Foundations for a Disequilibrium Theory of the Business Cycle

Building on *The Dynamics of Keynesian Monetary Growth* by Chiarella and Flaschel (2000), this book is a key contribution to business cycle theory, setting out a disequilibrium approach with gradual adjustments of the key macroeconomic variables. Its analytic study of a deterministic model of economic activity, inflation and income distribution integrates elements in the tradition of Keynes, Metzler and Goodwin (KMG). After a qualitative analysis of the basic feedback mechanisms, the authors calibrate the KMG model to the stylized facts of the business cycle in the US economy, and then undertake a detailed numerical investigation of the local and global dynamics generated by the model. Finally, topical issues in monetary policy are studied in small macro-models as well as for the KMG model by incorporating an estimated Taylor-type interest rate reaction function. The stability features of this enhanced model are also compared to those of the original KMG model.

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Foundations for a Disequilibrium Theory of the Business Cycle

Qualitative Analysis and Quantitative Assessment

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Foreword

The authors of this book, Carl Chiarella, Peter Flaschel and Reiner Franke, have been engaged in a major research programme in macroeconomic analysis for an extended period of time, arguably dating from the mid-1980s if not earlier. This has resulted in a series of papers and books by them and several others in various combinations, including Willi Semmler, Toichiro Asada, Gang Gong, and still more. This group is scattered in various parts of the globe, principally in the cities of Bielefeld, Beijing, New York, Sydney and Tokyo. While the output of this group is the result of visits to each other's institutions (particularly those of Flaschel to the University of Technology, Sydney) and meetings at various international conferences, the intellectual centre of their enterprise has been the Faculty of Economics at Bielefeld University. It is here that Flaschel, Franke and Semmler are, or have been at various times, located, and where the group has held an almost annual workshop on their developing research agenda over the last decade. Hence I feel it is appropriate to neologize here and dub the results of their collective efforts to constitute an emerging school of macroeconomic thought 'the Bielefeld School'. This book can then be characterized as representing a significant phase in the development of this Bielefeld School.

The authors themselves have in earlier work provided their own label for the core model they have developed and studied: the 'Keynes–Metzler–Goodwin' (KMG) model. This book more directly compares this model to other macroeconomic approaches, both those of a more New-Classical orientation as well as most substantially with those of various New-Keynesian formulations, especially the recently emerging synthesis due to Michael Woodford along with Glenn Rudebusch and Lars Svensson. At one point, in reference to James Tobin's later work, they suggest that their model could be considered to be derived from an 'Old-Keynesian' perspective, and it does draw on the basic IS-LM framework still used by many policymakers, with an added aggregate supply component. However, they generally stick to their use of the KMG label in describing it.

The basic elements in this approach involve allowing for substantial real effects to arise from financial markets, which they argue is the Keynes part. The Metzler part involves allowing an important role for inventory adjustments, something that is much less common in many current macroeconomic models. Finally, the Goodwin part emphasizes the importance of income distribution, particularly wage dynamics operating through a modified Phillips curve setup. In sharp contrast to both the New-Classical and New-Keynesian approaches they abjure the rational expectations assumption in modelling inflationary expectations. They do allow inflation expectations to play a central role in their model, but view them as operating in a more generalized 'inflation climate' that gradually adjusts over time. Rather than just a trend-chasing adaptive expectations mechanism they also assume a tendency for reversion to a normal level over time, a pattern they label 'regressive expectations'. These are models fundamentally of disequilibrium dynamics with gradually adapting processes.

Another central element that distinguishes their approach from many others is the assumption of nonlinearity in the investment function. While this may further separate them from many of the New-Classical and Keynesian modellers, this draws upon the influence of earlier economists who worked at the time of Keynes, such as Kalecki or – in his aftermath – such as Kaldor and Hicks, with both Metzler and Goodwin part of that group as well. This links them with the more general literature on models of complex dynamics arising from nonlinear models, which both Chiarella and Franke have separately contributed to in the past. In this KMG approach, instability arises from the nonlinearities being sufficiently great to trigger Hopf bifurcations and resulting endogenous limit cycle behaviour. However, these nonlinearities also provide bounds to the dynamics of the system.

There are two principal extensions that this book presents. The first is an effort to reach out more directly to policymakers by an effort to calibrate their model to fit parameter values relevant to the US economy. The second (in the final two chapters) is the introduction of a Taylor rule to endogenize policy feedback and the determination of interest rates. In this they are directly confronting the efforts of Woodford, and also of Rudebusch and Svensson, who have seen the Taylor rule as a way to eliminate indeterminacy in their models. They label this extension of their basic model the KMGT model.

Their final chapter examines the stability characteristics of this KMGT model. There they de-emphasize the nonlinearity of the investment function, which allows for endogenous cycles no longer to arise from a Hopf bifurcation. They even consider the matter of cycles due to exogenous shocks on an otherwise stable system in a Frischian manner. A final curious implication from this model is a heightened importance of the Metzlerian aspect of the system in determining the pattern of its dynamics.

At this point I would like to raise a point about a lacuna in this otherwise generally comprehensive book. This is the relationship of this Bielefeld School to those of the various branches of Post-Keynesian macroeconomics. They do not directly draw upon or cite any of the current prominent Post-Keynesian economists. However, it can be argued that their approach can be viewed as a sophisticated formulation of certain Post-Keynesian elements or trends. Certainly, Goodwin as well as Kalecki have been much admired by many Post-Keynesians, and the idea that money has real effects is an idea accepted by most Post-Keynesians. However, they do not obviously focus on endogenous money per se as do Paul Davidson and Basil Moore, even though their use of a Taylor rule effectively makes money endogenous. Also, they have been more precisely mathematical than have been many of the Post-Keynesians. Nevertheless, certain Post-Keynesians have developed models that have some definite similarities to what this school does, with Philip Arestis and Peter Skott coming to mind most particularly, notably in combining financial models with real effects with distributional shares dynamics that can generate endogenous cycles. Thus, I have no problem describing the Bielefeld School as representing effectively a highly sophisticated Post-Keynesian approach. Certainly, there is no doubt that they belong to the more general Keynesian approach, arguably much more so than the New-Keynesians, who use the questionable rational expectations assumption.

Thus the authors of this book should be applauded. They have moved a distinctive and policy-relevant approach to macroeconomic analysis forward decisively. Their careful synthesis of realistic dynamic elements and their careful analysis of the sensitivity and stability characteristics of their model in a policy context is much to be admired. In this book the Bielefeld School achieves a genuine culmination of great depth and breadth.

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Preface

In this book we build on a theoretical approach the foundations for which were laid in the work *The Dynamics of Keynesian Monetary Growth: Macrofoundations* by two of the present authors. In that work we considered a hierarchically structured sequence of macrodynamic models, starting from Tobinian neoclassical monetary growth and its historical counterpart, the Keynes–Wicksell monetary growth models, leading then via Keynesian IS-LM growth dynamics to a model type that has been labelled the Keynes–Metzler–Goodwin (KMG) growth dynamics. In the present book we will extend the baseline KMG model in various directions, analysing it in a much more detailed way than in previous work and, most importantly, studying it also from the empirical and the numerical point of view. Special emphasis is placed on the dynamic feedback relationships and on endogenously generated business cycle fluctuations in a growth context. In the initial stages the study concentrates here on the private sector and, essentially, abstracts from policy issues. Gaining thereby basic insights into the stabilizing and destabilizing forces in the economy, modern discussions of monetary policy are also integrated later.

As shown in the work by Chiarella and Flaschel, the KMG model type manages to avoid a variety of problems associated with the traditional IS-LM growth model, such as the boundedness in the responsiveness of aggregate demand, multiple IS-LM equilibria, or discontinuities in phase space dynamics. The model achieves this by allowing for disequilibrium on the goods market, taking the implied inventory changes into account and introducing gradual adjustments towards desired inventories as well as the concept of expected sales. All this is formulated along Metzlerian lines and so constitutes the M-component of the KMG approach. From the higher-dimensional viewpoint of the Metzlerian disequilibrium adjustment process, the problems that many advanced IS-LM models are facing appear, in fact, rather misleading. Our model's Metzlerian component can thus be regarded as a useful or even indispensable device for the general modelling architecture, though its mechanisms are not at the heart of the economy.

The outstanding theoretical features of the KMG approach to macrodynamics are the relationships to Keynes' (1936) *General Theory* and to Goodwin's (1967) seminal paper on the interaction of growth and income distribution; these are the K- and G-components in the model. Concerning the K-component, the present book is still close to traditional macroeconomics in its description of consumption and investment behaviour. In a first stage, the interest rate is also determined by a familiar LM equilibrium condition, where the money supply is assumed to grow at a constant rate. It can in this respect be said that, while government and a central bank are present in the model, they conduct a neutral policy, so that the private sector can be studied in a kind of vacuum. In a second stage, we take up the recent New-Keynesian research agenda and follow the modern practice of studying monetary policy rules – i.e. interest rate rules of a Taylor type.

The real innovation of our modelling framework lies, nevertheless, in a new approach to the wage-price spiral as an extension of both Keynes' and Goodwin's views on this matter. As it is formulated and combined with aggregate demand, this building block can be usefully compared to the traditional Keynesian AS-AD dynamics (Old Neoclassical Synthesis) as well as to the currently fashionable New-Keynesian theory of staggered wage and price settings (New Neoclassical Synthesis). Underlining the agents' gradual reactions to the disequilibria they perceive, the wage-price dynamics in our model are, however, radically different from the Neoclassical Syntheses (Old and New), with respect to modes of operation and the implications for the macrodynamic system into which it is embedded. The role of an elaborate wage-price spiral in the course of the business cycle is thus one major focus of interest in this book, from the theoretical point of view as well as empirically, where in our numerical simulations we seek to calibrate the model's cyclical behaviour to the stylized facts of the business cycle fluctuations observed in the world's major economy, namely the US economy.

In sum, the book takes up the work begun in Chiarella and Flaschel (2000a) and provides detailed qualitative, quantitative and empirical studies of a mature version of the traditional Keynesian approach, which were then still out of reach. As an alternative to the New-Keynesian macroeconomics, it puts forward an approach to disequilibrium dynamics that aims to shed light on the study of demand-constrained modern market economies that, in particular, are subject to sometimes more and sometimes less virulent adjustments in wages and prices.

A number of professional colleagues deserve special thanks. There are, first of all, our co-authors in several related published and unpublished works, Toichiro Asada, Willi Semmler and Peter Skott, who in many ways have contributed to the present project through stimulating discussions on various aspects of the subject matter of this book as well as on related research projects. We furthermore thank Richard Day, Duncan Foley and Reinhard Neck for a variety of stimulating comments at various stages in the development of the present work and related topics. We are grateful for comments and criticisms we have received from numerous participants at presentations of aspects of the material of this book at international conferences and research seminars. Of course, none of the aforementioned is responsible for the remaining errors in this work, neither with respect to form nor substance. We are indebted to three anonymous referees who read the original version of the manuscript (chapters 2 to 6) and offered many, even detailed, suggestions for its improvement. We also wish to acknowledge the sustained financial support for this project from the School of Finance and Economics at the University of Technology, Sydney. Finally, we would like to thank Chris Harrison and Lynn Dunlop of CUP for all that they have done to make the publication process go as smoothly as it has.

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Notation

Steady-state or trend values are indicated by a superscript 'o'. When no confusion arises, letters F, G, H may also define certain functional expressions in a specific context. A dot over a variable $x = x(t)$ denotes the time derivative, a caret its growth rate: $\dot{x} = dx/dt$, $\hat{x} = \dot{x}/x$. In the numerical simulations, flow variables are measured at annual rates.

As far as possible, the notation tries to follow the logic of using capital letters for level variables and lower-case letters for variables in intensive form, or for constant (steady-state) ratios. Greek letters are most often constant coefficients in behavioural equations (with, however, the notable exceptions being π, ω, ξ and ϕ).

B	outstanding government fixed-price bonds (priced at $p_b = 1$)
C	real private consumption (demand is generally realized)
E	number of equities
F	neoclassical production function in chapter 2; otherwise generic symbol for functions defined in a local context
G	real government expenditure (demand is always realized)
I	real net investment of fixed capital (demand is always realized)
I_N^d	desired real inventory investment
$\mathcal{J}$	Jacobian matrix in the mathematical analysis
K	stock of fixed capital
L	employment – i.e. total working hours per year (labour demand is always realized)
L^s	labour supply – i.e. supply of total working hours per year
M	stock of money supply
N	inventories of finished goods
N^d	desired stock of inventories
S	total real saving: $S = S_f + S_g + S_h$
S_f	real saving of firms (unintended inventory changes)
S_g	real government saving
S_p	real saving of private households
T	total real tax collections

T^c	real taxes of asset holders
W	real wealth of private households
Y	real output
Y^d	real aggregate demand
Y^e	expected real aggregate demand
Y^n	output at normal use of capacity: $Y^n = y^n K$
a_y	abbreviates a sum of coefficients in chapter 6, section 3, subsection 3: $a_y = c_p + \gamma + s_c\delta - (1 - s_c)\theta_c$
c_p	consumption coefficient of agents without income from economic activities; see chapter 4, section 2, subsection 1, eq. (4.4)
e	employment rate (w.r.t. hours): $e = L/L^s$
f_x	functional relationship representing the determination of variable x, $\dot{x}$ or $\hat{x}$
f_{xy}	partial derivative of function f_x with respect to variable y
g^o	steady-state growth rate of real variables
g_ℓ	growth rate of labour supply: $g_\ell = \hat{L}^s$ (a constant)
g_m	growth rate of money supply: $g_m = \hat{M}$ (a constant)
g_z	growth rate of trend labour productivity: $g_z = \hat{z}^o$ (a constant)
i	nominal rate of interest on government bonds; federal funds rate in chapters 8 and 9
k^s	fixed capital per (efficiency units of) labour supply: $k^s = K/z^o L^s$
ℓ	labour intensity (in efficiency units): $\ell = z^o L/K = 1/k^s$
m	real balances relative to the capital stock: $m = M/pK$
n	inventory–capital ratio: $n = N/K$
p	price level
p_e	price of equities
q	return differential: $q = r - (i - \pi)$
r	rate of return on fixed capital, specified as $r = (pY - wL - \delta pK)/pK$
s_c	propensity to save out of capital income on the part of asset owners
s_h	households' propensity to save out of total income (in chapters 2 and 3)
u	rate of capacity utilization: $u = Y/Y^n = y/y^n$
v	wage share (in gross product): $v = wL/pY$
w	nominal wage rate per hour
x_m	auxiliary variable in chapter 2: $x_m = z^o M/wK$
y	output–capital ratio: $y = Y/K$; except in chapter 1, section 3, where y denotes the output gap
y^d	ratio of aggregate demand to capital stock: $y^d = Y^d/K$
y^e	ratio of expected demand to capital stock: $y^e = Y^e/K$

y^n	normal output–capital ratio (a constant; no recourse to a neoclassical production function)
z	labour productivity – i.e. output per working hour: $z = Y/L$
z^o	trend value, or 'normal' level, of labour productivity
α	marginal product of capital in chapter 2: $\alpha = \alpha(y) = F_K(K, z^o L)$; symbol for policy parameters in Taylor rule in chapters 8 and 9
α_i	coefficient measuring interest rate smoothing in the Taylor rule
α_p	coefficient on inflation gap in the Taylor rule
α_u	coefficient on output gap in the Taylor rule
β_x	generically, reaction coefficient in an equation determining x, $\dot{x}$ or $\hat{x}$
β_y	adjustment speed in adaptive sales expectations
β_π	general adjustment speed in revisions of the inflation climate
β_{xy}	generically, reaction coefficient related to the determination of variable x, $\dot{x}$ or $\hat{x}$ with respect to changes in the exogenous variable y
β_{Iq}	responsiveness of investment (capital growth rate) to changes in q
β_{Iu}	responsiveness of investment to changes in u
β_{nn}	stock adjustment speed
β_{ny}	desired ratio of inventories over expected sales
β_{pu}	reaction coefficient of u in price Phillips curve
β_{pv}	reaction coefficient of $(1+\mu)v - 1$ in price Phillips curve
β_{we}	reaction coefficient of e in wage Phillips curve
β_{wv}	reaction coefficient of $(v - v^o)/v^o$ in wage Phillips curve
β_{zu}	responsiveness of (procyclical) labour productivity to changes in u
γ	government expenditures per unit of fixed capital: $\gamma = G/K$ (a constant, except for chapter 7, section 6, subsection 3)
δ	rate of depreciation of fixed capital (a constant)
$\eta_{m,i}$	interest elasticity of money demand (expressed as a positive number)
κ	coefficient in reduced-form wage-price equations: $\kappa = 1/(1 - \kappa_p \kappa_w)$
κ_p	parameter weighting $\hat{w}$ vs. π in price Phillips curve
κ_w	parameter weighting $\hat{p}$ vs. π in wage Phillips curve
κ_{wp}	same as κ_w, in chapter 5
κ_{wz}	parameter weighting $\hat{z}$ vs. $\hat{z}^o$ in wage Phillips curve (only chapter 5)

κ_π	parameter weighting adaptive expectations vs. regressive expectations in revisions of the inflation climate
μ	actual markup rate in chapter 5; same as μ^o otherwise
μ^o	target markup rate over unit labour costs
ξ	relative excess demand: $\xi = (Y^d - Y)/Y$
π	general inflation climate; except in chapter 1, section 3, where π denotes inflation
θ	same as θ_c (in chapters 2 and 3)
θ_c	tax parameter for T^c (net of interest): $T^c - iB/p = \theta_c K$
τ_w	tax rate on wages
ϕ	flexibility term in the nonlinear investment function in chapters 6 and 8: $\phi = \phi(u, q)$
ω	real wage rate, deflated by trend productivity: $\omega = (w/p)/z^o$

1 Competing approaches to Keynesian macrodynamics

1.1 Introduction

1.1.1 *General methodological remarks*

This book proposes a view of dynamic macroeconomic modelling that stresses the non-market-clearing approach. Here the focus is very much on dynamic adjustment processes amongst the principal markets and agents of the macroeconomy and the dynamic linkages between these. Our starting point is the *Keynes–Metzler–Goodwin* (KMG) model developed in earlier work of the authors together with other collaborators. The label is meant to highlight the key macroeconomic mechanisms introduced by the great economists referred to. The 'Keynes' refers to the causal nexus from financial to real markets, 'Metzler' to inventory dynamics and 'Goodwin' to the dynamics of distributive shares. It is our view that these are the core mechanisms which need to be at the heart of descriptive models of the macroeconomy.

An important aim of our analysis is to understand the dynamic interplay between these core driving mechanisms of the macroeconomy, in particular which are stabilizing and which destabilizing, and which parameters have the most influence in moving the economy back and forth between the regions of stability and instability. In the shock-driven models of modern macrodynamics a stabilizing effect is one that reduces the variance of some important state variables; however, here we are almost exclusively concerned with deterministic systems, and so the terms 'stability' and 'instability' are used in the sense that they refer to the local properties of the steady state.

For the KMG model that we work with, it can be mathematically proved that parameter variations that bring about instability are associated with a Hopf bifurcation. We will not be so concerned with regard to the details of this phenomenon but, rather, take it mainly as an indication that over a wider range of parameter values the dynamics are basically of a cyclical nature. Here, we are specifically interested in oscillations that occur at business cycle frequencies. For these investigations, however, we will need to resort to a numerical analysis.

Of course, oscillations in linear deterministic models will die out if the equilibrium is stable. This equally holds true if, as in the most elementary specifications of our building blocks, the model is (not linear but) 'quasi-linear'. On the other hand, the intrinsic nonlinearities (such as a multiplication of two variables) are also not sufficient to bound the explosive motion if the steady state is locally unstable. Hence, in order to generate persistent and bounded cyclical behaviour, we employ parameter combinations that imply instability and then introduce an extrinsic nonlinearity that takes effect in the outer regions of the state space, so that locally the system is spiralling outward and further away from the steady state it is spiralling inward. Since the KMG model, despite the various feedbacks from wage-price and inventory dynamics, is essentially still an investment-driven model, we will in this book focus concretely on a suitable nonlinearity in the investment function.

The present book adds two features to earlier work of the authors on the KMG model. First, it undertakes a very careful calibration of the model to the stylized business cycle facts of US data. The dynamic properties of the resulting calibrated model are studied in detail, especially stability regions in the space of key parameters. Second, in the final two chapters we take the LM block of versions of the model hitherto developed and replace it with a Taylor-type interest rate rule. This type of rule has, of course, become a – if not the – major policy tool of central banks worldwide, so in the interests of realism any model of the modern business cycle needs to incorporate it. To the resulting model we give the label *Keynes–Metzler–Goodwin–Taylor* (KMGT). The model could be taken by economists and policymakers inclined to the non-market-clearing approach and used as the basis of policy experiments and further empirical studies.

