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Accuracy analysis of structure with nearby interfaces within XFEM

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This paper presents the fundamental principle of the extended finite element method (XFEM) for electromagnetic field analysis. The accuracy analysis of structure with nearby interfaces within XFEM is presented. A numerical example applied to the parallel plate electrodes in 1-D static electric field is provided. Two types of meshing are used to analyse the accuracy of the meshing where the support of the enriched node are cut by more than one interface. [http://dx.doi.org/10.1063/1.4974983]

I. INTRODUCTION

A large number of applications can be seen that the quantities of electromagnetic field change rapidly over length scales which are small in comparison to the solution domain. Such examples include the high and steep electric field distribution at the slot-opening of stator of an electrical machine, the current and electric field distribution within a thin piece of a semiconducting material, such as SiC and ZnO, the exponential current distribution due to the skin-effect in a solid conductor subject to a high frequency magnetic field,[1] and the current and diamagnetic characteristics in thin superconducting tapes,[2] and the electric field very closely to the sharp tip of conductor, etc. To model such phenomena, the solutions typically involve discontinuities, singularities, high derivatives, or other non-smooth properties.

In past decades, the extended finite element method (XFEM), which was firstly proposed by Belytschko,[3] provides a mesh-independent approximation for non-smooth problems. Aiming to the approximation of non-smooth solutions, the traditional approach is to employ the polynomial approximation, which depends on meshes that conform to discontinuities and are refined near singularities and high gradients.[4] However, in extended finite element method, the strategy is to enrich a polynomial approximation space such that the non-smooth solutions can be modeled independent of the mesh. The enrichment is realized by appending special shape functions to traditional polynomial approximation. The enrichment method is based on Partition of Unity (PU) concept, including of the partition of unity method (PUM),[5] the generalized finite element method (GFEM),[6] and XFEM.[7] According to the GFEM, a local function, which is not necessarily polynomial, matches the assumed character of the solution and thus ensure good local approximation.[8] C. Lu and B. Shanker[9] analyze the high frequency electromagnetic problem by using GFEM. In XFEM, a local enrichment function, described as a discontinuous shape functions, is adding to the classical FEM through a partition of unity method.

XFEM provides a mesh-independent approximation for non-smooth problems. In the XFEM, the interface between materials is not aligned with the edges of finite elements. Thus, the accuracy description of interface location in the finite element domain is necessary for the enrichment approximation in the field analysis.

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XFEM can be very effective for the case when the support of each enrichment node is cut by only one interface. However, when the support of one enrichment node contains more interfaces due to the small sizes of media, such as the laminated cores in transformers and motors and the superconducting layers in HTS cables, it would encounter many numerical difficulties.[10]

II. EXTENDED FINITE ELEMENT METHOD

The classical FEM depends on the construction of meshes aligning with the interfaces and boundaries. The meshes are refined near domains which possess high variant field over very small space, discontinuity across an interface, etc. The accuracy is improved for smooth solutions, such as super convergent patch recovery. The path chosen in the XFEM is to enrich the approximation space of the FEM such that these non-smooth solution properties are accounted for correctly and independent of the mesh.

A. Interface description by using level set method

In the XFEM, the interface between materials is not aligned with the edges of finite elements. Thus, the accuracy description of interface location in the finite element domain is necessary for the enrichment approximation in the field analysis. The level sets are proven that it is not only useful for the interface description but also may be facilitated the enrichment function construction.

The level sets can provide smoother optimal boundaries and material interfaces for topology optimization. In this paper, level sets are introduced for the description of interface for the enrichment approximation.

The level set is to represent design domains and the material boundaries by continuum level set function. Comparison with the parameterization, the level set is given as an implicit equation of a high dimension function.[11,12] By solving the level set function, interface position is obtained.

A given domain Ω, which is shown in Fig. 1. This domain is composed of regions Ω_1 and Ω_2, $\Omega = \Omega_1 \bigcup \Omega_2$, the interface Γ, $\Gamma = \Omega_1 \bigcap \Omega_2$, the implicit level set function $\phi(x)$ is defined as

$$\begin{cases} \phi(x) > 0 & \text{in } \Omega_1 \\ \phi(x) = 0 & \text{on } \Gamma \\ \phi(x) < 0 & \text{in } \Omega_2 \end{cases} \tag{1}$$

where the zero level set $\phi(x) = 0$ describes the interface Γ.

The distance function may be defined as a typical level set function as

$$\phi(x) = \min \|x - x^*\| \tag{2}$$

where x^* is the closest point on the interface to the point x.

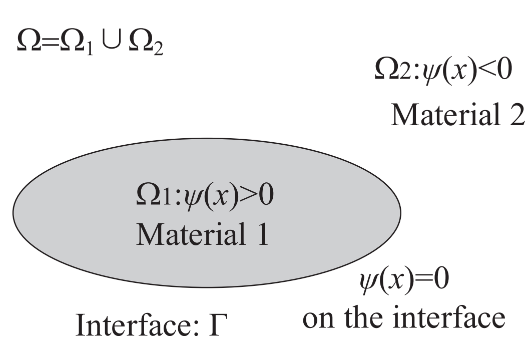


FIG. 1. The interface representation with level set function.

