Adaptive Sampling for Spatial Prediction in Environmental Monitoring using Wireless Sensor Networks: A Review

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***Abstract*—The paper presents a review of the spatial prediction problem in the environmental monitoring applications by utilizing stationary and mobile robotic wireless sensor networks. First, the problem of selecting the best subset of stationary wireless sensors monitoring environmental phenomena in terms of sensing quality is surveyed. Then, predictive inference approaches and sampling algorithms for mobile sensing agents to optimally observe spatially physical processes in the existing works are analysed.**

I. Introduction

Environmental issues such as pollution of land, water and air, drastic climate change, natural disasters, and resource depletion have topped the agenda in recent years. These are crucial not only to governments but also to environmental scientists. Therefore, it would be prudent to monitor spatially correlated environmental phenomena so as to ameliorate the knowledge and understanding of their economic, environmental, and health impacts and implications. For instance, determining distributional patterns of ecological phenomena can be utilized to design resource-specific exploitation plans [1]. Better understanding of physics underlying occurrence process of earthquakes [2] can generate early vital hazard warnings to society. Results from observing sulphur dioxide SO_2 in the air [3], which may affect the respiratory system, should be given to alert community council to reduce burning of sulphur-containing fuels in factories. In the agricultural field, monitoring nitrogen density in the soil can be used to regulate farm inputs that leads to mitigation of environmental pollution due to over-application of nitrogen fertilizer [4]. In addition to concerns mentioned above, there is required ubiquitous observations of temperature, humidity, rainfall, soil ingredients, and mono-nitrogen oxides NO_x in natural and built-up habitats [5], [6]. Nonetheless, since environmental measurements are a single scalar quantity that is only locally valid, it is required to make predictions about the process at unmeasured locations by using observations [7]–[9]. Therefore, there are currently strong motivations to monitor, model and predict the environmental field of interest that is often represented as complex phenomena. More particularly, if sensing equipment is effectively used in monitoring, modelling and predicting the spatial phenomena, which is referred to as adaptive sampling, results from visually representing the physical field are useful in making decisions regarding environmental issues.

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The typical task of wireless sensor networks (WSN) [10] consists of gathering measurements of a spatial field over a region of interest. However, for instance in a stationary wireless sensor network (SWSN), multiple wireless sensor nodes co-located within the vicinity of a phenomenon in a dense SWSN may generate similar data samples. This over-sampling problem has the potential to cause a sizeable redundancy in sensed data, data collection and analysis of long-term monitoring to be very expensive. Consequently, it is crucial to select the most informative subset of stationary wireless sensor nodes out of all potential ones, which should participate in the sensing task. The selection procedure is known as a *sensor selection* problem in which the resulting prediction is required to conform requirement of the highly sensing quality in realistic applications.

Nevertheless, if the networks of stationary wireless sensors are deployed in a changing environment, the stationary wireless sensor locations selected by a sensor selection algorithm are no longer the most informative over time. Hence, a wireless sensor network incorporating mobile robotic platforms is desirable. With a set of networked mobile sensor nodes, the mobile robotic wireless sensor networks (MRWSNs) are capable of providing services required not only for monitoring but also for exploring the environment. Effectively utilizing the MRWSNs to observe and predict the environmental fields is widely considered as a *sensor placement* problem.

This paper is to summarize all approaches in literature proposed to address the problems of sensor selection and sensor placement.

II. Sensor Selection in SWSNs

A. Criteria for Sensor Selection

In statistics, selecting observations has been considered as an experimental design problem [11]–[14]. The design objective is to derive the deployment of sensing devices by the use of model uncertainty, which could be formulated by complicated statistical techniques. The optimality criteria were constructed based upon the properties of the inverse moment matrix. For instance, D-optimality considers the determinant

[15], A-optimality examines the trace and E-optimality calculates the maximum eigenvalue [12].

