UNIVERSITY OF TECHNOLOGY SYDNEY
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Stimulated Brillouin scattering in nanophotonic waveguides and resonators

by

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Certificate of Authorship/Originality

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ABSTRACT

Stimulated Brillouin scattering in nanophotonic waveguides and resonators

by

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Dissertation directed by Associate Professor Christopher G. Poulton
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In this work, we theoretically and numerically study Stimulated Brillouin Scattering (SBS) in integrated waveguides and resonators. We review SBS process by using coupled equations and determine a broad range of SBS parameters including SBS gain, opto-acoustic overlap, optical forces and power conversion between pump and Stokes waves. For numeric analysis, in addition to performing simulations we write appropriate codes and employ different iterative techniques as well as root finding methods to analyze SBS in interested configurations.

We study silicon-chalcogenide slot waveguides as a robust candidate to enhance SBS. We explain how constructive contribution of radiation pressure and electrostriction can increase the SBS gain in this structure. We also optimize the waveguide geometry and determine the optimum pump power as well as waveguide length as a function of SBS figure of merit, using our analytic expressions. We also show that putting a silica layer on top of the waveguide lead to a significant increase in the opto-acoustic overlaps and therefore, rise the SBS gain while reducing the impact of nonlinear losses in this structure.

We explore SBS in integrated racetrack ring resonators in both regimes of amplifying and lasing. We use analytic and numeric approaches to demonstrate pump and Stokes evolution in designed rings and through the output. In addition we analyze the impact of nonlinear dispersion as well as thermal effects on SBS in rings. Finally, we determine the pump power to achieve Stokes amplification, the threshold pump power for lasing and the output Stokes power in the presence of linear and nonlinear optical losses.

Dedication

To Mudafiʿan-i Haram who defended the Ahl al-Bayt (a) and fought against takfiri terrorists.

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Sayyed Reza Mirnaziry
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List of Publications

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2. Sayyed Reza Mirnaziry, Christian Wolff, MJ Steel, Benjamin J Eggleton, and Christopher G Poulton. Stimulated Brillouin scattering in integrated ring resonators. *JOSA B*, 34(5):937–949, 2017.
3. Sayyed Reza Mirnaziry, Christian Wolff, Blair Morrison, MJ Steel, Benjamin J Eggleton, and Christopher G Poulton. Lasing in ring resonators by Stimulated Brillouin scattering in the presence of nonlinear loss. *Optics Express*, 25(20):23619–23633, 2017.

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1. Sayyed Reza Mirnaziry, Christian Wolff, MJ Steel, Benjamin J Eggleton, and Christopher G Poulton. Stimulated Brillouin scattering in silicon-chalcogenide slot waveguides. *CLEO/Europe EQEC 2015 Conference*, Munich, Germany.

We note that Chapters 4, 5 and 6 are written according to papers 1, 2 and 3, respectively with small modifications.

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Abbreviation

CW - Continuous Wave

SBS - Stimulated Brillouin Scattering

BSBS - Backward Stimulated Brillouin Scattering

SSA - Small Signal Approximation

TPA - Two Photon Absorption

FCA - Free Carrier Absorption

FCD - Free Carrier Dispersion

FSR - Free Spectral Range

SMF - Single Mode Fiber

Mid-IR - Mid- Infrared

WGR - Whispering Gallery Resonator