With its stress on the underlying macroeconomic forces of the economy and their interaction, the authors have characterized their approach in previous works as *macrofounded*. The authors still contend that this is the major advantage of the approach to business cycle modelling that they are advocating in this and other work. The approach thus stands in contrast to other currently more fashionable approaches, in particular real business cycle theory and the New-Keynesian approach. The common element of these two frameworks is the insistence on deriving all dynamic equations from microfoundations. In a pure form, this involves a representative agent solving an intertemporal expected utility-maximizing problem. The corresponding Euler equation, the market-clearing assumptions and the hypothesis of rational expectations yield

the dynamic structure of these models. Models of this kind still amount to a Robinson Crusoe economy, progress perhaps being that Friday has joined as his companion.[1] In a less pure form, these models are enriched by modifying the Euler equation or combining it with elements that intuitively or plausibly are meant to capture additional features such as, for example, other sectors in the economy or so-called backward-looking, boundedly rational agents. These models are microfounded in spirit, but no longer in all explicit details.[2]

Whilst it is, of course, good to obtain microfoundations for the postulated behavioural relationships, this approach carries with it certain disadvantages, in our view. Most importantly, the nature of the solution procedures for stochastic intertemporal optimization models makes it very difficult, if not impossible, to understand clearly the dynamic linkages and feedbacks between the various sectors and agents of the economy. It may in this respect be worth referring to the points made by Romer (2000) about the relevance of the IS-LM-AS model for analyzing short-run fluctuations, a model that in our terms could be viewed as a macrofounded model (though we emphasize that Romer himself does not employ that term). Romer sees two important advantages. First, prices do not adjust instantaneously to disturbances, and this seems to be a necessary feature of any model purporting to describe economic reality. Second, the microfounded approach does not at the end of the day lead to models that are more realistic than those based on intuitive or so-called 'ad hoc' arguments. As Romer (2000, pp. 7f.) summarizes it,'The tradeoff [when moving from the ad hoc assumption in IS-LM-AS to a relatively simple formulation based on intertemporal optimization] is similar for grounding the analysis of investment demand, money demand, price rigidity, and soon more strongly in microeconomic foundations: even the easiest models are dramatically harder than their IS-LM-AS counterparts, and not obviously more realistic.'

One might also go one step further and scratch at the halo of the expression 'microfoundations' as it has been used in the last three

[1] For example, Friday may be a rule-of-thumb consumer, as in the New-Keynesian models by Amato and Laubach (2003) or Gal et al. (2004).

[2] As a consequence, the conventional jump-variable techniques of this literature are less obvious in these models than in a purely optimizing framework. We recall that, in the early stages of the development of the jump-variable techniques for solving rational expectations models, some concerns were expressed about the lack of any theory to explain the jump in economic variables as well as about the arbitrariness in the selection of jump variables in larger-scale models. Some of these issues were articulated by Burmeister (1980). A nice quotation is also the following side remark by Blanchard (1981, p. 135) in his application of the jump-variable technique to the value of the stock market: 'Following a standard if not entirely convincing practice, I shall assume that q always adjusts so as to leave the economy on the stable path to equilibrium.'

decades against 'ad hoc' model building. We feel, in fact, sympathetic to Solow in his summary of the contemporarily predominant methodological approach: 'One could even question whether a representative agent model qualifies as microfoundation at all' (Solow, 2004, p. 660).[3]

A more specific point where we certainly depart from current fashions is in the handling of expectations. For almost three decades the rational expectations assumption has been accepted almost as an article of faith in some quarters. Interestingly, its hold on the economics profession has loosened over the last decade, with many papers on boundedly rational and heterogeneous agents appearing in a range of journals and books. Nevertheless, the grip of the rational expectations assumption is still almost vice-like in the reigning business cycle paradigms. However, we remain to be convinced that it is useful to build models of the economy where agents have the information and computational ability to form rational expectations or behave 'as if' they had such abilities. We believe that such an assumption is so far from reality that it does not serve even as some sort of baseline around which the economy moves. Rather, the formation of expectations under conditions of incomplete information, bounded rationality and limited computational ability is part of economic reality.

Apart from this negative judgement, four points should be mentioned with regard to the treatment of expectations in this book. First, we join the common – in fact, almost exclusive – practice in macrodynamic modelling of concentrating on the rate of inflation as the one and only variable about which expectations are formed.[4] Second, we will avoid the expression 'expected rate of inflation'. We, rather, introduce a variable π that in an uncertain environment the agents conceive as some average over a longer time in the future; it is not just the rate expected for the next period. Therefore, we prefer to use the term 'inflation climate' for π.

From this point of view it becomes, third, reasonable to consider the changes in π as revisions of a currently held opinion, which are made in a gradual manner in light of the most recent information about inflation.

[3] It would by no means inappropriate if we filled the next pages by quoting all the methodological remarks from this paper, which is an obituary of James Tobin where Solow reminds us of his seminal paper 'A general equilibrium approach to monetary theory' from thirty-five years ago. On this occasion we may say that we see ourselves in the tradition of Tobin's approach, about which Solow, to provoke contradiction we suppose, fears 'that it may soon be extinct, like some obscure Melanesian language whose native speakers are dying off (Solow, 2004, p. 659)

[4] Though it is hardly ever mentioned as a problem, we consider this a most serious shortcoming. Keynes' famous 'animal spirits' that are guiding entrepreneurs certainly refer to other, or at least additional, economic variables. Thus, in future work, we intend to take up the notion of a 'state of confidence' or a general 'business climate' as the expectational variable that should be centre stage in macrodynamic modelling. A first attempt in this direction was Franke and Asada (1994).

Formally (but only formally, we stress), this mechanism can be described as adaptive expectations. Though this adjustment principle has a bad reputation in some quarters, there is indeed widespread evidence from economics and the behavioural sciences that it is by no means that foolish and that it is indeed widely used by real economic agents (Flaschel et al., 1997, pp. 149–62 or, more extensively, Franke, 1999, give a compilation of such arguments). For the purposes of the present discussion, the following short citation from Mankiw (2001, p. C59) is illuminating enough. After noting how odd it is to assert that expectations about inflation are formed without incorporating all the news events that are so readily available in the modern world, he adds, 'Yet the assumption of adaptive expectations is, in essence, what the data are crying out for.'[5]

The fourth point is that we combine the 'adaptive expectations' with another relevant mechanism. While the former could also be characterized as chasing a trend, we additionally draw on a general idea from the asset markets, a fundamentalist view, so to speak, according to which the variable is expected to return to its normal level after some time. The adjustment mechanism that we will propose for our inflation climate π will thus be a weighted average of 'adaptive expectations' and these, as we call them, regressive expectations.

Returning to our interest in business cycle dynamics, we may also point out that the microfounded models are limited in the type of cyclical behaviour they can generate. The solution procedures usually involve a (log-)linearization of the Euler equations, otherwise it may be difficult to apply the solution methodology required to operationalize the rational expectations assumption.

Since linear dynamic models can make economic sense only in their regions of stability, exogenous stochastic processes are needed to generate persistent cycles. Attempts to calibrate these types of models often come down to tuning various types of exogenous stochastic processes. This problem is similar in kind to that of introducing suitable nonlinear mechanisms into our deterministic models to bound the explosive

[5] In our view, agents in the real world are not 'forward-looking', which is just another expression for rational expectations. They are 'backward-looking', to take up this currently fashionable term, in that they have only data from the past on the basis of which they can form expectations about the future. On the other hand, agents are sufficiently sophisticated to make use of econometric methods. While, being univariate, the adaptive expectations method is a particularly simple one, it would be more appropriate to assume that the agents adopt vector autoregressions to forecast future inflation. Then, in order to reduce at least the computational effort, one might try to short-circuit this general device by some simplified adjustment formulae where, however, reference is made not only to current inflation but also to some measure of the output gap, and perhaps the interest rate too.

motion. If this device may be viewed as the ad hoc feature of the macro-founded approach, then it may equally be argued that the open choice of exogenous stochastic processes may be seen as the ad hoc feature of the microfounded approach.

A final argument that we may give for developing further the macro-founded approach that we are advocating is that it still seems to be at the heart of the explicit or implicit modelling framework used by many policymakers. This is, no doubt, due to the fact that the microfounded approaches leave obscure the linkages between the different sectors and agents of the economy. But it is precisely these linkages that are of importance to policymakers.

1.1.2 A historical perspective

After elaborating on the many aspects of his new and, as he emphasized (Keynes, 1936, p. 3), *general* theory about the most fundamental macroeconomic relationships, Keynes (p. 313) purports in chapter 22 of *The General Theory* that this work should also be useful for a better understanding of the fluctuations that are summarized as business cycles, or, in his words, the trade cycle. The definite article 'the' already indicates that it is viewed as a systematic phenomenon (pp. 313f.):

> By a *cyclical* movement we mean that as the system progresses in, e.g. the upward direction, the forces propelling it upwards at first gather force and have a cumulative effect on one another but gradually lose their strength until at a certain point they tend to be replaced by forces operating in the opposite direction; which in turn gather force for a time and accentuate one another, until they too, having reached their maximum development, wane and give place to their opposite. We do not, however, merely mean by a *cyclical* movement that upward and downward tendencies, once started, do not persist for ever in the same direction but are ultimately reversed. We mean also that there is some recognisable degree of regularity in the time-sequence and duration of the upward and downward movements.

Hence, there must be deeper causes for this kind of cyclical behaviour. The most important cause Keynes identifies is investment and its key determinant, the marginal efficiency of capital (p. 313). The other two pillars of his theory are the marginal propensity to consume and the state of liquidity preference. Once these 'three main gaps in our existing knowledge' are filled, the complementary 'theory of prices [and wages] falls into its proper place as a matter which is subsidiary to our general theory' (pp. 31f.).

This approach to a theory of the trade cycle has not received full attention in the discussions that developed after the appearance of *The General Theory*, which in the main is probably due to the strong psychological factors that are penetrating the dynamic feedback mechanisms. So the concepts just mentioned provided only a loose theoretical frame for the more formal versions of Keynesian theory. In its striving for a rigorous design, modern macrodynamic modelling started out from more precise, and more limited, behavioural assumptions. This holds point in the 1950s and 1960s, as well as for the progress that the contemporary New-Keynesians claim to have made. In the remainder of this chapter we give a brief overview of these approaches from our point of view, and then locate our own approach with respect to these traditions. Since, in particular, price and wage formation are here not just a 'subsidiary' component, we emphasize the different assumptions and specifications concerning perfectly flexible or more sluggish prices and wages. It should also be remarked that this discussion – not only because of its brevity – loses sight of the systematic cyclical movements that Keynes had in mind. We will, however, return to this topic in the analysis of our own models later in the book.

We start, therefore, in the next section with a reconsideration of the old Neoclassical Synthesis, which we date as Stage I. Based initially on Patinkin's micro-oriented approach to macrodynamics and then further refined, this blend of Keynes and the Classics considered the original debate from the perspective of a larger modelling framework where all building blocks of the Keynesian approach are present, together with Classical and later Friedmanian supply-side arguments (marginal cost determination of the price level and an expectations-augmented money wage Phillips curve). A rigorous and almost canonical formulation was given to it by Sargent's advanced textbook (1979, chaps. 1–5). At the one end of the synthesis, the Classical version of the working of the macroeconomy was obtained by assuming enough flexibility in the real markets, in the first instance fully flexible wages and prices, while at the other end the Keynesian version emerged when real markets became less perfect and at least money wages were assumed to adjust in a delayed manner.

In section 1.3 we subsequently consider the basic components of the New-Keynesian approach, which we perceive as the Neoclassical Synthesis, State II. In section 1.4, still in a highly stylized fashion, the main ingredients of our own modelling framework are discussed. Here, the preceding sections 1.2 and 1.3 prove to be useful in two respects. First, the best perspective from which to understand and evaluate our work is to view it as introducing disequilibrium elements into the AS-AD setting of the Neoclassical Synthesis, Stage I, in order to remove certain central theoretical weaknesses. We will thus present our approach as a matured Keynesian macroeconomic model of disequilibrium dynamics. If it were not so risky in the overall competition for catchy and marketable labels, we might even be tempted to call it defiantly an Old-Keynesian approach.[6]

Second, the discussion of the Neoclassical Synthesis, Stage II, is useful since, interestingly, the reduced and sketchy way in which we try to characterize it allows us to recognize a close correspondence between our and the New-Keynesian modelling of, in particular, the wage-price and output dynamics. When stripped down to the bones, at first sight only the period-dating of these variables in the postulated relationships seems to be different. It will, however, also be worked out that this leads to radically different conclusions regarding the working of the economy.

1.2 Neoclassical Synthesis, Stage I: traditional AS-AD dynamics

We reconsider in this section what constituted the core of Keynesian macroeconomic theory until the beginning of the 1970s. This was, of course, the Neoclassical Synthesis, and we have already announced that, thirty years later and with a view to our discussion further below, we will also occasionally refer to it more precisely as the Neoclassical Synthesis, Stage I (NCS I). This body of theory organizes the description of a closed economy into three major building blocks: the IS and LM relationships for the goods and money market, which in combination yield the so-called AD curve; an AS curve derived from the marginal productivity principle for labour; and demand facing supply on the labour market. In its basic equilibrium formulation, prices (p) as well as nominal wages (w) are perfectly flexible, so that the economy is on its steady-state growth path.[7] For easier reference, let us denote this approach as NCS I(p, w). More recently it has found expression in the New-Classical economics and the equilibrium business cycle theory.

The agents' out-of-equilibrium behaviour has always been discussed verbally and also often formalized in small models, which, however, have mostly concentrated on selected issues. A first and most influential attempt to introduce disequilibrium adjustments into a complete macroeconomic model of NCS I was undertaken by Sargent (1979, chap. 5). We therefore find it appropriate to begin our review of Keynesian macrodynamics at this point.

[6] Inspired by the title of Tobin's (1992) article on the sense and meaning of less than perfect price flexibility.

[7] For a detailed presentation, see, e.g., Sargent (1979, chap. 2).

For a better comparison with the New-Keynesian models later on and their emphasis on monetary policy, it should be mentioned at this stage that all versions of NCS I that we are going to consider assume a neutral policy of Friedmanian type – that is, money supply is exogenous and grows at a constant rate.

1.2.1 *Keynesian AS-AD dynamics with rational expectations*

Sargent's (1979, chap. 5) economy comprises three sectors: households, firms and the government. The behavioural assumptions he employs are a good compromise between richness, where in some parts partial microfoundations are also provided, and parsimony, where stylized assumptions serve to keep the model analytically tractable. In particular, Sargent takes account of the budget equations for savings and asset accumulation; flows and stocks are thus explicitly related in a consistent manner.

The model departs from NCS I in only one respect: the assumption of perfectly flexible money wages w is abandoned and replaced with gradual adjustments. They are represented by an ordinary expectations-augmented wage Phillips curve, which is formulated in continuous time. Denoting inflationary expectations by π^e, measuring the demand pressure on the labour market by the deviations of the actual rate of employment $e = L/L^s$ from its exogenously given NAIRU level e^o (L is labour demand and L^s the labour supply), and specifying the speed of adjustment by a positive coefficient β_{we}, the wage Phillips curve reads

$$\hat{w} = \pi^e + \beta_{we}(e - e^o) \tag{1.1}$$

($\hat{w} = \dot{w}/w$ is the growth rate of w). Regarding expectations, π^e in (1.1) is viewed as capturing the price changes in the near future, even over the next short period, so to speak. If sluggish wages are to be the only departure from the equilibrium formulation of NCS, myopic perfect foresight has to be assumed in this respect. In the continuous-time setting we therefore have, for p the price level and $\hat{p}$ the current rate of inflation,

$$\pi^e = \hat{p} \tag{1.2}$$

To be precise, $\hat{p}$ has to be thought of as the right-hand time derivative; cf. Sargent (1987, p. 120).[8] Prices themselves, the perfect flexibility of

[8] In a further departure from NCS I(p, w), Sargent (1987, chap. 5.1) assumes gradual adjustments for expected inflation π^e, too. As will be worked out in chapter 2, section 4, this model has still some peculiar features, which can be seen as a weak reflection of the peculiar features that will arise in the presence of (1.2).

which is maintained, are supposed to be determined within a standard AS schedule based on marginal wage costs. Accordingly,

$$p = w/F_L(K, L) \tag{1.3}$$

where K is the capital stock, $F = F(K, L)$, the neoclassical production function (without technical progress), and $F_L = \partial F/\partial L$, the marginal product of labour.

The most important feature of the IS part of the model, which goes slightly beyond a principles textbook, is that (net) investment is no longer a function of the interest rate alone. Sargent instead conceives it as an increasing function of a return differential q, which is the difference between the real rate of return, r, of firms on their capital stock and the real rate of interest $i - \pi^e$ (i being the nominal interest rate).[9] With the neoclassical production function, r is given by the marginal product of capital $F_K = \partial F/\partial K$ minus the rate of depreciation of the capital stock. For the other other components of aggregate demand it is convenient to assume suitable fixed proportions to the capital stock as trend term (as they are detailed in chapter 2, section 2, of this book, for example). This leads to a simple multiplier relationship for output Y of the kind $Y = (1/s)(I + K)$, where I is investment and s the constant propensity to save of private households. Together, the model's IS block in intensive form is described by

$$y = (1/s)(I/K + \text{const.}) \tag{1.4}$$

$$I/K = f_I(q), \quad f_I' > 0 \tag{1.5}$$

$$q = r - (i - \pi^e) \tag{1.6}$$

$$r = F_K(K, L) - \delta \tag{1.7}$$

On the other hand, the LM equilibrium condition for the exogenous money supply M in a growing economy can be posed as

$$M = pY\, f_m(i), \quad f_m' < 0 \tag{1.8}$$

As far as the evolution of money, capital and the labour supply is concerned, it considerably eases the exposition if we here neglect the capacity effects of investment and assume that the capital stock K grows at the

[9] Sargent (1987, pp. 11–14) demonstrates that this expression is indeed close to Tobin's (average) q.

same constant rate g^o as the labour supply L^s, to which in turn the money supply adjusts:[10]

$$\hat{M} = \hat{K} = \hat{L}^s = g^o \tag{1.9}$$

Of course, long-run equilibrium inflation must be zero then, $\hat{p}^o = (\pi^e)^o = 0$, and g^o is the real as well as nominal steady-state growth rate (the steady-state values are generally indicated by a superscript 'o'). In sum, eqs. (1.1) to (1.9) constitute a model of the Neoclassical Synthesis, Stage 1, with perfectly flexible prices (p) and gradual wage adjustments ($\hat{w}$), though in a somewhat simplified form. For short, it may be referred to as NCS I($p, \hat{w}$).

The analysis of system (1.1) to (1.9) decomposes into two major steps. In the first step, we reveal the far-reaching implications of the marginal productivity principle for labour in eq.(1.3). Define, to this end, the labour–capital ratio $\ell = L/K$ and write the production function in intensive form, $f_y(\ell) := F(1, \ell) = F(1, L/K) = Y/K = y$, where $f_y'(\ell) = \partial F(1, L/K)/\partial(L/K) = \partial(KF(1, L/K))/\partial L = \partial F(K, L)/\partial L = F_L(L, K) > 0$ and $f_y''(\ell) < 0$. Inverting the function f_y gives the relationship $\ell = \ell(y) := f_y^{-1}(y)$, with the derivative $\ell'(y) = 1/f_y'[\ell(y)] > 0$.

Next, let ℓ^o be the labour intensity associated with full employment and notice that $\ell^s := L^s/K =$ const. by (1.9), and $\ell^s = \ell^o$ more specifically. This allows us to write the employment rate as $e = L/L^s = \ell/\ell^s = \ell(y)/\ell^o$.

From the manipulations above it is also seen that eq.(1.3) itself can be written as $p = w/f_y'[\ell(y)]$ in intensive form. Solving the equation for the real wage rate $\omega := w/p$, we have $\omega = f_y'[\ell(y)] = f_y'(e\ell^o)$, which in turn can be inverted as $(f_y')^{-1}(\omega) = e\ell^o$. The employment rate can thus be conceived as a function of the real wage, $e = e(\omega)$. The derivative of $(f_y')^{-1}$ being given by $d(f_y')^{-1}/d\omega = 1/f_y'' < 0$, we know that the employment rate is inversely related to the real wage, $e'(\omega) < 0$.