Recently, research attention to the sensor selection has concentrated on selecting observations in order to maximize the quality of parameter estimation [16]–[18] with a special focus on linear models that are often coupled with a stochastic measurement error term. The sensor selection metric can be formulated in a Bayesian framework [19] in which it is supposed to have the knowledge prior to carrying out the experiments. In equivalent words, combining the prior probability distribution of the parameter space with the observations, the design criteria can be derived. The sensor selection criteria are also defined based upon scalar functions of the Fisher information matrix or the Bayesian Fisher information matrix [18]. However, in the context of spatial prediction, the design objective frequently concerned is the quality of sensing, which is described as the accuracy of prediction or the uncertainty at unobserved locations of interest, after the observations are made. This requirement has been utilized to develop information-theoretic criteria [20]–[24].

B. Algorithms for Sensor Selection

In terms of sensor selection algorithms, one can simply process all direct enumerations of $\binom{n}{k}$ possible choices and pick the best subset of k sensors out of n potential ones having the minimal prediction error. It can be seen that this straightforward approach has practical implications depending on the values of n and k and thus motivates more structured methods. In [25], the global optimization techniques such as branch and bound were employed to exactly solve this problem. Nevertheless, since the sensor selection problem, which can be viewed as a combinatorial optimization problem, is NP-hard [26], [27], these accurate approaches are often computationally intensive [28], even with modest values of n and k, and not attractive in real world solutions.

In an effort to improve the model parameter estimation, there have been some interesting methods proposed. For instance, in [16], Joshi *et al.* proposed the heuristic method based on convex optimization [11] for the sensor selection problem. The heuristic approach in [16] utilizes a relaxation technique to convert a discrete optimization problem of sensor selection into a continuous optimization problem. Gupta *et al.* [29] in their work represented a stochastic sensor selection algorithm that selects sensor locations randomly by the use of a probability distribution. Maximum information in the estimation of the state variables can be obtained by a mixed-integer semi-definite program approach [22]. In [17], a binary particle swarm optimization technique was employed to choose a subset of sensors so that the error in parameter estimation is minimized.

Based on Bayesian experimental design, the information-theoretic approaches such as entropy [30] or mutual information [31], [32] were proposed to consider the prediction uncertainty of the random variables at unobserved locations in space. The greedy heuristic algorithms based on these information-theoretic models together with Gaussian processes proposed by Cressie [33] can obtain near-optimal solutions for the sensor selection problem. These algorithms were demonstrated in the works [7], [20], [26], [34], [35]. The premise behind the entropy approach is to minimize the uncertainty of conditional entropy of unobserved locations, given observations. Under Gaussian assumption, Ko *et al.* [26] proposed a greedy suboptimal algorithm by reorganizing the maximization of joint entropy of a chosen set as maximizing the determinant of the covariance matrix of random variables at chosen locations. However, as shown in [36], the entropy method tends to pick locations along the border of interested space causing sensed information waste. To address the drawbacks of the entropy approach, the work in [20] and our previous works in [21], [37] proposed a new method based on the mutual information. In this method a subset k from potential n sensor locations is selected such that the mutual information between the selected subset and the rest of the sensor locations is maximal. In the context of the sensor selection problem, this maximum mutual information is obtained indirectly.

The greedy approximation [38], [39] is a simple, but often used, algorithm for combinatorial optimization problems which can be directly applied in the sensor selection problem. By building up a solution piece by piece, the greedy approximation algorithm may complete the computational tasks in a few seconds. In every iteration, one sensor is chosen and moved to the selected set. Once a sensor is moved to the selected set it is impossible to remove it from the selected set during later iterations. Nevertheless, this prime disadvantage of the algorithm voids the solution to reach optimal values. Another prominent heuristic algorithm to solve combinatorial optimization problem is the genetic algorithm. The genetic algorithm imitates the evolutionary process of nature in which a solution deputizes for the organisms' genetic string. Yao *et al.* [40] in their work illustrated the use of the genetic algorithm to solve the sensor placement problem. Such an approach, however, can be very expensive for some computational costs when going over a population of individuals [41].

III. SENSOR PLACEMENT IN MRWSNS

In the wireless sensor network and robotic research community, mobile robotic sensor nodes have attracted much recent attention due to their vital impact on applications such as surveillance, environmental monitoring, wildlife detection and urban search and rescue operations [42]. The mobile ability of robotics can be utilized to improve performance in WSNs such as node localization, data collection, data aggregation and detection and reaction of failed nodes.