B. Enrichment function

In the discretized domain, I is the set of all nodes, I^* is the set of the enriched nodes, $I^* \in I$. The approximation of a potential function $u(x)$ may be defined as[5]

$$u^h(x) = \sum_{i \in I} N_i(x)\, u_i + \sum_{i \in I^*} N_i^*(x) \cdot [\psi(x)]\, a_i \tag{3}$$

where the first term on the right-hand side is the standard FE approximation, the second the enrichment, which extends the standard FE approximation, i the index number of FE nodes contained in set I, or the enriched nodes contained in I^*, which is a subset of I. N_i and $N_i{}^*$ are standard FE shape functions, in general, $N_i = N_i^*$. The coefficients u_i belong to the standard FE part and a_i are additional nodal unknowns. The function $\psi(x)$ is called an enrichment function according to special and detailed knowledge for the solution problem. The products $Ni\,(x){\cdot}\psi(x)$ are local enrichment functions because their supports coincide with the supports of typical FE shape functions, leading to sparsity in the discrete equations.

The enrichment function for weak discontinuity may be chosen as follows.

$$\psi(x) = \sum N_I\, |\phi_I| - \left|\sum N_I \phi_I\right| \tag{4}$$

III. NUMERICAL EXAMPLE

Fig. 2 shows a parallel plate electrode containing three dielectrics. The distances d_1, d_2, and d_3 of three dielectrics are 0.0489 m, 0.0012 m and 0.0489 m. The middle one may be regarded as a thin layer because the width of middle one is 2.5% of that of the others. The permittivity of three dielectrics are $\varepsilon_1 = 10\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$, and $\varepsilon_1 = 10\varepsilon_0$, respectively. The imposed voltage U_0 is 10 V.

In this example, two types of meshes shown in Fig. 3 and Fig. 4 are used, respectively. In Fig. 3, there are 11 nodes and 10 elements. The enriched nodes are node 5, 6, and 7. The support of node 6 is cut by two interfaces. In Fig. 4, there are 13 nodes and 12 elements. The enriched nodes are node 5, 6, 8, and 9, whose supports are cut by only one interface.

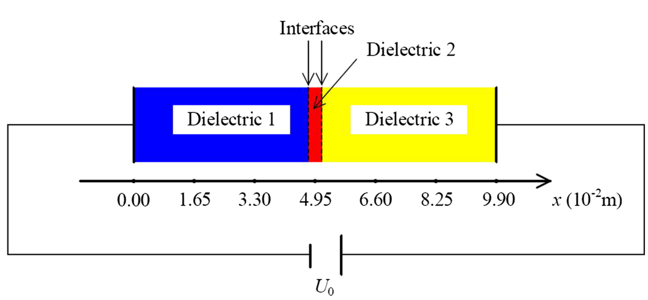


FIG. 2. Parallel plate electrode containing three dielectrics.

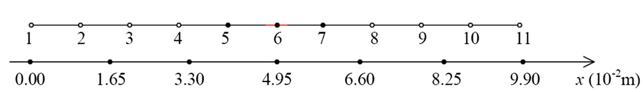


FIG. 3. Mesh 1 with 11 nodes and 10 elements.

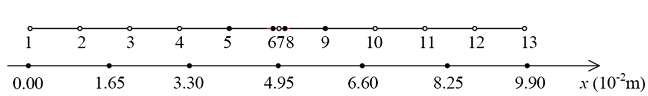
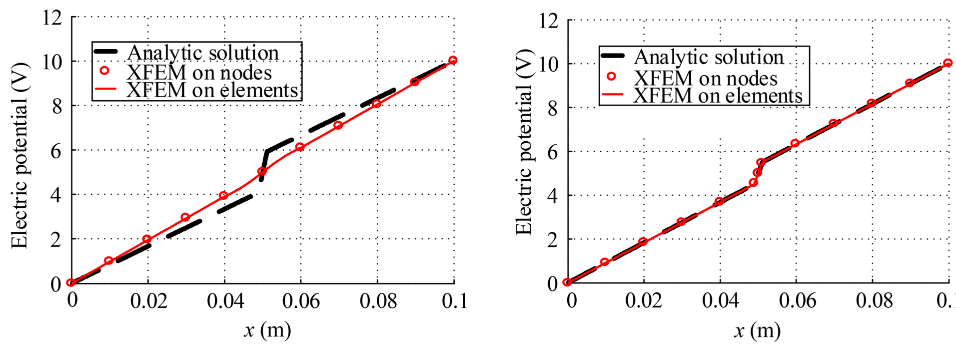


FIG. 4. Mesh 2 with 13 nodes and 12 elements.



(a) Mesh 1 (b) Mesh 2

FIG. 5. Electric potential distribution.

In this example, two interfaces between different materials can be described by using the following global level set function.

$$\phi(x) = \begin{cases} x - 0.0489, & (x \le 0.0495) \\ 0.0511 - x, & (x > 0.0495) \end{cases} \tag{5}$$

where $\phi(x) = 0$ means x on the interfaces between two dielectrics.

The electric potential distrubutions of the parallel plate electrode with two types of meshes are shown in Fig. 5(a) and Fig. 5(b). Compared with mesh 1 where the support of enriched node 6 is cut by two interfaces, the mesh 2 where the support of each enriched node cut by only one interface has the higher accuracy. The global level set function may not give the accurate description for the multi-interfaces.

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