Now substitute this function and the hypothesis of myopic perfect foresight (1.2) into the wage Phillips curve (1.1). With $\hat{\omega} = \hat{w} - \hat{p} = \hat{w} - \pi^e$, we then get the upshot of the first step of the analysis, a scalar differential equation in the real wage rate ω alone,

$$\dot{\omega} = \omega\beta_{we}\,[e(\omega) - e^o], \quad \text{where } e'(\omega) < 0 \tag{1.10}$$

Equation (1.10) has a unique stationary point ω^o that brings about full (or normal) employment, $e(\omega^o) = e^o$, which is obviously globally asymptotically stable. Since we can recover from ω the employment

[10] Sargent (1987, chap. 5.2) allows for, in our notation, $\hat{K} = I/K = f_I(q)$. This makes the analysis more technical, while the main conclusions are preserved.

rate e, labour intensity ℓ and the output–capital ratio y, the stability property applies to the whole real sector of the economy. Note in addition that this holds regardless of what may happen to the nominal variables, and that the IS relationships (1.4) to (1.7) have played no role so far. Hence three conclusions emerge: (1) the model dichotomizes into a real and a nominal part; (2) the real sector is completely determined by the supply-side of the economy; and (3), as will be generally expected in mainstream approaches to economic theory, the real sector is a global shock absorber.

The second step of the analysis must be concerned with the nominal magnitudes, goods prices p and money wages w. To simplify the presentation, suppose the dynamics has already settled down on its steady-state values. Since Y, by (1.9), will then constantly grow at the same rate as M, the ratio M/Y stays put and the only remaining variable on which the LM rate of interest in (1.8) depends is the price level p. Of course, the interest rate increases with p, so that we have

$$i = i_{LM}(p) \quad \text{where } i'_{LM} > 0 \qquad (1.11)$$

Observe, furthermore, that the return rate r in (1.7) is dependent on real variables only. The steady-state assumption for the real sector makes it therefore a constant, $r = r^o$. Since $f_I(q) = I/K = \hat{K} = g^o$ in this equilibrium, the return differential q must also attain a fixed value q^o.[11] It follows that $q = r^o - i_{LM}(p) + \pi^e = q^o$.

Finally, we return to the myopic perfect assumption in (1.2), $\hat{p} = \pi^e$, and substitute it in the previous equation. Solving for $\hat{p} = \dot{p}/p$ and multiplying the result by p, the price level is seen to be governed by the differential equation

$$\dot{p} = p[q^o - r^o + i_{LM}(p)], \quad \text{where } i'_{LM} > 0 \qquad (1.12)$$

This equation, too, has a unique equilibrium value p^o. Unlike eq. (1.10), however, it is unstable; since the right-hand side of (1.12) is strictly increasing in p, the price level dynamics are explosive. Such an unstable economy would, of course, not be meaningful.

Recognizing this, two alternative conclusions can be drawn. Either the model is not well enough designed and some building blocks should be respecified, or the description of the model is not yet complete. The usual reception of similar models, or models with similar properties,

[11] q^o may well be positive, which could be interpreted as a risk premium of fixed investment, which yields the return r, *vs.* purchasing government bonds, which yields $i - \pi^e$ as its real rate of return.

leans towards the second conclusion, understanding it as calling not for another behavioural relationship to be built into the model but for an additional assumption to be invoked. This is the assumption of rational expectations, which in a somewhat related framework has been introduced by Sargent and Wallace (1973). For our present purpose, the *rational expectations hypothesis* may be summarized as follows.

> Looking into the future, the agents in a dynamic economic system behave in such a way that, among the a priori possible solutions of the formal equations, all eventually unbounded trajectories fail to realize.

The postulate takes effect in the initial conditions of the dynamics, where it implies that not all variables can be treated as predetermined at starting time $t = 0$, say. Some of the variables are instead permitted to take on special values that ensure the boundedness of all state variables if the system starts from there. These variables are commonly said to 'jump' onto a suitable set (a stable manifold often) that contains this value. If these variables and their jumps make economic sense, then the model is still considered to be well defined. For example, a capital stock cannot jump within or at the beginning of a short period, but an expectational variable may.

The procedure itself, and thus implicitly the theoretical assumption from which it derives, is also raised to the status of a 'technique'. Accordingly, we now have to subject our system to the 'jump variable technique'.

Regarding the unstable differential equation (1.12), where the real sector was supposed to be already in its equilibrium position, the jumping procedure is trivial. There is only one dynamic variable left, the price level p, and the only possibility to keep it within bounds is to let it directly jump to its equilibrium values p^o, from which the system has come to rest.

Things are more complicated if the real sector is out of equilibrium and adjustments are still taking place there. We can here refer to Sargent (1987, p. 122), who, assuming a suitable log-linearity in the LM equation (1.8), is able to compute the time path of p in explicit terms. He arrives at an equation that expresses the current price level as a function of the entire *future* paths of the money supply and other variables of the real sector, where, as he adds in parentheses (p. 122), 'we are again imposing a terminal condition' that formally suppresses a monotonically diverging term like const.$\cdot e^{\alpha t}$ ($\alpha =$ const. > 0). This corresponds to the situation of an n-dimensional linear differential equations system that has one unstable and $n-1$ stable eigenvalues λ_i (with Re $\lambda_i > 0$ and Re $\lambda_j < 0$, respectively); and, given the $n-1$ initial values of the predetermined variables, the price level, as prescribed by Sargent's formula, jumps onto the system's $(n-1)$-dimensional stable manifold. Note that, since the real wage ω is a predetermined variable, the money wage rate w is a jump variable too (though it need not show up in the mathematical analysis).

It is worth pointing out that in this type of model, NCS I$(w, \hat{p})$, money is neutral not only in the long run but also in the short run, the latter in the sense that an unanticipated permanent shock to the money supply leaves the real sector unaffected if the price level jumps immediately by the same percentage as the change in M. Besides, it is remarkable that the inflation thus occurring in the economy goes unnoticed by the private agents, since they are exclusively concerned with $\pi^e = \hat{p}_+$, the right-hand derivative $(p_{t+h} - p_t)/hp_t$ of the price level with $h \to 0$, which in our simplified setting can always remain at zero.

Somewhat polemically, this type of jump experiment might also be considered to be a static equilibrium argument in dynamic disguise. In any case, the behaviour of jump variables is more appealing if the sudden change in the money supply is anticipated. Say, in a stationary economy ($g^o = 0$ for simplicity), the central bank announces at time $t = 0$ that at time $T > 0$ it will raise the money supply from M_o to M_1, which increases the equilibrium price level from p_o to p_1 (omitting superscript 'o' for a moment). The point is that, under these circumstances, the price level does not jump, at $t = T$, from p_o to p_1.

In general terms, this would enable some agents to make capital gains, a possibility that would be realized by other agents, who, in turn, would exploit this situation themselves – and so on. Consistency then requires the price level to jump at an earlier time, already at $t = 0$ when the change in M was announced, and by a lesser amount; from then on the price level steadily increases until it reaches the new equilibrium value at precisely $t = T$.[12] Hence, the economy experiences inflation before anything has happened to the money supply, and no more inflation at all after the change has taken place. This attractive feature for a theory that invokes the Rational Expectations Hypothesis (and additional arbitrage arguments) is illustrated in figure 1.1.

Nevertheless, the kind of jumps occurring in NCS I$(p, \hat{w})$ look quite strange from a Keynesian perspective. Recall that money wage adjustments are governed by the wage Phillips curve (1.1), and that this relationship was introduced to take account of an empirical regularity. The latter observation means that, unlike in the New-Keynesian versions,

[12] See Turnovsky (1995, p. 73) for a formal reasoning.

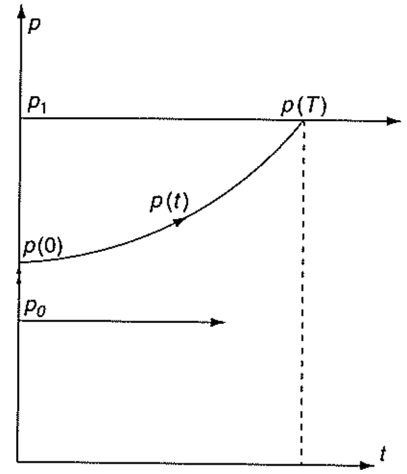

Figure 1.1: An anticipated monetary policy shock in NCS I$(p, \hat{w})$

this type of Phillips curve is still, in modern terms, backward-looking. In discrete time with an adjustment period of length h it would read

$$(w_{t+h} - w_t)/w_t = h[\pi^e_t + \beta_{we}(e_t - e^o)]$$

where all variables dated t are given. This makes clear that, at the time when NCS I$(p, \hat{w})$ was designed, the money wage rate w was, or should have been, thought of as a predetermined variable.

On the other hand, in the experiment described above the money wage rate had to jump, because the price level had to jump while the level of the real wage rate had to be preserved. It follows that the discontinuity in w has its reason outside the wage Phillips curve; in an ad hoc manner, we might say, the Phillips curve was cancelled in one point in time. It is in this sense that the model and the jump variable technique applied to it exhibit an inconsistency.

A deeper conceptual explanation of this failure of NCS I$(p, \hat{w})$ can be sought in the feature that it, within a framework that admits some sluggish adjustments in the nominal variables, attempts to integrate the assumption of demand-constrained firms with the assumption that firms are price takers. Essentially, this type of model falls back on the Neoclassical Synthesis that is operated under the assumption that prices and wages are both perfectly flexible; that is, we are essentially back in NCS I(p, w).[13]

[13] Further details of the anomalies that rational expectations may give rise to in models with IS-LM-AS plus Phillips curve, where the real wage remains a predetermined variable, can be found in Flaschel et al. (1997, chap. 8 & 9).

1.2.2 *Further scenarios of the wage-price dynamics*

Introducing dynamic elements into the equilibrium formulation of the Neoclassical Synthesis, Stage I, as it was done in NCS I($p, \hat{w}$), is, according to our evaluation in the previous subsection, not a very promising step towards modelling Keynesian disequilibrium dynamics. The first unsatisfactory feature was that the economy dichotomizes into real and nominal sectors. Since this is mainly due to the assumption that prices at every point in time are determined by the marginal productivity principle for labour, it is straightforward to ask if a more plausible model results if the assumption is relaxed, while nevertheless maintaining the principle as a benchmark case. In this way we get an alternative version of NCS I where not only wages but also prices are supposed to adjust in a gradual manner. Hence, in this respect, prices and wages are now put on equal footing. Even without a deeper analysis, this seems more reasonable than their methodologically unequal treatment in NCS I($p, \hat{w}$).

The basic idea is that the price level p is predetermined in the short period, and firms raise it in the next period when it is currently below its natural reference value, which is still given by nominal marginal wage costs, or the competitive price p_c,

$$p_c = w/F_L(K, L) \tag{1.13}$$

Conversely, firms tend to reduce prices if p exceeds p_c. In meanwhile obvious notation, we refer to this type of model with gradual adjustments in both p and w as NCS I($\hat{p}, \hat{w}$).

In the precise specification of this idea we cannot maintain the myopic perfect assumption $\pi^e = \hat{p}$ from (1.2) and at the same time use the same expression for reference inflation, namely π^e, as in the wage Phillips curve (1.1). $\hat{p}$ would then show up on the left-hand side as well as on the right-hand side of such a relationship, which would not be meaningful in the present setting. Rather, we already let ourselves be guided by a common device in the New-Keynesian Phillips curve literature. Combining, as it is called, forward-looking and backward-looking elements in discrete time and neglecting the discount factor (which occurs if the price Phillips curve is explicitly derived from microeconomic fundamentals), it reads

$$\hat{p}_t = \phi_p \hat{p}_{t+1} + (1-\phi_p)\hat{p}_{t-1} + \beta_p \cdot \text{demand pressure}$$

where, retaining the symbol, $\hat{p}_t = (p_t - p_{t-1})/p_{t-1}$ and ϕ_p $(0 \le \phi_p \le 1)$ measures the weight of the forward-looking inflation component. Here we still disregard this kind of forward-looking behaviour and set $\phi_p = 0$, so that only the first difference $\hat{p}_t - \hat{p}_{t-1}$ remains. For our purpose we translate it directly into the time derivative of the rate of price inflation. The role of the demand pressure is taken over by the percentage deviation of nominal marginal wage costs from the current price p. With a constant adjustment speed β_p, gradual price (or, rather, inflation) adjustments are therefore described by

$$d\hat{p}/dt = \beta_p(p_c/p - 1) \tag{1.14}$$

In order to give real wages a more direct bearing on investment and thus (negatively) on aggregate demand, we slightly respecify the real rate of return r in (1.7). We drop the marginal product of capital there and define r now as the rate of profit $r = (pY - wL - \delta pK)/pK$. Recalling that labour intensity $\ell = L/K$ was shown to be an increasing function of the output–capital ratio, $\ell = \ell(y)$ with $\ell'(y) > 0$, the profit rate can be written as

$$r = y - \omega\ell(y) - \delta \tag{1.15}$$

Lastly, the dimension of the model can be reduced if it is assumed that the money supply M grows in line not with real capital K as in (1.9) but with price-valued capital pK. The proportions being already those of the steady-state ratio m^o, we have

$$M/pK = m^o \quad (= \text{const.}) \tag{1.16}$$

Our simplified version of NCS I($\hat{p}, \hat{w}$) is thus complete. In sum, it is given by eqs. (1.1), (1.2), (1.4) to (1.6), (1.8), (1.9) and (1.13) to (1.16) (eq. (1.9) now without the first equality, of course).

The stability analysis has to begin with the model's IS-LM part. By (1.16), the LM equation becomes $m^o = yf_m(i)$, from which it is clear that the LM interest rate is an increasing function of the output–capital ratio, $i = i(y)$ with $i'(y) > 0$. Plugging this together with (1.5), (1.6) and (1.15) into the goods market equilibrium equation (1.4) gives $y = (1/s)f_I[y - \omega\ell(y) - \delta - i(y) + \hat{p}]$ as the condition for the output–capital ratio in the IS-LM equilibrium. It is easily seen that the solution y is a function of the real wage rate and the rate of inflation, and that (at least near the long-run equilibrium position) the partial derivatives have the expected sign:[14]

$$y = y(\omega, \hat{p}), \quad y_\omega = \partial y/\partial\omega < 0, \quad y_\pi = \partial y/\partial\hat{p} > 0$$

[14] Note that in the long-run equilibrium, where the marginal productivity principle holds, $\partial(y - \omega\ell(y))/\partial y = 0$; see the remarks leading to eq. (1.10). Hence $f_I[..]$ unambiguously decreases with y.

From the analysis in the previous subsection we know that $p_c = w/F_L = w/f_y'(\ell)$, where $f_y = F(1, L/K)$ with $f_y' > 0$, $f_y'' < 0$. Hence $p_c/p = \omega/f_y'(\ell)$. On the other hand, (1.1) and (1.2) give rise to $\hat{\omega} = \hat{w} - \hat{p} = \pi^e + \beta_{we}(e - e^o) - \hat{p} = \beta_{we}(e - e^o)$, and we also know that the employment rate is an increasing function of the output–capital ratio, $e = e(y)$. NCS I$(\hat{p}, \hat{w})$ can therefore be reduced to a two-dimensional differential equations system. Abbreviating $\ell(\omega, \hat{p}) = \ell[y(\omega, \hat{p})]$ and $e(\omega, \hat{p}) = e[y(\omega, \hat{p})]$,

$$\dot{\omega} = \omega\beta_{we}[e(\omega, \hat{p}) - e^o] \tag{1.17}$$

$$d\hat{p}/dt = \beta_p\{\omega/f_y'[\ell(\omega, \hat{p})] - 1\} \tag{1.18}$$

It is clearly seen that the real wage impacts on the rate of inflation and vice versa. Thus, making both prices as well as wages a dynamic variable has overcome the dichotomy of NCS I$(p, \hat{w})$. In this sense, NCS I$(\hat{p}, \hat{w})$ is a superior variant of Keynesian dynamic modelling.

It is easily established that system (1.17), (1.18) has a unique equilibrium position ω^o, $\hat{p}^o$. Evaluating the Jacobian $\mathcal{J}$ at that point, three entries are unambiguously signed. The sign of the fourth entry j_{21} is given by the sign of the expression $f_y' - \omega f_y''\ell' y_\omega = \omega(1 - y_\omega f_y''/f_y')$. We limit the discussion to the most relevant case of a positive sign. It results if the curvature of the production function is shallow enough, or if the IS-LM output response y_ω is sufficiently weak, which in turn can be caused by sufficiently weak investment reactions, f_I' small. The Jacobian of (1.17), (1.18) then has the following sign pattern:

$$\mathcal{J} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} = \begin{bmatrix} - & + \\ + & + \end{bmatrix}$$

Since $\det\mathcal{J} < 0$, the equilibrium of (1.17), (1.18) is a saddle-point. This indicates that, again, the jump variable technique might be used in order to place the dynamics on the stable manifold. Notice, however, that in contrast to NCS I$(p, \hat{w})$, it would now be applied to the rate of inflation and no longer to the price level itself. Nevertheless, we have already pointed out in the methodological remarks that this is not a convincing design for a Keynesian disequilibrium dynamics.

The local instability result can be understood against the background of the traditional Keynesian feedback mechanisms: the Keynes effect, the Mundell effect and the real wage (or Rose) effect. They are discussed in detail as the book proceeds, especially in chapter 2, section 7, chapter 3, section 8, chapter 7, section 3, subsection 2, and chapter 7, section 4, subsection 2. A graphical exposition of the latter two effects is given in figures 3 and 4 further below. Briefly, we may mention that the

simplifying assumption (1.16) has cancelled the stabilizing Keynes effect (which involves real balances effects from changes in the price level and their impact on the LM interest rate and aggregate demand). So we can summarize that, while the real wage effect is stabilizing (reflected by $j_{11} < 0$ in the Jacobian), it is dominated by the destabilizing Mundell effect (entry $j_{22} > 0$ in $\mathcal{J}$), irrespective of the adjustment speed β_{we} and β_p.

This interpretation also gives a hint as to how the instability could alternatively be dealt with. We could try to build a suitable nonlinearity into the investment function that is able to tame the explosive tendencies in the outer regions of the state space. In this way the dynamics would be confined to a compact region and persistent fluctuations would be obtained.[15]

Another route of research is to find conditions for local stability by changing one or two assumptions in the model, while still preserving the basic structure; in particular, the treatment of wages and prices. To begin with, an immediate approach is to drop the assumption of myopic perfect foresight, $\pi^e = \hat{p}$, and assume a gradual adjustment of π^e towards current inflation (which we will argue at another place can indeed be meaningful). As already mentioned, this was another model of the NCS I$(p, \hat{w})$ variety that was put forward by Sargent (1987, chap. 5.1), which we will carefully reconsider in chapter 2. A typical result in this and many other small-scale models with the same mechanism is that local stability prevails if the adjustments of π^e are sufficiently sluggish. In this book we will, moreover, encounter the result in the model of chapter 3, which – going beyond Sargent – could be classified as an elaborated NCS I$(\hat{p}, \hat{w})$ textbook model.[16] Interestingly, in the more advanced model studied in Part II, the stability result will then have to be somewhat qualified.

To indicate quite another approach to the stability issue that maintains the myopic perfect foresight assumption, the present economy could also be stabilized by a more active monetary policy. In anticipation of the discussions of the Taylor rule, assume the central bank changes the money supply in such a way that the interest rate rises or falls if the rate of inflation rises or falls, respectively. The response may even be supposed to be so strong that the same holds true for the real rate of interest $i - \pi^e = i - \hat{p}$. Accordingly, the LM equation (1.8) is dismissed and the interest rate is represented by a function of the inflation rate, $i = i(\hat{p})$ with $di/d\hat{p} > 1$.

This interest rate policy undermines the Mundell effect, in that an increase in $\hat{p}$ directly diminishes investment $I/K = f_I[r - (i - \hat{p})]$. As a

[15] Although it has to admitted that the present model might be too small to achieve this feature in a reasonable manner.

[16] Examples of the same kind of result from the literature (in a Tobinian vein) are Hadjimichalakis (1971), Benhabib and Miyao (1981) and Hayakawa (1984).

consequence, the IS-LM equilibrium response $\partial y/\partial \hat{p}$ is now negative, and so are the entries j_{12} and j_{22} in the Jacobian $\mathcal{J}$ of the resulting economy, which ensures a positive determinant of $\mathcal{J}$ and a negative trace. The equilibrium is therefore stable.

