^{++}	the updated estimate of s_1^{+}
X	a matrix, or a matrix of training inputs
x	a scalar
X^{*}	a matrix of test inputs
$X^{\top}$	the transpose of matrix X
X^{-1}	the inverse of matrix X
x_i	the i-th element of $\mathbf{x}$
iff	if and only if