The Invariance of State Estimation for Robot Navigation

by

Teng Zhang

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Faculty of Engineering and Information Technology
University of Technology Sydney

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Declaration of Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Date: Feb 22 2018

Abstract

UNIVERSITY OF TECHNOLOGY SYDNEY

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We are living in an era that are being changed by mobile robots such as unmanned aerial vehicles and self-driving cars. State estimation for navigation is one of the fundamental problems in mobile robot's applications. This work entirely focuses on two problems of state estimation for robot navigation, i.e., simultaneous localization and mapping (SLAM), and visual-inertial navigation systems (VINS).

The SLAM problem asks whether it is possible for a robot to build a map of an unknown environment and simultaneously work out its own location within the map. The VINS problem aims at the estimates of a robot's pose and velocity by the on-board sensor funsion of a camera and an inertial measurement unit (IMU). After data association, both SLAM and VINS need a back-end solver to estimate the state of a robot and the environment, which mainly includes two methods: extended Kalman filter and optimization.

The main contribution of this thesis is the invariance theory, which proposes the basic principles for state estimation, i.e., the actual estimates should be invariant under unobservable (deterministic or stochastic) transformations. The invariance theory does not only provide compact, elegant, profound explanations and insights of extended Kalman filter and optimization in SLAM and VINS, but also help to design new algorithms to improve the existing methods.

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Acronyms & Abbreviations

1D	One-Dimensional
2D	Two-Dimensional
3D	Three-Dimensional
CAS	Centre for Autonomous Systems
UTS	University of Technology Sydney
SLAM	Simultaneous localization and mapping
VINS	Visual-Inertial navigation system
EKF	Extended Kalman filter
GN	Gauss-Newton
LM	Levenberg-Marquart
Dogleg	Powell's Dogleg
IMU	Inertial measurement unit
MAV	Micro aerial vehicle
UAV	Unmanned aerial vehicle

Nomenclature

	General Notations
$\mathbb{GL}(n)$	the general linear group of degree n
$\mathbb{R}^3$	The 3-dimensional Euclidean space
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{SO}(3)$	The special orthogonal group
$\mathbb{SE}(3)$	The special Euclidean group
$\mathbf{R} \in \mathbb{SO}(3)$	the orientation or the rotation
$\mathbf{p} \in \mathbb{R}^3$	the position