Apart from the purely Classical situation with perfectly flexible prices and wages, we have so far considered gradual wage adjustments combined with perfectly flexible prices and the case where both wages and prices react gradually to a perceived disequilibrium. For systematic reasons a fourth case remains to be considered, namely the combination of gradual price adjustments with perfectly flexible wages, or NCS I$(\hat{p}, w)$. Besides, this version can be conceived as a prelude to the contemporary New-Keynesian baseline model, which assumes continuous clearing of the labour market and staggered price setting by firms.

In the present framework, with a given normal rate of employment e^o, labour market equilibrium means that the output–capital ratio is already at its steady-state level y^o, which equals $f_y(\ell) = f_y(e^o\ell^o)$ (recall that, generally, $e = \ell/\ell^o$). The IS-LM equilibrium output relationship then degenerates to $y^o = (1/s)f_I[y^o - \omega\ell(y^o) - \delta - i(y^o) + \hat{p}]$. Since the argument of the function f_I must be constant, too, we get $\hat{p} - \omega\ell(y^o) =$ const., implying that the real wage moves in line with the rate of inflation, $\omega = \omega(\hat{p})$ and $d\omega/d\hat{p} > 0$. In this way eq. (1.18), which drives the rate of inflation, becomes

$$d\hat{p}/dt = \beta_p[\omega(\hat{p})/f_y'(e\ell^o) - 1] \qquad (1.19)$$

and this is the only law of motion left in the economy.

Clearly, eq. (1.19) is purely explosive. As in NCS I$(p, \hat{w})$ above, it follows that to obtain stable dynamics one would have to resort to the jump variable technique. However, even if we had no objections to this procedure as a matter of principle, the model is still conceptually questionable for two other reasons. First, it is not only the rate of price inflation that would have to jump but, with it, the level (!) of real wages as well. Second, given that the labour market is supposed to be in equilibrium all the time irrespective of possible shocks to real wages and inflation, what variable is then to be viewed as clearing this market? We conclude from these observations that NCS I$(\hat{p}, w)$ is also an unsatisfactory, if not inconsistent, approach to Keynesian dynamics.

Focusing on the determination of wages and prices in the economy, table 1.1 summarizes the four scenarios of the Neoclassical Synthesis, Stage I, that we have considered. Within our small models stripped down to two dimensions, it has been argued that only the two versions on the diagonal of the table appear to be a useful basis for a further analysis of

Table 1.1: *Four variants of the Neoclassical Synthesis, Stage I (NCS I)*

	Equilibrium prices	Gradual price adjustments
Equilibrium wages	NCS I(w, p) Classical AS-AD version	NCS I$(w, \hat{p})$ Later: New-Keynesian baseline model
Gradual wage adjustments	NCS I$(\hat{w}, p)$ Textbook Keynesian AS-AD version	NCS I$(\hat{w}, \hat{p})$ Later: mature Keynesian models

traditional AS-AD growth dynamics. We uphold our negative evaluation of NCS I$(w, \hat{p})$ in the upper right corner, but have also indicated that it has later been revived in a microfounded and appreciably refined form that led to a standard and now orthodox model of contemporary macroeconomics, the New-Keynesian baseline model. This approach can even be viewed as the Neoclassical Synthesis, Stage II. Avoiding all the technical effort a full-fledged analysis would require, we will therefore try to reveal its basic mechanisms in the next section.

The classical AS-AD equilibrium version in the first diagonal entry of table 1.1 has later developed into the New-Classical economics and the equilibrium business cycle theory, a theoretical framework that we will not be concerned with here. The lower diagonal entry alludes to the version of NCS I that with its dynamic adjustments of both prices and wages will be most fruitful for us. In fact, all the models studied from chapter 3 onwards can be construed as arising from this approach – in a much more elaborated and appropriate form, of course, as we will claim. Hence the brief characterization that NCS I$(\hat{w}, \hat{p})$ provides the basis for our (and perhaps other) 'mature' Keynesian models. A first introduction to the wage-price dynamics that will be underlying Parts II and III of the book, as they emerge from the discussion in the present section, is given in section 1.4 below.

1.3 Neoclassical Synthesis, Stage II: New-Keynesian macrodynamics

Starting out from the concepts of the Neoclassical Synthesis at the beginning of the 1970s, the preceding section has discussed four model versions that distinguish perfectly flexible wages and/or prices vs. gradual adjustments of wages and/or prices. One of these versions could be seen as a first analogue of the present and topical New-Keynesian view on macrodynamics. Apart from the specific building blocks that we will have to discuss, in general three features may be mentioned that make this approach stand out: the state-of-the-art, mathematically rigorous microfoundations; the 'forward-looking' elements; and its close connection to issues of monetary policy. The reformulation of the traditional Keynesian AS-AD dynamics along these lines has by now gained so much popularity that we have classified it as the Neoclassical Synthesis, Stage II (NCS II).

A survey of its achievements and problems can, for example, be found in Galì (2000) and King (2000). Walsh (2003, chaps. 5 & 11) provides an advanced textbook presentation of the New-Keynesian baseline model, including an extensive discussion of monetary policy matters. The stress on monetary policy is even stronger in Woodford's (2003) book, which contains the most detailed analysis of this, as it may also be called, Neo-Wicksellian type of modelling.

In the present section we take up the New-Keynesian baseline model and a certain type of extension. In the baseline model, goods prices are determined in a staggered fashion while wages are assumed to clear the labour market instantaneously. On the other hand, the extended version studied by us allows for both gradual price and wage adjustments. These models are formulated in a most elementary way that, in particular, abstracts from the otherwise important stochastic perturbations. We can therefore discuss immediately their basic implications, their potential and weaknesses, without great technical effort.

The simplified presentation also permits us to relate the models to the deterministic and continuous-time modelling of the matured Keynesian macrodynamics that will be the main subject of the book. For convenience, a direct comparison can be made to a stripped-down version of the book's dynamic AS-AD approach, which will be put forward in section 1, subsection 4.

We limit ourselves to an assessment of the basic properties of the private sector in the different frames of reference, largely in isolation from any policy interference. For each model version we will therefore also consider a perfectly neutral monetary policy, which means here that the (nominal) interest rate is pegged at its steady-state value. As a rule, economies in the New-Keynesian theory exhibit quite unsatisfactory features, a finding that strongly emphasizes the role of monetary policy in the current macrodynamic literature. In contrast to the Neoclassical Synthesis, Stage I, with its exogenous money supply, it has to be noted that now monetary policy, which sets out to remedy the possible shortcomings, takes the form of an interest rate reaction function (mostly a version of the famous Taylor rule). This means that the interest rate

itself becomes a policy variable; the money supply and the LM curve can even completely disappear from the scene.

1.3.1 *The baseline model with perfect wage flexibility*

Following the presentation in Walsh (2003, chap. 11.1), the New-Keynesian baseline model is made up of three components. The demand side is obtained from a log-linear approximation to the representative household's Euler condition for optimal consumption. This gives rise to an expectational output relationship, where, however, fixed investment by firms is still absent.[17] Equations of this kind are often referred to as a dynamic IS curve in the literature. Next, the supply-side of the economy is represented by inflation adjustments occurring in a setting of monopolistic competition, where individual firms adjust prices in a staggered overlapping fashion. This yields the New-Keynesian Phillips curve. The third component is monetary policy in the form of an interest reaction function. We postpone this issue a little while and provisionally assume that the nominal interest rate is simply pegged at its steady-state value i^o.

To be in line with the New-Keynesian standard notation, we slightly change the meaning of two symbols in this section (temporarily). Thus, π at present denotes the rate of price inflation itself (not expected inflation), while (instead of the output–capital ratio) y stands for the output gap, the percentage deviation of output from its equilibrium level.[18] Furthermore, let E_t be the expectation operator based on information available at time t, and β a discount factor, $\beta \leq 1$.[19] With $\varepsilon_{y,t}$ and $\varepsilon_{p,t}$ being stochastic demand and supply shocks, respectively, the first two components of the baseline model can be written as

$$y_t = E_t y_{t+1} - \beta_{yi}[i_t - E_t \pi_{t+1} - (i^o - \pi^o)] + \varepsilon_{y,t} \qquad (1.20)$$

$$\pi_t = \beta E_t \pi_{t+1} + \beta_{py} y_t + \varepsilon_{p,t} \quad [\pi_t = (p_t - p_{t-1})/p_{t-1}] \qquad (1.21)$$

[17] Investment can be incorporated, but this complicates the model considerably (showing that an extension of the most elementary microfoundations is no easy matter); see Woodford (2003, chap. 5.3, pp. 352ff.). A clear and concise summary of a more ambitious private sector is also given in section 3 by Smets and Wouters (2003).

[18] Since Walsh himself confines the analysis to a stationary economy, the equilibrium level Y^o of output can be normalized at unity and we have $y = (Y - Y^o)/Y^o \approx \ln Y - \ln Y^o = \ln Y$.

[19] Interpreted as the representative household's discount factor, β (the usual notation, which we maintain) is strictly less than unity. However, as Mankiw (2001, pp. C51f.) derives the New-Keynesian Phillips curve from staggered price setting along the lines of Calvo (1983), β can also be equal to one.

It goes without saying that these equations presuppose rational expectations. As the shocks are normally distributed around zero, the steady-state values of inflation and the output gap are $\pi^o = 0$, $y^o = 0$. From the dating convention for the rate of inflation π_t of period t it can, moreover, be inferred that, if the model treats π_t as a jump variable, this is tantamount to treating the price level of period t as a jump variable. Since the current price level is the control variable of the optimizing firms, π_t should indeed be a jump variable. Similarly, the output gap is directly determined by private consumption, which is the control variable of the optimizing household. Hence y_t, too, should be a jump variable.

To reveal the basic dynamic properties of this private sector it suffices to study the deterministic counterpart of (1.20), (1.21). Solving (1.21) for $\pi_{t+1} = E_t\pi_{t+1} = (\pi_t - \beta_{py}y_t)/\beta$, substituting it in (1.20) and using $\pi^o = 0$, we get

$$y_{t+1} - y_t = \beta_{yi}(i_t - i^o) - (\beta_{yi}/\beta)(\pi_t - \beta_{py}y_t)$$
$$\pi_{t+1} - \pi_t = (1/\beta - 1)\pi_t - (\beta_{py}/\beta)y_t$$

Before investigating this system we should, however, pause for a moment and consider the inflation adjustments that derive from the New-Keynesian Phillips curve (1.21), for this purely forward-looking determination of the inflation rate has undergone severe empirical criticism. Mankiw (2001, p. C52) summarizes it in a particularly strong statement: 'Although the New Keynesian Phillips curve has many virtues, it has also one striking vice: It is completely at odds with the facts.'

The basic problem of the New-Keynesian Phillips curve can be seen without much econometrics. Neglecting the discount factor in the inflation adjustment equation, i.e. putting $\beta = 1$, we have the relationship $\pi_{t+1} - \pi_t = -\beta_{py}y_t$. It predicts that the rate of inflation rises if output is below its natural (or trend) level, and vice versa. This kind of reaction may appear counter-intuitive. Actually, it contradicts the basic principles taught in macroeconomic textbooks. For example, Taylor's (2001, chap. 24) discussion of his inflation adjustment rule can be readily formalized as

$$\pi_{t+1} - \pi_t = \tilde{\beta}_{py}y_t \qquad (1.22)$$

with no minus sign on the right-hand side. Taylor supports the relationship by a diagram (on p. 569), the message of which he sums up as: 'The data show that inflation falls when real GDP is below potential GDP, and inflation rises when real GDP is above potential GDP.' Blanchard (2000, chap. 8, p. 154) calls a relationship like (1.22) an

'accelerationist' Phillips curve. He illustrates its validity in another diagram that (instead of the output gap) refers to the unemployment rate. He also provides a straightforward estimation that for annual inflation changes in the United States between 1970 and 1998 gives the equation $\pi_t - \pi_{t-1} = 6.5\% - 1.0 \cdot \text{unemployment}_t$. These elementary observations suffice to explain Mankiw's verdict.[20]

It may be added that (1.22) implies countercyclical (!) motions of the price level (relative to its trend). This feature is by now an established stylized fact of the business cycle, not only in the United States. We will deal with it in chapter 5; to anticipate, the countercyclicality in the time series of the price level is nicely brought out in figure 2.

After this brief critical evaluation of the New-Keynesian Phillips curve, we return to the two-dimensional difference equations system in y_t and π_t. Its analysis and that of the other systems below is simplified if we consider their continuous-time analogues, where we replace only the difference operator with the time derivative.[21] The private sector specified so far is then succinctly described by the two differential equations

$$\dot{y} = \beta_{yi}(i - i^o) + (\beta_{py}\beta_{yi}/\beta)y - (\beta_{yi}/\beta)\pi \quad (\text{where } i = i^o) \qquad (1.23)$$
$$\dot{\pi} = -(\beta_{py}/\beta)y + (1/\beta - 1)\pi \qquad (1.24)$$

According to the remark on (1.20), (1.21), y and π should be both jump variables, which requires both eigenvalues of the Jacobian of (1.23), (1.24) to have positive real parts. In this deterministic setting both variables would then directly jump into the equilibrium point of the system. Thus, $y^o = 0$, $\pi^o = 0$ constitute the uniquely determined starting point of the optimal solution path. The trajectory itself is here rather uninteresting since, without shocks, output and prices remain at their initial levels.

It is, however, straightforward to show that the Jacobian of (1.23), (1.24) is negative, so that the equilibrium point of this system is a saddle

[20] Mankiw (2001, pp. C54ff.) takes a slightly different angle and formulates his argument in terms of impulse-response functions to a monetary contraction. One reply from the New-Keynesian side to this kind of criticism is that here the output gap is specified as deviations from a smooth trend, which is an ad hoc measure with no theoretical justification. It would be more appropriate to conceive it as output minus the concept of flexible price output (which is unsmooth and quite volatile), though this variable is neither observable nor does it constitute the present New-Keynesian notion of the output gap. This problem, in turn, can be solved (as it is claimed) by returning to the more fundamental version of the Phillips curve, which refers to marginal wage costs instead of an output gap; see Galí and Gertler (1999) and Galí et al. (2001).

[21] Hence, the differential equations thereby obtained should not be interpreted as the limit of a sequence of discrete-time economies with adjustment period h, where h shrinks to zero. Though deterministic and continuous-time formulations are absent in the New-Keynesian references given above, they are occasionally found useful at other places. An example is Fuhrer and Moore (1995).

where one eigenvalue is negative and the other positive.[22] As a consequence, the system could start anywhere on the one-dimensional stable manifold to get to $y^o = 0$, $\pi^o = 0$. In other words, we have a situation of indeterminacy. In this sense, the model turns out to be ill-defined. To be more precise, a stochastic framework could handle indeterminacy by invoking the concept of 'sunspot equilibria'. But indeterminacy is certainly an undesirable feature for an elementary model at the beginning of a theory.

We can, therefore, sum up that a basic New-Keynesian model with neutral monetary policy holding the nominal interest rate constant is as inconsistent as its forerunner NSC I($\hat{p}$, w) set up in the preceding subsection, where in the presence of the LM curve the real balances ratio M/pK was supposed to remain fixed.

The incompatibility of the number of unstable eigenvalues and the number of jump variables in (1.23), (1.24) is the place where monetary policy can be seen to set in. Suitable reactions of the interest rate to deviations of output and inflation from equilibrium can help the model economy out of the dilemma. Specifically, this is achieved by a (now standard) Taylor rule, which – parameterizing Taylor's (1993a, p. 202) formulation – we write down as[23]

$$i = (i^o - \pi^o) + \pi + \alpha_\pi(\pi - \pi^o) + \alpha_y y \qquad (1.25)$$

Zero policy coefficients, $\alpha_\pi = \alpha_y = 0$, pursue a policy of a constant real rate of interest, while with $\alpha_\pi > 0$ the central bank increases not only the nominal but also the real interest rate as inflation rises. A positive output gap coefficient, $\alpha_y > 0$, is another facet of a central bank leaning against the wind.

Plugging the interest rate rule (1.25) into (1.23) and taking account of $\pi^o = 0$ transforms the dynamic IS curve into

$$\dot{y} = \beta_{yi}(\alpha_y + \beta_{py}/\beta)y + \beta_{yi}(1 + \alpha_\pi - 1/\beta)\pi \qquad (1.26)$$

The optimality conditions for the changes in y and π are now given by (1.26) and (1.24). The Jacobian $\mathcal{J}$ of this system has a positive trace, since both diagonal entries are positive. The determinant is calculated as

$$\det \mathcal{J} = \beta_{yi}[\alpha_y(1 - \beta) + \alpha_\pi \beta_{py}]/\beta$$

22 The original discrete-time system can also be shown to be a saddle-point; so the simpler continuous-time system leads to no distortion.

23 Of course, π^o, which is here zero, corresponds to the target rate of inflation, which Taylor sets at 2 per cent. For the policy coefficients he chooses the well-known values $\alpha_\pi = \alpha_y = 0.50$.

which is positive as soon as $\alpha_y > 0$ or $\alpha_\pi > 0$. As the latter can be taken for granted, the New-Keynesian baseline model with the Taylor rule (1.25) has a uniquely determined optimal solution trajectory. This kind of monetary policy has indeed succeeded in ruling out indeterminacy. It may, furthermore, be stressed that the outcome that the private sector is not workable by itself is a general result in New-Keynesian baseline approaches. King (2000), Walsh (2003) and Woodford (2003) provide a variety of examples where a Taylor-type rule is employed in order to get determinacy. And also, conversely, monetary policy is in this theoretical field the only device that is considered to take care of determinacy if it is endangered.

While the model is now consistent, one may nevertheless question its usefulness. It has already been remarked about eqs. (1.23), (1.24) that the dynamic trajectories are rather dull. In the deterministic setting nothing else happens after the agents have chosen the optimal levels of inflation and output (the latter via consumption). Some motions would be observed in a stochastic setting, but the shocks are purely transitory there: as soon as the random noise stops, inflation and the output gap would immediately jump back to their steady-state values. In the New-Keynesian baseline model there is thus no inertia at all, so the macroeconomic dynamics are still quite poor.

It might at first appear surprising that a model in which the concept of staggered prices has originally been set out to capture inertial pricing behaviour cannot generate persistence. Following Mankiw (2001, p. C53), the puzzle resolves if the distinction between inertia in the price level and inertia in the inflation rate is made. Because individual prices are adjusted intermittently in the New-Keynesian models, the price level adjusts slowly to shocks. But the rate of inflation – the change in the price level – can adjust instantly (just as the capital stock adjusts slowly, while net investment may be a jump variable that can change immediately to changing conditions).

1.3.2 Staggered wages and prices

One extension of the New-Keynesian baseline model is particularly important for us. In light of the central role that the real wage rate has played in the Neoclassical Synthesis, Stage I, wage formation may also be here made explicit. In fact, wage-setting agents can be, and have been, incorporated into the model analogously to the treatment of the goods market, by way of assuming monopolistic competition among suppliers of differentiated types of labour. Erceg et al. (2000) and, following them, Woodford (2003, chap. 4.1) combine this idea of wage determination with an advanced version of the earlier price determination (see also Walsh, 2003, chap. 5.5). Apart from an interest in extensions of the New-Keynesian workhorse model as such, it will also be informative to compare the resulting wage and price Phillips curve relationships with the Phillips curves that we employ in the book's mature Keynesian models, which will be subsequently briefly introduced in section 1.4.

After the dust of processing the microeconomic foundations of staggered wage and price setting has settled, Woodford (2003, p. 225) comes up with the following two equations that the joint evolution of wages and prices must satisfy:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \beta_{wy} y_t - \beta_{w\omega}(\ln \omega_t - \ln \omega_t^n) \tag{1.27}$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \beta_{py} y_t + \beta_{p\omega}(\ln \omega_t - \ln \omega_t^n) \tag{1.28}$$

where π_t^w denotes wage inflation, $\pi_t^w = (w_t - w_{t-1})/w_{t-1}$, and π_t^p price inflation (the superscript 'p' has now been added for a more pronounced contrast to wage inflation). The output gap enters the determination of price as well as wage inflation, since in this framework there is no room for unemployment.

What is most remarkable in these relationships is the influence of the real wage rate $\omega = w/p$, i.e. its percentage deviations from the natural real wage (the equilibrium real wage when both wages and prices are fully flexible). Observe that ω has a negative bearing on wage inflation and a positive bearing on price inflation.

The full model is given by eqs. (1.27), (1.28), the Euler condition of the private households (1.21) and the Taylor rule (1.25).[24] The previous two jump variables y_t and $\pi_t = \pi_t^p$ are thus joined by π_t^w, which is likewise a jump variable. As this is tantamount to treating the price level p_t and the nominal wage rate w_t as jump variables (cf. the remark on (1.21) above), the real wage rate ω_t is a jump variable too. In other words, the model still includes no predetermined variable. This means that the critical remark at the end of the preceding subsection continues to apply: also, this extended New-Keynesian model will not be able to generate dynamics with some meaningful persistence in the time series (apart from persistence in the exogenous random shocks).

The rest of the present subsection is thus exclusively concerned with consistency: is the model's determinacy ensured? To avoid the technical

[24] The explicit introduction of wages does not affect the condition for optimal consumption of households; see Erceg et al. (2000, p. 291).

subtleties, the analysis is again similarly sketchy as above. To begin with (1.27), (1.28), in a deterministic setting with rational expectations we have

$$\pi_{t+1}^w = [\pi_t^w - \beta_{wy} y_t + \beta_{w\omega}(\ln \omega_t - \ln \omega_t^n)]/\beta$$

$$\pi_{t+1}^p = [\pi_t^p - \beta_{py} y_t - \beta_{p\omega}(\ln \omega_t - \ln \omega_t^n)]/\beta$$

To ease the exposition, let us now directly work with $\beta = 1$. This has the advantage that the first differences of the inflation rates, $\pi_{t+1}^a - \pi_t^a$ $(a = w, p)$, depend merely on y_t and $\ln \omega_t$. Neglect furthermore exogenous variations of the natural real wage and put $\chi_t = \ln \omega_t - \ln \omega_t^n = \ln \omega_t - \ln \omega^n$. The continuous-time analogue of (1.27), (1.28) then reads

$$\dot{\pi}^w = -\beta_{wy} y + \beta_{w\omega} \chi \tag{1.29}$$

$$\dot{\pi}^p = -\beta_{py} y - \beta_{p\omega} \chi \tag{1.30}$$

The change in χ derives from its definition. The change in y is obtained from substituting (1.25) and the above expression for π_{t+1}^p in the Euler condition $y_{t+1} - y_t = \beta_{yi}(i_t - i^o) - \beta_{yi}\pi_{t+1}^p$. Thus,

$$\dot{y} = \beta_{yi}[(\alpha_y + \beta_{py})y + \alpha_\pi \pi^p + \beta_{p\omega}\chi] \tag{1.31}$$

$$\dot{\chi} = \pi^w - \pi^p \tag{1.32}$$

Together, the economy's dynamic first-order conditions translated into continuous time are described by the Jacobian matrix $\mathcal{J}$ of (1.29) to (1.32), which has a simple determinant:

$$\mathcal{J} = \begin{bmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \beta_{yi}\alpha_\pi & \beta_{yi}(\alpha_y + \beta_{py}) & \beta_{yi}\beta_{p\omega} \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{1.33}$$

$$\det \mathcal{J} = -\alpha_\pi \beta_{yi}(\beta_{p\omega}\beta_{wy} + \beta_{py}\beta_{w\omega})$$

Since all variables are jump variables, the determinacy of the model's optimal solution path requires that all four eigenvalues of $\mathcal{J}$ have positive real parts. To check this, factorize the characteristic polynomial $P(\lambda)$ with respect to the four eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. We get the following equation that an eigenvalue λ must satisfy:

$$P(\lambda) = \lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_o = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)(\lambda - \lambda_4) = 0$$

Consider first the case $\alpha_\pi = 0$, which implies $\lambda_1 = 0$ because of det $\mathcal{J} = 0$. It is well known that the term a_1 in the polynomial is given by $a_1 = -\sum_k \mathcal{J}_k$, where $\mathcal{J}_k$ are the four third-order principal minors. Multiplication in $P(\lambda)$ then gives for the coefficient associated with $\lambda = \lambda^1$

$$-\lambda_2\lambda_3\lambda_4 = a_1 = -(\mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 + \mathcal{J}_4) > 0$$

The positive sign obtains because $\mathcal{J}_1 < 0$, $\mathcal{J}_2 < 0$, $\mathcal{J}_3 = \mathcal{J}_4 = 0$. Hence, at least one eigenvalue has a negative real part. Again, as in the baseline model, the economy would be ill-defined in the absence of monetary policy, when $\alpha_\pi = \alpha_y = 0$.

Similarly, as before, we therefore ask: can the Taylor rule with $\alpha_\pi > 0$ and $\alpha_y \geq 0$, or at least with a suitable choice of the two policy coefficients, ensure determinacy? This time the answer is a definite 'no'. The impossibility result follows directly from the negative sign of the determinant of $\mathcal{J}$ as $\alpha_\pi > 0$. For then we have $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = \det \mathcal{J} < 0$, implying that at least one eigenvalue continues to have a negative real part.

Erceg et al. (2000) treat the problem of monetary policy in terms slightly different from indeterminacy. They include random errors in their model, but only shocks to the Pareto-efficient steady-state values. The main proposition (on pp. 296ff.) states that, however the interest rate is set, no more than one of the three variables output gap, price inflation and wage inflation can have zero variance when the exogenous shocks have non-zero variance. The reason is that the output gap would remain at zero and wage and price inflation would remain constant only if the aggregate real wage rate were continuously at its Pareto-optimal level. The latter, however, moves in response to each of the exogenous shocks considered.

The significance of this result is accentuated by a complementary proposition that examines the limiting cases of perfectly flexible wages and prices, respectively. It says that, with staggered price contracts and perfectly flexible wages, monetary policy can completely stabilize price inflation and the output gap, thereby attaining the Pareto-optimal social welfare level. And, conversely, monetary policy can achieve the same goal in the presence of staggered wage contracts and perfectly flexible prices (Erceg et al., 2000, p. 298).

At a general level, the following quote from Erceg et al. (2000, pp. 305f) is a good conclusion of the discussion on the extended baseline model: 'While considerations of parsimony alone might suggest an exclusive focus on either staggered price setting or staggered wage setting, the inclusion of both types of nominal inertia makes a critical difference in the monetary policy problem.' The way in which

the difference becomes 'critical' and the continual prevalence of the parsimonious formulations cast serious doubts, in our view, on the usefulness of hard-core New-Keynesian theory, an approach entirely based on agents who, in accordance with the intertemporal infinite-horizon optimization problems they are supposed to solve, are purely forward-looking – as was, for example, specified in eqs (1.27), (1.28) above.

1.3.3 *Combining forward-looking and backward-looking behaviour I*

Empirical criticism, such as Mankiw's (2000) verdict quoted above on the New-Keynesian Phillips curve with its purely forward-looking expectations, has fostered the idea that inertia in the rate of interest may be generated if some – as they are called – backward-looking elements are introduced. One idea is to augment the microfoundations by a backward-looking indexation of prices (or wages) that takes the form $\ln p_t = \ln p_{t-1} + \gamma_p \pi^p_{t-1}$, where γ_p is the indexation rate for prices that are not reoptimized ($0 \leq \gamma_p \leq 1$; see Woodford, 2003, p. 234). Equation (1.28) thus becomes

$$\pi^p_t - \gamma_p \pi^p_{t-1} = \beta E_t(\pi^p_{t+1} - \gamma_p \pi^p_t) + \beta_{py} y_t + \beta_{p\omega}(\ln \omega_t - \ln \omega^n_t) \qquad (1.34)$$

(and likewise for wages). Obviously, the formal analysis can remain perfectly the same as before if we introduce the auxiliary variable $\tilde{\pi}^p_t := \pi^p_t - \gamma_p \pi^p_{t-1}$, which is again a jump variable. However, even if, for example, after a purely transitory shock to inflation at $t = 0$ this variable immediately returns to its zero equilibrium value at $t = 1$, this does not yet hold true for the inflation rate π^p_t itself. Now it exhibits some persistence since $\pi^p_1 = \tilde{\pi}^p_1 + \gamma_p \pi^p_o = 0 + \gamma_p \pi^p_o$ and, repeating this argument forward in time, $\pi^p_t = \gamma^t_p \pi^p_o$. Hence, the higher the indexation rate the higher the degree of persistence in inflation.[25]

More common in the literature on optimal monetary policy in particular is another specification that is almost, but not exactly, identical to (1.34). It starts directly from the New-Keynesian Phillips curve (1.21), and makes no more explicit reference to the underlying microfoundations. The combination of forward-looking and backward-looking elements is, rather, formulated in a straightforward manner as a weighted

[25] The IS curve can be treated in a similar way as (1.34) if the terms $u(C_t)$ in the representative household's intertemporal utility function are replaced with $u(C_t - \eta C_{t-1})$, where the parameter η measures the degree of 'habit persistence' ($0 \leq \eta < 1$).

average of the two corresponding inflation rates. The device is likewise applied to the IS equation (1.20). Together we thus have[26]

$$y_t = \phi_y E_t y_{t+1} + (1-\phi_y) y_{t-1} - \beta_{yi}[i_t - E_t \pi_{t+1} - (i^o - \pi^o)] + \varepsilon_{y,t} \qquad (1.35)$$

$$\pi_t = \phi_p \beta E_t \pi_{t+1} + (1-\phi_p)\pi_{t-1} + \beta_{py} y_t + \varepsilon_{p,t} \qquad (1.36)$$

The weights ϕ_y, ϕ_p determine the extent to which behaviour looks forward and backward ($0 \le \phi_y, \phi_p \le 1$).[27] Clearly, with $\phi_y = \phi_p = 1$, we are back to the New-Keynesian baseline model (1.20), (1.21). At the other end, $\phi_y = \phi_p = 0$, agents are said to to be purely backward-looking (except for the expectations $E_t\pi_{t+1}$ in (1.35), if they are maintained).

Generally with hybrid expectations, $0 < \phi_y, \phi_p < 1$, forward-looking and backward-looking agents are conceived to coexist. It is, however, not clear what the microfoundations look like in such a world and why these relationships should lead to just the equations as they are specified in (1.35), (1.36), though they might appear fairly appealing. Why should the naive, backward-looking agents survive at all? Why are they not outperformed and ousted by the fully rational forward-looking agents?[28] The view on these issues is usually pragmatic, unless agnostic. The following quote from Leeper and Zha (2001, p. 85) is certainly representative for applications of (1.35), (1.36): 'We are less concerned with whether backward-looking behavior can be sensibly rationalized in an optimizing framework than we are with extracting the model's implications.' Is it unfair to say that the hybrid specification (1.35), (1.36) has only 'soft' microfoundations?

It is a priori not obvious, either, which variables should be regarded as predetermined in (1.35), (1.36), and it is also not always explicitly stated in the presentation of these equations. Nonetheless, the consequence

[26] Wages are again neglected in this subsection, as they are in almost all the literature that integrates forward-looking and backward-looking behaviour by similar principles as in eqs. (1.35) and (1.36). In the following we therefore return to the previous notation $\pi_t = (p_t - p_{t-1})/p_{t-1}$ for the rate of price inflation, dropping the superscript 'p' of section 1.3.2.

[27] Note that Woodford's equation (1.34) can be rearranged as (omitting the superscript 'p') $\pi_t = \phi_{p1}\beta E_t \pi_{t+1} + \phi_{p2}\pi_{t-1} + \beta_{py}/(1+\beta\gamma_p) y_t + \cdots$, but unless $\beta = 1$ the two coefficients ϕ_{p1} and ϕ_{p2} do not exactly add up to unity as in (1.36).

[28] Besides habit persistence in the scientific community, this is precisely Milton Friedman's (1953, especially pp. 21f.) defence of rational expectations as a positive hypothesis about observable behaviour: agents need not consciously formulate and solve complex optimization problems and make sure in sophisticated ways that their beliefs and decisions are mutually consistent. The vision is, rather, that evolutionary pressure and imitation induce them to act *as if* they did meet the epistemic conditions of the rational expectations hypothesis, while other behaviour will be driven out of the market.

appears to be that now y_t and π_t are treated as predetermined, and $E_t y_{t+1}$ and $E_t \pi_{t+1}$ are jump variables.[29]

One approach to obtaining the time paths of (1.35), (1.36), after an interest rate rule has been incorporated, is a suitable transformation of these equations such that the changes in the predetermined variables are described by an ordinary (backward-looking) difference equation of the form

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = P \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + Q \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{p,t} \end{bmatrix} \qquad (1.37)$$

The problem, of course, is to derive the two matrices $P, Q \in \mathrm{I\!R}^{2\times 2}$ of the reduced form from the structural equations, where determinacy of the model prevails if P turns out to be stable (both eigenvalues within the unit circle). There are (at least) two procedures to determine P and Q, both of which require so much effort that, as a rule, one has to resort to numerical methods. One procedure, for example, amounts to finding the solution of a quadratic matrix equation for P. This also shows that the structural coefficients in (1.35), (1.36) enter the reduced-form solution (1.37) in complicated (and 'unpredictable') ways. The appendix to chapter 8 presents more details of how (1.35), (1.36) and P in (1.37) can be dealt with.

Definite results on (1.35), (1.36) in combination with the Taylor rule are not too difficult to obtain if all agents are assumed to be backward-looking: $\phi_y = \phi_p = 0$. Even if the expectations $E_t\pi_{t+1}$ in (1.35) are maintained, the structural equations can be transformed into (1.37) without sophisticated methods. The resulting entries of the matrix P are, however, so unwieldy that we omit this case. To illustrate the stabilizing potential of monetary policy when inflation and the output gap are predetermined, it suffices to suppose a lagged influence of output in the Phillips curve, which proves to be more convenient, while the forward-looking element in (1.35) can be preserved. Thus, we set up the following deterministic version of a dynamic IS curve and an accelerationist Phillips curve, combined with the Taylor rule:

$$y_t = y_{t-1} - \beta_{yi}[i_t - E_t\pi_{t+1} - (i^o - \pi^o)] \qquad (1.38)$$

$$\pi_t = \pi_{t-1} + \beta_{py} y_{t-1} \qquad (1.39)$$

$$i_t = (i^o - \pi^o) + \pi_t + \alpha_\pi(\pi_t - \pi^o) + \alpha_y y_t \qquad (1.40)$$

[29] One often learns this from elaborations in a technical appendix of a working paper. For example, we have found such a clear and direct statement in Ellingsen and Söderström (2004, p. 15). Incidentally, this paper demonstrates that more advanced models may look more than one period ahead in the future; see our sketches in the appendix to chapter 8.

What the Taylor rule (1.40) should achieve in this case is that, whatever values of y_t, π_t are given by history (and demand and supply shocks) in period $t=0$, in the absence of further shocks the economy will eventually return to its steady-state position $(y^o, \pi^o) = (0, 0)$. Correspondingly, with exogenous shocks imposed on (1.38) to (1.40), the variances of y_t and π_t would remain finite.

After a few elementary manipulations, the reduced-form equations of (1.38) to (1.40) result as

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \mathcal{J} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} (1-\alpha_\pi\beta_{yi}\beta_{py})/h(\alpha_y) & -\alpha_\pi\beta_{yi}/h(\alpha_y) \\ \beta_{py} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \tag{1.41}$$

where $h(\alpha_y) := 1 + \beta_{yi}(\alpha_y - \beta_{py})$. The stability of $\mathcal{J}$ can be examined with the aid of the Schur criterion. Defining and computing

$$a_1 := -\text{trace}\,\mathcal{J} = -1 + (\alpha_\pi\beta_{yi}\beta_{py} - 1)/h(\alpha_y)$$
$$a_2 := \det\,\mathcal{J} = 1/h(\alpha_y)$$

the criterion states that the two eigenvalues of $\mathcal{J}$ are inside the unit circle if and only if[30]

$$1 - a_1 + a_2 > 0, \quad 1 + a_1 + a_2 > 0, \quad 1 - a_2 > 0$$

The second inequality is satisfied unambiguously, the third one if $\alpha_y - \beta_{py} > 0$, and a sufficient (but by no means necessary) condition for the first inequality to be satisfied is $1 - \alpha_\pi\beta_{yi}\beta_{py} > 0$. It can thus be concluded that the central bank can stabilize the economy if it adopts the Taylor rule (1.40) with a sufficiently small (!) inflation gap coefficient: and a sufficiently large output gap coefficient: $\alpha_\pi < 1/\beta_{yi}\beta_{py}$ and $\alpha_y > \beta_{py}$.

Obviously, the time paths generated by (1.41) will then also exhibit a certain degree of persistence. The economy (1.38)–(1.40) is in this respect far more satisfactory than the New-Keynesian baseline model and its wage-price extension we have considered, though it does not meet the New-Keynesian standards regarding the microeconomic underpinnings. Using an estimated model with more lags from the literature, we will actually argue in chapter 8, section 4, that models of this type even produce too much persistence – mainly in the rate of inflation, which is due to the accelerationist specification of the Phillips curve in (1.39).

As has been indicated in the remark on (1.35), (1.36) concerning the predetermined variables, mixed forward-looking and backward-looking

[30] In this convenient form the criterion is quoted in Gabisch and Lorenz (1989, p. 45). a_1 and a_2 are the coefficients of the characteristic polynomial $\lambda^2 + a_1\lambda + a_2$, which are known to be given by $-$trace $\mathcal{J}$ and det $\mathcal{J}$, respectively.

systems are much more difficult to handle. We must therefore content ourselves with giving a brief impression that these economies may have meaningful properties. To this end, we again resort to a continuous-time approximation. As it is still more heroic than before, it may be taken as a purely heuristic device. The result that the analysis of the mixed economies is more involved than in the New-Keynesian baseline model also makes itself felt in the fact that the continuous-time approximation will no longer be two-dimensional but four-dimensional. In this respect, the basic structure of the discrete-time economy with its two predetermined and two jump variables still shines through.

Thus, consider the deterministic version of (1.35), (1.36). To avoid clumsy expressions, set $\beta = 1$ in (1.36) right at the beginning. Rearranging the terms and letting Δ denote the (backward) difference operator, the two equations can be written as

$$\phi_y(\Delta y_{t+1} - \Delta y_t) = (1 - 2\phi_y)\Delta y_t + \beta_{yi}[i_t - \pi_{t+1} - (i^o - \pi^o)]$$
$$\phi_p(\Delta\pi_{t+1} - \Delta\pi_t) = (1 - 2\phi_p)\Delta\pi_t - \beta_{py}y_t$$

The heroic approximation, now, is to replace the second-order differences with the second-order time derivatives (which is only meaningful for $0 < \phi_y, \phi_p < 1$). In addition, in the first equation we substitute for π_{t+1}, which equals $\Delta\pi_{t+1} + \pi_t$, the term $\dot\pi + \pi$.[31] Substituting the Taylor rule for i_t, we get

$$\ddot y = [\alpha_y\beta_{yi}y + (1 - 2\phi_y)\dot y + \alpha_\pi\beta_{yi}\pi - \beta_{yi}\dot\pi]/\phi_y$$
$$\ddot\pi = [(1 - 2\phi_p)\dot\pi - \beta_{py}y]/\phi_p$$

The equations can be transformed into a four-dimensional first-order differential equations system by setting $x = \dot y$ and $\rho = \dot\pi$. It then reads

$$\begin{bmatrix} \dot y \\ \dot x \\ \dot\pi \\ \dot\rho \end{bmatrix} = \mathcal{J} \begin{bmatrix} y \\ x \\ \pi \\ \rho \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha_y\beta_{yi}/\phi_y & (1-2\phi_y)/\phi_y & \alpha_\pi\beta_{yi}/\phi_y & -\beta_{yi}/\phi_y \\ 0 & 0 & 0 & 1 \\ -\beta_{py}/\phi_y & 0 & 0 & (1-2\phi_p)/\phi_p \end{bmatrix} \begin{bmatrix} y \\ x \\ \pi \\ \rho \end{bmatrix} \tag{1.42}$$

[31] It appears more appropriate to solve (1.36) for $\pi_{t+1} = (1 - \phi_p)\Delta\pi_t/\phi_p - \beta_{py}y_t/\phi_p - \pi_t$ and make use of this expression. We nevertheless prefer the other option, since it yields a zero determinant of the Jacobian matrix for the benchmark coefficient $\alpha_\pi = 0$. This is not only helpful for the subsequent analysis but also, perhaps, more trustworthy. (The determinant in the alternative case has the same sign as $\alpha_\pi + 2$.)

Of course, y and π must maintain their role as predetermined variables. While in the original (deterministic) model y_{t+1} and π_{t+1} are jump variables, this part can here be taken over by the time derivatives $\dot{y} = x$ and $\dot{\pi} = p$. Hence, determinacy requires that the matrix $\mathcal{J}$ has two eigenvalues with negative and two with positive real parts.

From the sign of the determinant, det $\mathcal{J} = \alpha_\pi \beta_{yi} \beta_{py} / \phi_y \phi_p$, it can be inferred immediately that indeterminacy prevails if monetary policy is inactive – that is, if the interest rate is fixed at $i = i^o$: this case corresponds to $\alpha_\pi = -1$, so that det $\mathcal{J} < 0$ and the relation $\lambda_1 \lambda_2 \lambda_3 \lambda_4 = \det \mathcal{J}$ for the four eigenvalues tells us that the number of eigenvalues with positive real parts cannot be even.

Suppose for the rest of the analysis that forward-looking behaviour has a greater weight in the economy, so that $\phi_y > 1/2$, $\phi_p > 1/2$. Consider first the special case $\alpha_\pi = 0$, where the determinant vanishes and one eigenvalue λ_1 is zero. Three of the principal third-order minors are zero, the fourth is unambiguously positive, $\mathcal{J}_3 = \beta_{yi}[\beta_{py} - \alpha_y(1 - 2\phi_p)]/\phi_p > 0$. By the same argument as in the preceding subsection this gives us the condition $-\lambda_2 \lambda_3 \lambda_4 = -(\mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 + \mathcal{J}_4) < 0$ for the other three eigenvalues. Assuming without loss of generality that $\lambda_2 > 0$, both λ_3 and λ_4 have either positive or negative real parts. From $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 =$ trace $\mathcal{J} < 0$ it follows that they are negative.

The real parts of λ_2, λ_3, λ_4 do not change their sign when α_π is slightly increased above zero. Since det $\mathcal{J} > 0$ then the equation $\lambda_1 \lambda_2 \lambda_3 \lambda_4 =$ det $\mathcal{J}$ now tells us that λ_1 is real and positive. In sum, two eigenvalues of $\mathcal{J}$ have positive and two have negative real parts. We can therefore conclude that, at least if $\phi_y > 1/2$, $\phi_p > 1/2$ and the central bank's inflation gap coefficient α_π is not too high, determinacy of the economy is ensured.

Basically, models with hybrid expectations are still in line with he New-Keynesian baseline model in that they preserve the feature of jumping variables. On the other hand, an important conceptual change to observe is that the output gap and the rate of inflation turn from jump variables into predetermined variables. Interestingly, in the original discrete-time and stochastic formulation of the hybrid expectations model, the meaning of the term 'jump variable' becomes somewhat eroded, in particular if, for computing the time paths of the economy, one refers to the reduced-form presentation (1.37) that governs the dynamics of – exclusively – the predetermined variables.

The great advantage of the hybrid expectations over the baseline model is that, by allowing the key variables output and inflation to be predetermined (without introducing additional variables such as the capital stock), it provides much greater scope for a reasonable persistence in the time series. This explains the fact that the vast majority of New-Keynesian-oriented models on monetary policy issues with an empirical background (however remote) disregard the baseline model and work with some version of hybrid expectations.

1.3.4 *Combining forward-looking and backward-looking behaviour II*

While introducing a weighted average of forward-looking and backward-looking components into the New-Keynesian framework considerably widens the scope for gaining satisfactory trajectories, these models, even in their less advanced versions, are already fairly complex. One may therefore look for simplifications that preserve the basic concepts but make the model easier to analyse. Such simplifications could also be more readily accepted now since, as we have already indicated, the microfoundations of the hybrid expectations models are not as 'firm' as in the New-Keynesian baseline model.

A useful simplification one occasionally finds in the literature does not change the variables entering eqs. (1.35) and (1.36) in the preceding subsection but does change the time index of some of them. Specifically, the output gap in the IS equation and the rate of inflation in the Phillips curve are shifted one period forward in time, on the left-hand side as well as on the right-hand side of the equations. Presupposing also $\beta = 1$ and, of course, ϕ_y, $\phi_p < 1$, we have

$$y_{t+1} = \phi_y E_t y_{t+1} + (1 - \phi_y) y_t - \beta_{yi}[i_t - E_t \pi_{t+1} - (i^o - \pi^o)] + \varepsilon_{y,t} \quad (1.43)$$

$$\pi_{t+1} = \phi_p E_t \pi_{t+1} + (1 - \phi_y) \pi_t + \beta_{py} y_t + \varepsilon_{p,t} \quad (1.44)$$

The prevailing view seems to be that this modification of (1.35), (1.36) does not need a comprehensive justification. In Lansing and Trehan (2003, p. 250), for example, it is understood that equations such as (1.43) and (1.44) serve 'to loosely approximate some commonly-used specifications in the literature'. In the precursory working paper (2001, p. 4), the authors speak of a timing convention that, with respect to inflation, changes a New-Keynesian Phillips curve to a 'Neoclassical'-style Phillips curve.[32] The undogmatic attitude of papers employing (1.43) and (1.44) is also exemplified by remarks that the same or other authors have obtained similar results on the basis of the New-Keynesian dating, where the advantage of (1.43) and

[32] For further details on the two Phillips curve setups, reference is made to Roberts (1995). The modified dating is also mentioned in Woodford (2003, p. 150).

(1.44) is that they are simpler and may even admit an analytical treatment.[33]

The first gain from eqs. (1.43), (1.44) is that they relieve the user of any further thoughts about predetermined variables; inflation π_t and output gap y_t are definitely predetermined. After a few manipulations, the system is seen to be equivalent to the following ordinary stochastic difference equations:[34]

$$y_{t+1} = y_t - \frac{\beta_{yi}}{1-\phi_y}\left[i_t - \pi_t - \frac{\beta_{py}}{1-\phi_p}y_t - (i^o - \pi^o)\right] + \varepsilon_{y,t} \qquad (1.45)$$

$$\pi_{t+1} = \pi_t + \frac{\beta_{py}}{1-\phi_p}\pi_t + \varepsilon_{p,t} \qquad (1.46)$$

Obviously, once y_o, π_o are given at time $t=0$, these equations can be directly iterated forward and there is also no more necessity to think about 'jump' variables. The feature is, of course, maintained if the Taylor rule $i_t = i(\pi_t, y_t)$ from (1.40) is applied to the interest rate.

A nice property of (1.43), (1.44) is also that one can immediately see the bearing that forward-looking expectations have on the economy. An increase in the forward-looking component, represented by rising weights ϕ_y and ϕ_p in the reduced-form equations (1.45), (1.46), reinforces the reactions of y_{t+1}, π_{t+1} to the deviations of y_t, π_t from equilibrium. The reactions would even tend to infinity as the weights approach unity. The effects of ϕ_y and ϕ_p may be similar in the New-Keynesian setting of (1.35), (1.36), but this is much harder to verify there.

The fact that we can obtain the time paths of (1.43), (1.44) without any sophisticated methods does not yet mean that this economy (as a linear system) is meaningful. In this respect, it still has to be checked that the deterministic counterpart of (1.45), (1.46) is asymptotically stable. However, an analysis of these difference equations is certainly easier than in the New-Keynesian case of hybrid expectations, where first a matrix such as P in (1.37) would have to be computed. To illustrate the strong prospects of (1.43), (1.44) for stability, it here suffices for us to consider the (deterministic and) continuous-time analogue of (1.45), (1.46). Substituting the Taylor rule (1.40) into (1.45), putting the first

[33] Lansing and Trehan can explicitly compute the policy coefficients α_π and α_y that minimize an intertemporal loss function of the central bank.

[34] The derivation is based on the rule that an equation such as $z_{t+1} = \phi E_t z_{t+1} + \beta x_t + \varepsilon_t$ (with $E_t \varepsilon_t = 0$ and β and x being row and column vectors, respectively) can be transformed into $z_{t+1} = \beta x_t/(1-\phi) + \varepsilon_t$. To see this, take expectations of both sides of the structural equation, solve it for $E_t z_{t+1}$, which yields $E_t z_{t+1} = \beta x_t/(1-\phi)$, and substitute this expression back into the original equation.

differences of y and π on the left-hand side, replacing them with the time derivatives and abbreviating $\tilde\beta_{py} = \beta_{py}/(1-\phi_p)$, we get

$$\begin{bmatrix}\dot y\\ \dot\pi\end{bmatrix} = \mathcal{J}\begin{bmatrix}y\\ \pi\end{bmatrix} = \begin{bmatrix}-\beta_{yi}(\alpha_y - \tilde\beta_{py})/(1-\phi_y) & -\alpha_\pi\beta_{yi}/(1-\phi_y)\\ \beta_{py}/(1-\phi_p) & 0\end{bmatrix}\begin{bmatrix}y\\ \pi\end{bmatrix} \qquad (1.47)$$

Again, the private sector on its own with the interest rate fixed at $i = i^o$ is not viable (as a linear system), since the equilibrium of (1.47) is then a saddle point. Just note that this case is captured by setting $\alpha_\pi = -1$, which renders entry j_{12} positive and so leads to det $\mathcal{J} = 0 - j_{12}j_{21} < 0$.

On the other hand, $\alpha_\pi > 0$ yields $j_{12} < 0$ and thus a positive determinant, while $\alpha_y > \beta_{py}/(1-\phi_p)$ takes care of a negative trace of $\mathcal{J}$. It follows that, by adopting the Taylor rule in a reasonable way (i.e. choosing a positive inflation gap coefficient and a sufficiently strong response to the output gap), the central bank can stabilize the economy. It may also be observed that this holds for any degree to which the agents are forward-looking; only the output gap coefficient α_y must sufficiently increase as ϕ_p, the weight in the Phillips curve, increases towards unity.

Convergence of the deterministic economy is ensured, but it takes time after a shock until the two state variables y and π reach their steady-state values. In other words, the model generates inertia in inflation and the output gap. It thus shares this property with the hybrid expectations as they were specified in (1.35), (1.36), whereas the analysis of the reduced-form system (1.47) requires far less effort than that of the four-dimensional system (1.42).

With the neoclassical dating convention, there is also some scope to drop the assumption of perfectly flexible wages and reintroduce the nominal wage adjustments from the New-Keynesian framework of subsection 1.3.2. In contrast to the impossibility result there obtained, monetary policy may now be able to stabilize an economy with uniquely determined time paths. The range of the policy coefficients α_π and α_y achieving this might even be established analytically.

Thus, reconsider the (deterministic) New-Keynesian Phillips curves (1.27) and (1.28) for wage and price inflation (π^w and π^p) and subject them to the neoclassical dating. Setting $\beta = 1$ and assuming that the natural real wage ω^n is time-invariant, the equations become

$$\pi^w_{t+1} = \phi_w E_t \pi^w_{t+1} + (1-\phi_w)\pi^w_t + \beta_{wy}y_t - \beta_{w\omega}(\ln\omega_t - \ln\omega^n) \qquad (1.48)$$

$$\pi^p_{t+1} = \phi_p E_t \pi^p_{t+1} + (1-\phi_p)\pi^p_t + \beta_{py}y_t + \beta_{p\omega}(\ln\omega_t - \ln\omega^n) \qquad (1.49)$$

Besides (1.48), (1.49), the economy is described by the neoclassical-type IS equation (1.43), the Taylor rule (1.25) and a definitional equation determining the changes in the real wage rate. Certainly, wage inflation and the real wage are predetermined too, so that we have four dynamic equations for the four predetermined variables π^w, π^p, y and $\chi = \ln \omega - \ln \omega^o$. To set up the deterministic continuous-time counterpart, solve (1.49) for π^p_{t+1} and substitute this in (1.43). The time derivatives for π^w, π^p, y are then computed in the same way as before, and the one for χ is just eq. (1.32). Together, the following four-dimensional system of ordinary differential equations is obtained:

$$\dot{\pi}^w = (\beta_{wy} y - \beta_{w\omega} \chi)/(1 - \phi_w) \tag{1.50}$$

$$\dot{\pi}^p = (\beta_{py} y + \beta_{p\omega} \chi)/(1 - \phi_p) \tag{1.51}$$

$$\dot{y} = -\beta_{iy}[(\alpha_y - \tilde{\beta}_{py})y + \alpha_\pi \pi - \tilde{\beta}_{p\omega}\chi]/(1 - \phi_y) \tag{1.52}$$

$$\dot{\chi} = \pi^w - \pi^p \tag{1.53}$$

where $\tilde{\beta}_{pa} := \beta_{pa}/(1 - \phi_a)$, $a = y, \omega$ in (1.52). It is instructive to compare this system to the equations (1.29) to (1.32), which approximate the original New-Keynesian model with staggered wages and prices. The difference in the first three constituent equations is plain too see: in all three of them a sign reversal has taken place.

The purely forward-looking New-Keynesian approach has therefore been changed in two fundamental respects. First, the introduction of hybrid expectations with their backward-looking component transforms jump variables into predetermined variables. Second, the neoclassical dating convention relates the same variables and their time derivatives to each other as in the baseline approach, where, however, we find that a negative impact turns into a positive effect and vice versa.

In the remainder of this subsection we examine whether the present system (1.50)–(1.53) differs from the New-Keynesian wage-price baseline model in a third respect as well. Analysis of the New-Keynesian system (1.29)–(1.32) (and other results from the literature) has revealed that there was no way in this model for monetary policy to bring about determinacy. The corresponding question for the predetermined variables in (1.50) to (1.53) is whether, with a suitable choice of the policy coefficients in the Taylor rule, the central bank can ensure the asymptotic stability of the equilibrium point.

A stability analysis of (1.50)–(1.53) has to study the Jacobian matrix of this system. It exhibits the same pattern of entries as the matrix $\mathcal{J}$ in (1.33) from the New-Keynesian economy, except that, as we have just seen, most of the signs are reversed. Although it is a 4×4 matrix, the many zero entries still make it possible to apply the Routh–Hurwitz stability conditions and get out definite results.[35] On the other hand, here we prefer to save ourselves the admittedly tedious calculations and concentrate on the essential distinction from the New-Keynesian case. For this purpose we can content ourselves with a numerical demonstration that the central bank can, in fact, succeed in stabilizing (1.50)–(1.53).

There are not too many parameters to which numerical values are to assigned. From the estimated (purely backward-looking) model by Rudebusch and Svensson (1999), which we shall discuss in chapter 8, section 4, we borrow $\beta_{yi} = 0.09$ and $\beta_{py} = 0.15$. Deliberately, we set β_{wy} slightly higher than β_{py}, choosing $\beta_{wy} = 0.20$. The real wage effect in the two Phillips curve, we feel, should not be very strong, so put $\beta_{p\omega} = \beta_{w\omega} = 0.05$. Since in subsection 1.3.3 we were almost forced to assume $\phi_y > 1/2$, $\phi_p > 1/2$ (in order to have a negative trace for $\mathcal{J}$ in (1.42)), we here choose a lower degree to which the agents are forward-looking; let us say $\phi_y = \phi_p = \phi_w = 0.25$.

It remains to choose two values for the policy coefficients α_π and α_y, compute the eigenvalues of system (1.50)–(1.53) and note whether they imply stability or not. Doing this for a great many combinations of the two parameters, the (α_π, α_y) policy parameter plane is subdivided into two areas: a region containing the pairs α_π, α_y entailing stability, and the parameters in the rest of the plane, which imply instability. The outcome is shown in figure 1.2.[36]

The stability conditions acquired from figure 1.2 appear quite plausible. For every value of α_π that is not too large, sufficiently high values of α_y guarantee stability. On the other hand, for $\alpha_\pi \geq 0.20$ roughly, α_y must not be too large, either. Incidentally, Taylor's proposed coefficients $\alpha_\pi = \alpha_y = 0.50$ are contained within the stability region. Varying the forward-looking weights, it is furthermore found that a greater influence of the forward-looking agents has a certain destabilizing tendency: the stability region shrinks as ϕ_y, ϕ_p, ϕ_w are uniformly increased. The stability region even tends to vanish as the weights approach unity. Non-uniform variations of the three weights (especially ϕ_w) can produce additional interesting effects.[37]

The main point we wish to make, however, is that, in contrast to the purely forward-looking version of the New-Keynesian approach, the present combination of output and wage-price dynamics constitutes a

[35] The pattern of zero entries is not much different from the 4×4 Jacobian matrix to which the Routh–Hurwitz conditions were successfully applied in Franke and Asada (1994, pp. 280ff., 293f.).

[36] Pairs in the dotted area induce local stability; instability prevails outside.

[37] A salient feature of our parameter setting is that, with equal weights ϕ_p and ϕ_w, the coefficient β_{wy} exceeds β_{py}. Stability is still possible if β_{wy} falls short of β_{py}, bur the stability regions have another shape then.

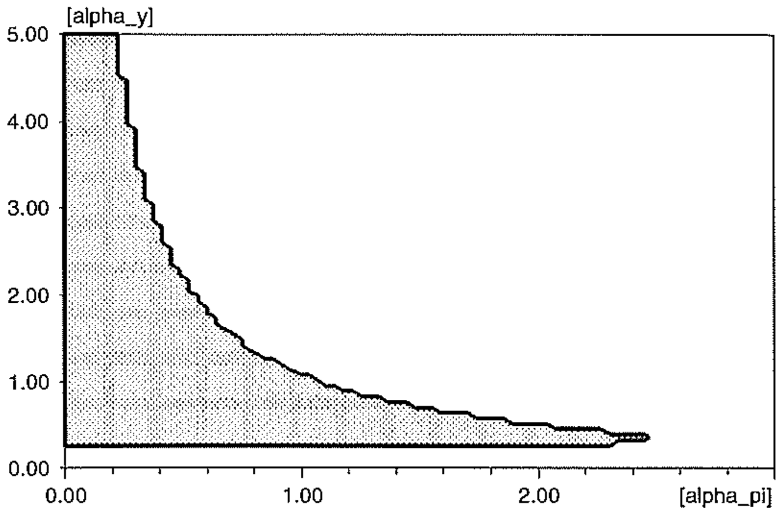

Figure 1.2: The parameter diagram of (α_π, α_y) for system (1.50)–(1.53)

model with a reasonable scope for inertial behaviour and for monetary policy.

1.4 Keynesian DAS-AD dynamics and the wage-price spiral

1.4.1 The D(isequilibrium)AS-AD approach to the wage-price spiral

Our approach to Keynesian wage-price dynamics is rooted in the Neoclassical Synthesis, Stage I; in the version that advances both a Phillips curve for money wages and a Phillips curve for changes in the price level. Originally, marginal pricing still served as a benchmark, but it was no longer required to apply in every instant of time. In other words, firms were allowed to be off their supply curve; hence, the approach can be characterized as a disequilibrium AS approach, designated by the acronym DAS for short.

A simple model of this type was put forward for illustrative purposes in subsection 1.2.2, and in table 1.1 in the same subsection we indicated that it could be seen as a precursor to later, as we called them, 'mature' Keynesian models. In fact, the wage and price Phillips curves that we will widely employ in this book have been developed over the years from these beginnings. Nevertheless, we leave the genesis by which we arrived

at the present specification to one side. After the discussion of the New-Keynesian treatment of wage and price formation and, in particular, the neoclassical dating convention, it is instead expedient to refer to the two New-Keynesian Phillips curves from the preceding subsection. For convenience they are reproduced here:

$$\pi^w_{t+1} = \phi_w E_t \pi^w_{t+1} + (1-\phi_w)\pi^w_t + \beta_{wy} y_t - \beta_{w\omega}(\ln \omega_t - \ln \omega^o) \qquad (1.48)$$

$$\pi^p_{t+1} = \phi_p E_t \pi^p_{t+1} + (1-\phi_p)\pi^p_t + \beta_{py} y_t + \beta_{p\omega}(\ln \omega_t - \ln \omega^o) \qquad (1.49)$$

Our own approach can be presented as starting out from these equations and then modifying them in six respects.

(1) According to eq. (1.48), the anticipation of next period's inflation entering the determination of the change in money wages is the rate of wage inflation itself. In contrast, we hold that price inflation is the relevant term here. That is, we replace $E_t\pi^w_{t+1}$ in (1.48) with $E_t\pi^p_{t+1}$. On the other hand, we continue to assume that these future inflation rates are perfectly foreseen, so that we can directly substitute π^p_{t+1} for $E_t\pi^w_{t+1}$.

(2) Analogous reasoning applies to the price Phillips curve, which means that π^w_{t+1} is substituted for $E_t\pi^p_{t+1}$ in (1.49).

(3) The idea of the entries π^w_t and π^p_t in (1.48), (1.49) was to represent backward-looking behaviour. This was even more clearly expressed in the original hybrid expectations (1.36), where in the price Phillips curve this rate was dated $t-1$. Here we introduce a new variable π^c, which we conceive as a general *inflation climate*, and substitute π^c_t for both (!) π^w_t in (1.48) and π^p_t in (1.49). The climate is predetermined in period t and is later supposed to change between the periods in a way that can be described as gradual adjustments towards the current rate of inflation (a target that is possibly combined with other reference rates of inflation). As the story is currently told, the replacement of π^w_t and π^p_t with π^c_t can be viewed as being motivated by the fact that, in modern terminology, adjustments of this kind are called backward-looking.

(4) Regarding the key terms in the Phillips curve, our perspective is that wage and price inflation are influenced by the demand pressure on the respective markets. The corresponding measure for the labour market is (not the output gap but) the rate of employment e_t – i.e. its deviation from the NAIRU level e^o (which we assume to be exogenously given and fixed).

(5) The output gap (denoted by y) in the price Phillips curve could be retained in this respect. In the models we consider it is, however,

more appropriate to employ the output–capital ratio as a proxy for the demand pressure on the goods market, which we likewise, from now on and in the rest of the book, denote by the letter y. That is, we refer to the deviations of y_t from its equilibrium level y^o. In the small model below it is assumed to be fixed, for simplicity.[38]

(6) Lastly, the role and the signs of the influence of real wages is maintained, except that we drop the logarithm and identify the natural real wage rate ω^n, which in principle may vary, with the fixed value of the real wage ω^o that prevails in the steady state.

In sum, these remarks give rise to the following version of a wage and a price Phillips curve, where, obviously, κ_w and κ_p are the two weights $(0 \le \kappa_w, \kappa_p \le 1)$ corresponding to ϕ_w and ϕ_p in (1.48), (1.49):

$$\pi^w_{t+1} = \kappa_w \pi^p_{t+1} + (1-\kappa_w)\pi^c_t + \beta_{we}(e_t - e^o) - \beta_{w\omega}(\omega_t - \omega^o) \quad (1.54)$$

$$\pi^p_{t+1} = \kappa_p \pi^w_{t+1} + (1-\kappa_p)\pi^c_t + \beta_{py}(y_t - y^o) + \beta_{p\omega}(\omega_t - \omega^o) \quad (1.55)$$

It goes without saying that, as in the New-Keynesian specification (1.48), (1.49) of hybrid expectations under the neoclassical dating convention, the two rates of inflation π^w_t and π^p_t are treated as predetermined. However, because the same variable π^c_t is supposed to enter the price and the wage Philips curve, eqs. (1.54) and (1.55) cannot encompass the former, not even in the most special case when $\kappa_w = \kappa_p = 0$ and the determination of the inflation climate degenerates to identifying it with current inflation. For $\pi^c_t = \pi^w_t$, (1.54) is then identical to (1.48), but not (1.55) with (1.49); and the other way round for $\pi^c_t = \pi^p_t$.

The important thing to note in (1.54), (1.55) is that income distribution is supposed to have a bearing on both the changes in prices and money wages. To be in line with (1.48), (1.49), income distribution is here represented by the real wage rate. Later in the book, from chapter 3 onward, in the context of our reasoning there we will find it more reasonable to employ the wage share for the same purpose. The sign of the impact of real wages is the same as in (1.48), (1.49): negative in the wage Phillips curve and positive in the price Phillips curve.[39]

Since in particular the additional influence of a wage term in the two Phillips curves, with the postulated signs, is not a standard feature in the literature, it should first be checked that the approach of eqs. (1.54) and

[38] Generally also the concept of potential output may be taken as a benchmark, which in models with a neoclassical production function is given by the output level that maximizes short-run profits and so is varying with labour intensity $\ell = L/K$.

[39] We will later also replace the output–capital ratio y with the rate of capacity utilization u. This is, however, rather inessential, since with our specification of the latter y and u will differ only by a proportionality factor.

(1.55) is not outright counter-factual. In Chen et al. (2004) the equations have therefore been subjected to an econometric test, combining them with two adjustment equations for the utilization of capital (u_t, taking the role of y_t, in a dynamic IS equation) and of labour (e_t, being directly coupled with u_t). The inflation climate was simply specified in a backward-looking manner, as a moving average over the past twelve quarters with linearly declining weights. Though these four equations are a relatively small system compared to an unrestricted VAR with twelve lags, it was found that the null hypothesis of imposing these structural restrictions cannot be rejected in favour of the latter. Hence, the small system can also be econometrically defended as one parsimonious representation (among several or many others, of course) of the true data-generating process. The estimation of its wage-price part, on the basis of quarterly data for the US economy from 1965 to 2000, resulted in the following numerical equations, where the role of the real wage ω_t is here supposed to be taken by the wage share, denoted as v_t:[40]

$$\pi^w_{t+1} = 0.65\pi^p_{t+1} + 0.40\pi^c_t + 0.11(e_t - e^o) - 0.09(v_t - v^o)$$

$$\pi^p_{t+1} = 0.35\pi^w_{t+1} + 0.67\pi^c_t + 0.03(u_t - u^o) + 0.07(v_t - v^o)$$

Two features of the estimates are worth pointing out. First, although the coefficients on the first two terms of the equations were unrestricted, they are not only positive but also their sum is not significantly different from one. This justifies their interpretation as weights in setting up a rate of benchmark wage and price inflation, $\kappa_w \pi^p_{t+1} + (1-\kappa_w)\pi^c_t$ and $\kappa_w \pi^w_{t+1} + (1-\kappa_w)\pi^c_t$, respectively. Second, all coefficients come out with the theoretically required signs and are, furthermore, significant. In particular, this holds for the impact of the wage share. Thus these non-standard terms, too, are corroborated by the data. In sum, the estimation lends confidence to the concept of our two Phillips curves in (1.54), (1.55) and their additional reference to a wage term.

So far, the Phillips curves have been considered in their structural form. Here it has to be taken into account that they cannot be directly used to gauge the impact of the gap variables $e_t - e^o$, etc., on the two inflation rates. For example, though not being present in (1.54), the demand pressure on the goods market $y_t - y^o$ still has a bearing on wage inflation, via its influence on π^p_{t+1}, which in turn enters the determination of π^w_{t+1}. These effects can be made explicit, and their relative strength can be assessed, if we compute the reduced form of (1.54), (1.55). To

[40] Actually, in the models from chapter 3 on we ourselves will employ the wage share rather than the real wage.

this end it has to be ruled out that the weights κ_w and κ_p are both unity. Substituting (1.55) in (1.54) and solving the resulting equation for π^w_{t+1}, and correspondingly so to obtain π^p_{t+1}, the reduced-form equations for wage and price inflation are seen to read,

$$\pi^w_{t+1} = \pi^c_t + \kappa[\beta_{we}(e_t - e^o) + \kappa_w\beta_{py}(y_t - y^o) + (\kappa_w\beta_{p\omega} - \beta_{w\omega})(\omega_t - \omega^o)] \tag{1.56}$$

$$\pi^p_{t+1} = \pi^c_t + \kappa[\beta_{py}(y_t - y^o) + \kappa_p\beta_{we}(e_t - e^o) + (\beta_{p\omega} - \kappa_p\beta_{w\omega})(\omega_t - \omega^o)] \tag{1.57}$$

$$\kappa := 1/(1 - \kappa_w\kappa_p)$$

These two Phillips curve relationships are clearly traditional in their dependence on demand pressure and the inflation climate. Regarding the latter, note that it corresponds to what defines an expectations-augmented Phillips curve and is there usually called expected inflation. With a view to its (still outstanding) determination in a backward-looking manner, we have preferred to call it a general inflation climate, which, being in a non-perfect world with less than perfect foresight, we think of as being adopted by boundedly rational agents.

On the other hand, eqs. (1.56) and (1.57) are more advanced than the usual reduced-form Phillips curves, in that there are two separate curves the dependent variables of which are each determined by the conditions on both the labour and the goods market. Apart from that, we have the additional influence of the real wage rate (or the wage share later in the book). Whether eventually π^w_{t+1} and π^p_{t+1} are negatively or positively affected by that variable is contingent on the relative size of the coefficients $\beta_{w\omega}$, $\beta_{p\omega}$, κ_w, κ_p.

Relating (1.56), (1.57) to the New-Keynesian specifications discussed in subsection 1.3.4, it may be observed that no current rate of inflation enters the right-hand side, neither π^w_t nor π^p_t. This means that our approach does not include, even as a special case, the accelerationist-type Phillips curves that result from the neoclassical dating; see eqs. (1.50), (1.51) for the continuous-time formulation in the same subsection. As we have remarked on (1.54), (1.55), under exceptional conditions either the wage or the price Phillips curve might be of an accelerationist type, but not both.

The actual wage-price dynamics could nevertheless come close to what would be generated by accelerationist Phillips curves, depending especially on the adjustment rules that are supposed to govern the changes of the inflation climate. To some extent this issue will be investigated in the calibrations of chapter 5, where a common wage Phillips curve

(1.54) is combined with a simple accelerationist price Phillips curve, on the one hand, and with the price Phillips curve of eq. (1.55) on the other hand. It will there be shown that these two building blocks can produce very similar cyclical features of wages, prices and income distribution.

1.4.2 *Feedback-guided stability analysis: example 1*

Clearly, two separate Phillips curves for wages and prices imply a theory of income distribution, at least as far as real wages are concerned.[41] In this respect it is a great advantage of our approach that, although the inflation climate π^c has an important bearing on the rates of inflation, income distribution remains unaffected by it. Passing over to continuous time and again writing $\hat{w}$ for π^w, $\hat{p}$ for π^p, it is immediately seen from the reduced-form equations (1.56), (1.57) that π^c cancels out in the determination of the changes in the real wage, $\hat{\omega} = \hat{w} - \hat{p}$. Specifically, we get

$$\dot{\omega} = \omega\kappa\{(1 - \kappa_p)[\beta_{we}(e - e^o) - \beta_{w\omega}(\omega - \omega^o)] - (1 - \kappa_w)[\beta_{py}(y - y^o) + \beta_{p\omega}(\omega - \omega^o)]\} \tag{1.58}$$

If the coefficients $\beta_{w\omega}$, $\beta_{p\omega}$ are positive and κ_w, κ_p less than one, eq. (1.58) yields a negative feedback of ω directly on itself – that is, a stabilizing effect. We note that this effect is different from what is implied by the New-Keynesian laws of motion (1.29), (1.30), (1.32) in subsection 1.3.2. Ignoring the output gap for this kind of partial argument, we there obtain a second-order differential equation for $\chi = \ln\omega - \ln\omega^o$, namely $\ddot{\chi} = \dot{\pi}^w - \dot{\pi}^p = (\beta_{w\omega} + \beta_{p\omega})\chi$. Since the two roots of the characteristic polynomial $\lambda^2 + (\beta_{w\omega} + \beta_{p\omega}) = 0$ are purely imaginary, $\mathrm{Re}\,\lambda_{1,2} = 0$, the direct auto-feedback of real wages is neutral.

The traditional argument regarding real wage effects is a bit more indirect. It says that an increase in ω reduces aggregate demand and so output and employment, which in turn lowers price as well as wage inflation. It thus depends on the relative flexibilities of wages and prices whether the real wage rate in the second stage, so to speak, rises or falls. These flexibilities are usually seen to be represented by the slope coefficients in the two Phillips curves, by β_{we} and β_{py} in our notation. Equation (1.58), however, demonstrates that the seemingly innocent weights κ_w and κ_p also take effect.

Reconsidering the discussion of the Neoclassical Synthesis, Stage I, we can build a small model reflecting these stability effects. If we abstract

[41] The wage share is additionally influenced by possible variations of labour productivity over the cycle, which we will account for from chapter 3 on.

from other dynamic feedbacks, they are even decisive. In setting up the IS-LM part of the model, it will be understood that the expected rate of inflation in the specification of the real interest rate is now replaced with the inflation climate, so that the return differential in (1.6) reads $q = r - (i - \pi^c)$. The other equations we here employ are (1.4), (1.5) and (1.15), while the interest rate is determined in the LM equation (1.8) under the additional assumption $\hat{M} = \hat{K}$ for the money supply, or we simply peg it at $i = i^o$. In the former case we have $pyf_m(i) = M/K =$ const. in (1.8), which shows that the interest rate is an increasing function of the output–capital ratio, $i = i(y)$ with $i' > 0$; otherwise $i' = 0$, of course.

Freezing furthermore the inflation climate at its steady-state value $\pi^o = 0$, temporary equilibrium on the goods market is described by the condition $y = (1/s)f_I[y - \omega\ell(y) - \delta - i(y) + \pi^o]$ (recall that $\ell = L/K$ is labour intensity, a rising function $\ell = \ell(y)$ of the output–capital ratio). It has already been observed in subsection 1.2.2 that $\partial(y - \omega\ell(y))/\partial y = 0$ in the steady-state position. This ensures that (at least locally) output is inversely related to the real wage, $y = y(\omega)$ with $y_\omega = \partial y/\partial\omega < 0$.

The employment rate $e = L/L^s = \ell/\ell^s = \ell[y(\omega)]/\ell^s$ is a function of $\ell^s = L^s/K$ and the real wage, and so likewise responds negatively to an increase in ω – i.e. $e = e(\omega, \ell^s)$ with $e_\omega = \partial e/\partial\omega = \ell' y_\omega/\ell^s < 0$. Regarding the denominator of e, we assume that the labour supply grows at a constant rate equal to the real growth rate in long-run equilibrium, g^o. Thus, $\hat{\ell}^s = \hat{L}^s - \hat{K} = g^o - I/K = g^o - f_I$, or, more explicitly,

$$\dot{\ell}^s = \ell^s\{g^o - f_I[y(\omega) - \omega\ell[y(\omega)] - \delta - i[y(\omega)] + \pi^o]\} \tag{1.59}$$

Substituting $y = y(\omega)$ and $e = e(\omega, \ell^s)$ in (1.58) and solving it for the time derivative $\dot{\omega} = \hat{\omega}\cdot\omega$, we have a differential equations system in the two (predetermined, of course) variables ω and ℓ^s. Evaluated at the steady state, the Jacobian is easily computed as

$$\mathcal{J} = \begin{bmatrix} \omega\kappa[-(1-\kappa_p)\beta_{we}|e_\omega| + (1-\kappa_w)\beta_{py}|y_\omega| - \tilde{\beta}_\omega] & -\omega\kappa(1-\kappa_p)\beta_{we}/\ell^s \\ \ell^s f_I'(1 - |y_\omega| i') & 0 \end{bmatrix}$$

$$= \begin{bmatrix} ? & - \\ + & 0 \end{bmatrix}$$

where $\tilde{\beta}_\omega := (1-\kappa_p)\beta_{w\omega} + (1-\kappa_w)\beta_{p\omega} \geq 0$; besides $\kappa_p < 1$, we have in the sign pattern assumed that the interest rate reactions to output changes are not too extreme, so that $i' = di/dy < 1/|y_\omega|$ ($i' = 0$ if i is fixed at i^o).

As det $\mathcal{J} > 0$ under these circumstances, the steady state is stable if and only if trace $\mathcal{J} < 0$ – that is, if entry j_{11} is negative. Hence, stability prevails if the composite term $\tilde{\beta}_\omega$, representing the direct real wage auto-feedbacks, is dominant, or already if $(1-\kappa_p)\beta_{we}$ exceeds $(1-\kappa_w)\beta_{py}$.

In the present setting we are told that higher wage flexibility in the form of a steeper slope β_{we} in the wage Phillips curve is favourable for stability, whereas higher price flexibility in the form of a steeper slope β_{py} in the price Phillips curve tends to be destabilizing. Of course, this can only be a very preliminary message. An immediate objection is that we have limited ourselves to just one channel through which real wages affect aggregate demand, namely its negative impact on investment demand. If differentiated savings propensities of 'capitalists' and workers are allowed for, then rising wages would raise consumption on the part of workers, which could well dominate the other negative effect(s). We would thus obtain a different sign pattern in the Jacobian.

Another aspect is of general significance. It has to be realized that results on the, say, stabilizing effect of a parameter increase may not necessarily carry over to more comprehensive models. The reason is that β_{we}, β_{py}, etc. could also reinforce or weaken a second or third stabilizing or destabilizing mechanism. To anticipate two central results from this book's extensive numerical stability investigation, the characterization (suggested here) of β_{we}, $\beta_{w\omega}$, $\beta_{p\omega}$ as stabilizing and β_{py} as destabilizing may be compared with our final and succinct summary of parameter stability effects in the full Keynes–Metzler–Goodwin model, which is table 7.3 (chapter 7, section 5, subsection 3) for the version with LM curve, and table 9.6 (chapter 9, section 5, subsection 3) for the version with a Taylor-type interest rate reaction function.[42]

Besides, chapter 7, section 3, designs and studies carefully another two-dimensional submodel that concentrates on real wage effects, but discards IS and instead includes elements of the Metzlerian goods market disequilibrium part. The stability results there obtained are then directly confronted with the corresponding findings in the full KMG model. These remarks may give a first impression that we wish to understand basic feedback mechanisms and their consequences for economic stability, whereas we are at the same time aware that we cannot rely too much on them if the theoretical framework is extended. In this way, however, we also learn more about what is important, or peculiar, in the richer model economy.

1.4.3 *Feedback-guided stability analysis: example 2*

Other feedback mechanisms that, besides the real wage effects, will be dealt with at various places in the book are the Keynes effect and the Mundell effect. The Keynes effect is well known for its stabilizing potential, which

[42] The counterpart of coefficient β_{py} is there β_{pu}.

involves the price level and its impact on real balances in the LM curve. As high prices above normal are tantamount to relatively low real balances, they raise the nominal interest rate and diminish investment (and perhaps consumption). Reinforced through the multiplier, this decreases output and finally puts downward pressure on the price level to return to normal.[43]

The Mundell effect works through expected inflation, or, in our framework, the general inflation climate. An increase in π^c reduces the real rate of interest and so increases investment. The induced increase in output raises inflation rates, which, perhaps with some delay, causes the inflation climate to move upward. Taken together, a positive feedback loop comes into being. In particular, the destabilizing effect will be stronger, and therefore tend to outweigh the stabilizing Keynes effect, the more rapidly the inflation climate adjusts to actual inflation.

To identify these effects within a small model in pure form, we cancel the real wage dynamics. For example, if we put $\kappa_w = 1$ and $\beta_{we} = \beta_{wv} = 0$, the real wage in (1.58) remains constant, $\dot{\omega} = 0$. Regarding its level, we assume $\omega = \omega^o$.

To make the role of the price level more pronounced, let the money supply move one to one with real capital, $\hat{M} = \hat{K}$. The LM condition (1.8) then becomes $pyf_m(i) = M/K =$ const., from which it can be immediately inferred that the LM interest rate is an increasing function of both the output–capital ratio and the price level, $i = i(y, p)$ and $i_y > 0$, $i_p > 0$ for its partial derivatives. Inserting this in the IS equilibrium condition, $y = (1/s)f_I[y - \omega^o \ell(y) - \delta - i(y, p) + \pi^c]$, and once again recalling that $\partial(y - \omega\ell(y))/\partial y = 0$ in the steady state, it is readily seen that the IS-LM output–capital ratio is a function of p and the inflation climate π^c, $y = y(p, \pi^c)$, with $y_p = \partial y/\partial p < 0$ and $y_\pi = \partial y/\partial \pi^c > 0$. These signs have just been verbally described as the output link in the chain of the Keynes and Mundell effects, respectively.

Substituting the output function in the continuous-time expression of the reduced-form price Phillips curve (1.57), we obtain

$$\dot{p} = p[\pi^c + \psi(p, \pi^c)], \quad \psi(p, \pi^c) := \kappa\beta_{py}[y(p, \pi^c) - y^o] \tag{1.60}$$

Obviously, $\psi_p = \partial\psi/\partial p < 0$ and $\psi_\pi = \partial\psi/\partial\pi^c > 0$.

Concerning the adjustments of the inflation climate, we assume gradual adjustments of π^c towards current price inflation,

$$\dot{\pi}^c = \beta_\pi(\hat{p} - \pi^c) = \beta_\pi\psi(p, \pi^c) \tag{1.61}$$

[43] In essence, this mechanism has already been pointed out by Keynes in chapter 19 of *The General Theory*, which was devoted to a discussion of 'Changes in money wages'. Indeed, it is the only mechanism that Keynes recognized as being of any significance to the alleged benefits of flexible prices.

where β_π measures the speed of these adjustments. The principle of (1.61) is commonly referred to as 'adaptive expectations', a term that, with our interpretation of π^c as a general climate variable, we consider to be not quite precise. In fact, in a disequilibrium context there are a number of theoretical and empirical arguments pointing out that partial adjustments towards a moving target are not as foolish as they are mostly held to be (see Flaschel et al., 1997, pp. 149–62 or, more extensively, Franke, 1999, for a compilation of such arguments). As we have already referred to Mankiw's (2001) paper on the Phillips curve, it is also interesting to quote his evaluation of adaptive expectations in this context. Afrer noting that these expectations are formed without taking important news into account that is available everywhere, he adds to this observation: 'Yet, the assumption of adaptive expectations is, in essence, what the data are crying out for' (p. C59).

Equations (1.60) and (1.61) make up an autonomous system in the two variables p and π^c, the Jacobian of which is given by

$$\mathcal{J} = \begin{bmatrix} p\psi_p & p(1+\psi_\pi) \\ \beta_\pi\psi_p & \beta_\pi\psi_\pi \end{bmatrix}$$

Entry $j_{11} = -p|\psi_p| < 0$ reflects the stabilizing Keynes effect. Its strength depends, in particular, on the responsiveness of the LM interest rate to price variations, which in combination with the IS equilibrium condition is here integrated in the derivative $\psi_p < 0$. The other diagonal entry $j_{22} = \beta_\pi\psi_\pi > 0$ represents the destabilizing Mundell effect, the strength of which is directly measured by the adjustment speed β_π.

In the present model these two effects are actually decisive for stability, since the determinant of the Jacobian is unambiguously positive, $\det\mathcal{J} = \beta_\pi p|\psi_p| > 0$, so that the stability condition reads $\mathrm{trace}\mathcal{J} = \beta_\pi\psi_\pi - p|\psi_p| < 0$. Perfectly in line with the above argument, stability therefore prevails if the destabilizing Mundell is sufficiently weak in comparison to the stabilizing Keynes effect. The condition is especially satisfied if the adjustment speed β_π is sufficiently low.

At the end of the preceding subsection we did, however, warn against premature conclusions from such small-scale submodels. In fact, in the more advanced models in the book, the wage share terms in the two Phillips curve, which replace the present real wage terms, become relevant and may spoil the nice picture just obtained. There are two reasons. First, although indirectly, these terms are also affected by output variations, the link being labour productivity Y/L, as $wL/pY = (w/p)/(Y/L)$. Second, corresponding to Okun's law, labour productivity (relative to its trend) will be assumed to move procyclically.

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On the whole, it then turns out that the response of inflation to output changes can be different from what holds true in the present limited model. The consequence will even be opposite stability effects: the Mundell effect can be stabilizing and the Keynes effect can be destabilizing. The conditions for and the likelihood of this to happen are discussed in great detail in chapter 3, section 8, and chapter 7, section 4, subsections 2 and 3. Again, we may emphasize that the results at which we will arrive in these more elaborated models can be better assessed against the background of the traditional and elementary feedback mechanisms here expounded.

Lastly, it should be considered what becomes of the Keynes effect in a modern framework (Part III of the book) that dispenses with the traditional determination of the interest rate through the LM curve and instead employs the concept of an interest rate reaction function. In a strict sense, there is then no more room for this effect. Alternatively, the notion of the Keynes effect might be extended: first, in that it relates to the rate of inflation rather than the price level; and, second, in that it is now directly and intentionally the central bank, and not 'the money market', that increases the interest rate in response to rising inflation.

1.4.4 D(isequilibrium)AS-D(isequilibrium)AD modelling

The notion of temporary goods market equilibrium is a useful tool to keep the dimensionality of models low. It comes with an assumption, however, that, at least in more ambitious models, is not unproblematic. The assumption, of course, is that of a downward-sloping IS curve. It is usually validated by the practically equivalent assumption of ultra-short-run stability of an underlying quantity adjustment process, which allows for the inequality of demand and supply, and for firms correspondingly changing their production levels. These activities are supposed to take place at the beginning of a period, within virtually no time at all and involving no inventory changes when output and demand are not matching. The process must therefore be viewed as a hypothetical, tâtonnement-like adjustment process.

Specifically, the assumption corresponds to Keynes' requirement that the marginal propensity to consume be less than unity. More generally, aggregate demand D may positively depend on m variables x^k, $D = D(x^1, \ldots, x^m)$, which in their turn are (directly or indirectly) increasing functions of output, $x^k = x^k(Y)$. Regarding $D = D[x^1(Y), \ldots, x^m(Y)]$, ultra-short-run stability then demands that, as Y rises, D rises less; formally, $\sum_k (\partial D/\partial x^k)(dx^k/dY) < 1$.

So, the first problem with IS equilibrium is the justifying background story of the ultra-short-run quantity adjustment process, which, when one comes to think about it, is conceptually not very convincing. On the other hand, it has become a convention and might be accepted as just that, especially when it serves to make a model analytically tractable.

The other problem is the stability condition itself. Leaving the realm of general mathematical analysis and passing over to concrete numerical issues, it may be found that the required boundedness of the term(s) $\partial D/\partial x^k$ is in fact too restrictive. One point is that a low responsiveness may imply unpleasant properties in numerical simulations. As a matter of fact, in Part I of the present book, which still works with the notion of IS-LM, our simulations of the cyclical dynamics we are interested in typically yield cycles with periods that are much too long to permit the interpretation of business cycles (see chapter 2, section 8, and chapter 3, section 10). Our view is that stronger output reactions would shorten the expansion and contraction phases, and that these faster reactions should, in the first instance, be brought about by a higher responsiveness of investment to the return differential (a higher coefficient f_I' in the notation above). Unfortunately, this is precluded by the ultra-short-run stability assumption, which we therefore experience as a straitjacket for this approach.

Other models may have problems with other implications of the boundedness requirement. It is already the case that a back-of-the-envelope calculation can often show that, a priori, the admitted order of magnitude is implausibly low. Generally, we conclude that as Keynesian models, or the topics that are studied by numerical methods, become more detailed and more ambitious it becomes increasingly desirable to get rid of the IS device. In other words, a disequilibrium AD part of such models becomes desirable.

Abandoning the IS concept means allowing for goods market disequilibrium to prevail over one or several short periods. Recognizing that goods are non-perishable and that rationing is not a universal phenomenon in the economy, the imbalances of output and demand have to be buffered by inventories. So an inventory dynamics is added to the model. On the other hand, production by firms adjusts gradually now, not instantaneously, to demand. For these decisions firms have to form sales expectations, and they have to take into account the deviations of the actual stock of inventories from an optimal or desired level. A new model building block incorporating these features will be designed later in chapter 4, section 2, subsection 2, and will then be underlying in the rest of the book, Parts II and III.

According to the remarks above, and in extension of the title of section 1.4, the modelling framework that thus emerges can be classified as a DAS-DAD approach to Keynesian modelling, where the additional letter 'D' stands for 'disequilibrium'. Alluding to the expression used in table 1.1 (subsection 1.2.2) on the four variants of the Neoclassical Synthesis, Stage I, this will then really be a 'mature' Keynesian model.

1.5 Plan of the book

1.5.1 Part I: Textbook Approaches

Part I of the book begins, in chapter 2, by reconsidering (and slightly extending) Sargent's (1979, 1987) textbook model of the conventional AS-AD growth dynamics, which integrates into a consistent whole what has been – and still is – regarded by many economists as being representative of Keynesian macroeconomic theory. The key features of this model are: savings and investment are independent decisions; an interest-bearing financial asset is explicitly considered; the rate of monetary expansion is a control variable; goods and financial markets are in continuous temporary equilibrium; employment of labour is determined by output meeting aggregate demand and so will generally differ from the level of labour supply; firms are nevertheless always on their supply curve – that is, the marginal productivity principle for labour applies and serves to determine the price level and, a fortiori, the rate of inflation; the rise of nominal wages is governed by an expectations-augmented Phillips curve; and inflationary expectations are formed by a combination of adaptive and regressive expectations.

The analysis of this chapter completes Sargent's rather sketchy treatment of the dynamic system thus defined. The ensuing study of the local dynamics, in particular, overturns Sargent's general optimism concerning the stability of the steady-state growth path. Moreover, singling out three basic feedback mechanisms and investigating their relative strength by means of numerical simulations, the system's potential for cyclical dynamics is demonstrated.

From chapter 3 on, the book dispenses with the marginal productivity principle for labour because of its counter-factual implications in a business cycle context. An alternative representation of the production technology allows for variations in capacity utilization and for procyclical fluctuations in (detrended) labour productivity. While, in the short term, firms produce to satisfy current demand, prices remain invariant and are changed only between these periods. Their adjustments are described by a second Phillips curve relationship. A novel feature here is that price inflation reacts not only to a measure of demand pressure but also to deviations of the present markup on average cost from a target markup rate. The influence of this second factor, which involves the wage share and later turns out to be of some consequence for the usual (Keynes and Mundell) effects on stability, is justified a priori on theoretical grounds as well as by referring to the cyclical implications for the real wage dynamics.

After a conceptual discussion and the mathematical investigation of local stability conditions, a numerical stability analysis is performed, which is based on a partial (here still back-of-the-envelope) calibration of the model's key parameters. A sensitivity analysis shows that, despite being only three-dimensional, the model is already so complex that a number of important reaction coefficients cannot always be unambiguously identified as being either stabilizing or destabilizing. A more careful analysis of the basic feedback loops also reveals that the effects more or less known from the literature do not suffice to explain stability or instability; there are more circuitous mechanisms involving certain cross-effects of the state variables that can play a major role in this respect. In these interactions one can also identify the forces that may bring an economic expansion to an end, so that typically the system exhibits cyclical behaviour. Lastly, a global simulation run sketches the resulting comovements of the key economic variables.

1.5.2 Part II: Analytical Framework: Theory and Evidence

Part II of the book is an extensive study of our Keynes–Metzler–Goodwin model. As is introduced in chapter 4, the model emerges from that presented in chapter 3 with the addition of some specification details; in particular, it also includes an influence of the wage share in the wage Phillips curve. The major conceptual innovation, however, is that now goods market disequilibrium is allowed for. The rationale for this extension is based on certain theoretical reasoning and the fact that the IS temporary equilibrium design in the previous chapters puts severe restrictions on some numerical coefficients. The demand for finished goods is assumed to be satisfied from current production and existing inventories. The latter means that the model also has to keep track of the accumulation of stocks, and that firms' decisions about the level of production and inventory investment have to be newly specified (which constitutes the 'Metzlerian' component of the model). Apart from that, special care is taken of the model's accounting consistency in the macro context. Although in other respects the modelling is rather parsimonious, the reduced form of the dynamic system is already six-dimensional.

It may also be mentioned that the LM part and the assumption of a constant growth rate of the exogenous money supply is maintained throughout Part II. This conception has, in the meantime, become somewhat dated, and it will be abandoned in Part III. Our reasons for not immediately giving it up are explained in the next subsection.

The first achievement of the investigation of the KMG model is that a mathematical analysis can still set up economically meaningful conditions for local stability. The method of proof rests on a general principle, which begins with a suitable 3×3 submatrix of the system's Jacobian matrix and then constructs a cascade of stable submatrices of increasing order. The method may also have some interest beyond the present context, as it may equally be applied to other, and perhaps even more encompassing, disequilibrium models. In addition, it can be proved that if the system loses stability upon variations of a parameter then this occurs via a Hopf bifurcation. We take this result as an indication of the general potential of the KMG model for cyclical behaviour. A global analysis of this phenomenon, however, again requires a numerical approach, to which the ensuing chapters are devoted.

Chapter 5 concentrates on numerical simulations of the wage-price dynamics. Within a business cycle context, three alternative theories of inflation are combined with the extended money wage Phillips curve already mentioned and with adjustments of inflationary expectations. The first approach, almost directly, amounts to countercyclical motions of the price level; the second utilizes the extended price Phillips curve of the KMG model; the third considers formalized adjustments of a variable markup on unit labour costs. Assuming exogenous stylized sine wave oscillations of capacity utilization and the capital growth rate as the driving force, we are here interested in the cyclical features that the three modules imply for, in particular, the motions of the real wage rate, the wage share and the (detrended) price level. The corresponding cyclical statistics should come close to the stylized facts of the business cycle – that is, to the leads, lags and amplitudes of these variables that we gather from the empirical data of the US economy.

We find that in all three model variants this goal can be achieved by an appropriate choice of the numerical parameters. Hence, all three of them are suitable candidates for being incorporated into more comprehensive models of the business cycle.[44]

Chapter 6 combines the wage-price module of the KMG model with the other components of that model. Regarding their calibration, it is very

[44] In a comparison of the cyclical properties, the variable markup approach could be said to have a slight edge over the other two models, but this marginal advantage may easily be offset by arguments put forward by other model builders.

helpful that, as long as the motion of utilization and the capital growth rates are treated as exogenous, the former model part is independent of the dynamics of the latter (but not vice versa). The numerical values of the second module of chapter 5 can be taken over unaltered. On this basis, the rest of the KMG model is calibrated to the empirically observed fluctuations of the variables that are determined here. This completes the first stage of our calibration study, which also requires most of the effort. In the second stage, the sine wave motions of our two exogenous variables are replaced with their noisier empirical fluctuations. It turns out, nevertheless, that the main cyclical characteristics of the model variables are preserved, which confirms the usefulness of the stage 1 approach.

Finally, in stage 3, capacity utilization and the capital growth rate are endogenized, to which end the investment function is now included in the model. At medium levels of utilization, the influence of its two determinants (utilization and the return differential, profit rate minus real interest rate) is specified in a linear manner, whereas at higher rates of under- or overutilization the weights of the two determinants are plausibly shifting. This, on the whole, nonlinear function is numerically described by four parameters, and we are able to find values for them that achieve all that we want – not perfectly so, but to a satisfactory degree: (1) the model generates endogenous cycles around an unstable equilibrium that are bounded and persistent; in fact, the trajectories converge towards a globally unique limit cycle; (2) the oscillations occur at a business cycle frequency (a period of around eight years); (3) the variability and comovements of the variables are similar to those in stages 1 and 2.

The whole set of these numerical parameters constitutes our base scenario. Although, beginning with the fact that we are concerned with a deterministic economy, our calibration procedure is different from the methods of the real business cycle school, the results concerning a satisfactory match of the cyclical characteristics of the data can absolutely stand comparison with what has been obtained by this competitive equilibrium approach to business cycle modelling.

Chapter 7 performs a sensitivity analysis in a narrow and a wider sense. To begin with the latter, three two- or three-dimensional submodels are considered and their stability properties are related to those of the full KMG model. These submodels focus on the Metzlerian inventory dynamics, the wage-price dynamics, and the dynamics of the nominal variables. We identify the basic feedback mechanisms and examine their bearing on stability and instability in the submodel and then in the six-dimensional KMG model. At a general level, our findings here warn against premature stability conclusions from the low-dimensional models; not always, but occasionally, they may not carry over to the full model, even qualitatively. A more specific feature to which we should draw attention is that our calibration implies a negative output–inflation nexus, which is at first sight counter-intuitive. A more careful analysis seeks to reconcile it with intuition, or the habitual way of thinking.

The sensitivity analysis in a narrow sense investigates the impact of parameter variations on local stability. In order to provide a definite and concise message, our final aim is to classify thirteen central reaction coefficients or adjustment speeds as either stabilizing, destabilizing or ambiguous. Lastly, the effects of parameter changes on some selected topics of the global dynamics are looked into.

1.5.3 Part III: Monetary Policy

The earlier chapters assume a neutral monetary policy, in the form of a constant growth rate of the money supply. In modern macroeconomic modelling, however, central bank behaviour is increasingly described by an interest rate reaction function. To incorporate these recent developments, Part III reverses the causality of money and the rate of interest, and specifies a Taylor rule with interest rate smoothing. Accordingly, the interest rate undergoes partial adjustments in response to deviations of inflation and capacity utilization from their target values. Adding in this way the Taylor rule to the KMG model makes it a Keynes–Metzler–Goodwin–Taylor, or KMGT, model.

A Taylor rule in some form or another is certainly an appropriate modelling tool for the sake of realism. It might therefore be asked why it has not already been introduced in Part II of the book. We are well aware that the assumption of an exogenously growing money supply that, via LM, determines the rate of interest can be regarded as counterfactual. We have, nevertheless, maintained it in KMG for the following reason. A constant money growth rate is not only of interest as a historical (Friedmanian) recommendation to policymakers, it is also a most natural specification of a neutral monetary policy (that's why it was recommended, of course). Since the KMG model treats government spending and tax collections in a similar vein, the government sector as a whole, while being present in the model, behaves in a neutral manner. It is only the agents in the private sector that react to disequilibrium, so the KMG model is seen to study the private sector in a kind of vacuum. It makes sense to begin with such a theoretical construct in order to get an understanding of the basic stabilizing and destabilizing forces in the economy.

Moreover, if we subsequently reconsider the same effects, their strength and their direction, in the KMGT model and compare them to the effects in the KMG model, we get a better understanding of the general validity of these effects, on the one hand, and of the scope for an active monetary policy in the form of Taylor rules on the other.

After discussing the concept and estimation results of the Taylor rule, chapter 8 first seeks to reveal its logic and its stabilizing potential in a more elementary setting, with and without interest rate smoothing. We therefore temporarily leave the KMG framework altogether and put forward four low-dimensional prototype models in continuous time, consisting of an IS relationship, an accelerationist price Phillips curve and a version of a Taylor rule. Subsequently, an estimated quarterly 'backward-looking' model is studied. Originally put forward by Rudebusch and Svensson, it is well known in the literature and can be viewed as a discrete-time extension of the fourth of the prototype models with various lags. As they should be, the results from a numerical stability analysis of this ten-dimensional system are very compatible with the stability properties of the prototype models. As the Rudebusch–Svensson model is linear and the estimated parameters imply stability, demand and supply shocks serve to keep the economy going. For later comparison with our KMGT model, we note the most important dynamic properties, including the features of an impulse-response function.

An appendix sketches the kinds of problems that arise if such a backward-looking model is turned into a (hybrid) New-Keynesian 'forward-looking' model with the accompanying rational expectations hypothesis. Besides a compact introduction as to how linear rational expectations models can be treated, the appendix can be viewed as substantiating our evaluation of the New-Keynesian models in the methodological subsection 1.1.1, to the effect that 'the nature of the solution procedures ... makes it very difficult, if not impossible, to understand the dynamic linkages and feedbacks' in the economy.

Chapter 9 returns to KMG and incorporates the Taylor rule into it. The mathematical analysis in the KMGT model established in this way is more limited than in the KMG model. In particular, we find that a loss of stability as a parameter varies is no longer necessarily associated with a Hopf bifurcation.

For the numerical analysis the estimated policy coefficients of the Rudebusch–Svensson model for the Taylor rule are adopted (suitably adjusted to our continuous-time version). The numerical parameters from the calibration of the KMG model are maintained except for the coefficients of the nonlinear investment function (and one other parameter). Resetting them in an appropriate way makes the steady state unstable, and the system again generates endogenous, bounded and persistent oscillations with very similar cyclical statistics as in the KMG base scenario. In addition, we also consider an alternative set of investment coefficients that stabilize the steady state. Augmenting this system by demand and supply shocks comparable to those in the Rudebusch–Svensson model, the KMGT model can be more directly related to the Rudebusch–Svensson model and its dynamic properties. The richer structure of KMGT indeed pays off, in that some of its features prove to be more satisfactory than in that reference model.

Incidentally, this section also demonstrates that cyclical behaviour in KMG and KMGT is not necessarily dependent on an unstable equilibrium and extrinsic nonlinearities in some behavioural function(s). It can be generated (and match the data) just as well as in the orthodox business cycle models by employing the Frisch paradigm – i.e. imposing random shocks on an otherwise stable system.

After thus calibrating the KMGT model, we study the effect of changes in the three policy coefficients of the Taylor rule, in both the stable and unstable cases. Of course, we are also interested in the stability effects of the behavioural parameters of the private sector. In the end, we again arrive at a succinct characterization of these parameters as stabilizing, destabilizing or ambiguous. Finally, they are compared to the same characterization for the KMG model.

This brief overview of KMG and KMGT has summarized the positive results we reach. We will not conceal the fact that the properties of our model, even if the present framework is basically accepted, are not in every respect perfect, which is not too surprising since the dynamic feedbacks it takes into account are still limited. In particular, we would have liked more robustness in the variations of two or three parameters. To uncover more about these imperfections, however, the reader will have to investigate the fine detail of our analysis.

Part I

Textbook Approaches