With robots embedded in the sensor network, mobile robotic wireless sensor nodes provide a considerable benefit to connectivity, cost, reliability, and energy efficiency throughout the network as compared with a stationary wireless sensor network [43], [44]. Furthermore, the mobility of wireless sensors allows enhancement of the connectivity in a sparse WSN [45], [46]. However, the most widely utilized advantage of mobile robots in the WSNs is to efficiently improve data collection [47]–[50]. The combination of the new paradigm brings in new opportunities to reduce and better distribute the energy usage within the network. Particularly, mobile agents have the potential to decline the number of hops in data transmission, which justifies the reduction of energy used to transmit the data and avoids the funnelling effect in centralized WSNs.

A. Sensor Placement in WSNs

The term sensor placement has been used in various contexts of the WSNs. For instance, in the work by Fletcher *et al.* [51], an algorithm named Randomized Robot-assisted Relocation of Static Sensors (R3S2) is proposed to utilize mobile robots for the purpose of servicing the WSNs. Specifically, in R3S2, robots travel around the network to discover sensing holes that are not being covered due to unpredictable node failure, then move redundant sensors to the uncovered area. Similarly to the work [51] in terms of servicing systems, authors in [52] examine robot task allocation and robot task fulfilment in wireless sensor and robot networks. For example, the network will organize a group of robots to achieve a desired goal, while other moving robots will recharge batteries on the nodes in the regions of the network. Moreover, there are algorithms proposed in [53], [54] for placement of relocatable nodes in order to improve network connectivity. Considering a large-scale static WSN, [55], [56] propose approaches that also employ mobile robots to detect and report failed sensors and then replace these broken nodes.

B. Sensor Placement for Environmental Monitoring

In the context of monitoring spatial environmental fields, the sensor placement problem for predicting the spatial phenomena has been investigated, which has considerably contributed to a number of interesting approaches and algorithms. In [57], [58], locational optimization had been proposed in optimizing the mobile sensor network locations with respect to a known event probability density in the spatial environment. However, physical processes are not known a priori, and a density function can be only established when measurements are to be taken. Leonard *et al.* [59] employed a linear model to predict an ocean field and proposed a performance metric that minimizes uncertainty in a model estimate of the sampled field to derive a parameterized family of paths for the mobile sensor networks. By combining with coverage control [57], Martínez [60] derived a distributed prediction scheme based on a nearest neighbour interpolation approach for field estimation in mobile sensor networks. The primary disadvantage of the linear models in both [59] and [60] is that the model parameters must be known a priori. In terms of the compressive sensing framework, Huang *et al.* [61] maximized the entropy of next measurements to find the next most informative positions for networked mobile sensors to reconstruct an unknown sensing field. By defining a graph whose vertices and edges are considered as a single robot's visiting locations and moving paths, respectively, a path planning algorithm for a mobile robot was proposed in [62] so as to maximize information gained from measurements of a spatio-temporal phenomenon. Ouyang *et al.* [63] utilized a Dirichlet process mixture of Gaussian processes to model the continuous-valued spatial field and then proposed a decentralized multi-robot Bayes-optimal active learning policy for multiple robotic sensors so that the most informative partitions of a non-stationary phenomenon are to be sampled. The policy is then resolved by a greedy algorithm. The works in [64], [65], La *et al.* used consensus filters to propose a distributed sensor fusion algorithm for multiple mobile sensor nodes to automatically adjust their movements to obtain quasi uniform confidence of estimating and mapping a scalar field. By utilizing a Kalman filter for a downsampled system, [66] has developed the optimal sampling strategies in order to balance the estimation quality and the sensor network lifetime.

In terms of statistics, the spatially environmental phenomena are efficiently and effectively modelled by the Gaussian processes (GPs). Therefore, Suh *et al.* by their work [67] represented an environmental monitoring navigation strategy for a sensing robot, in which the information gain along the robot's trajectory is maximized. Considering a team of sensing agents, Graham *et al.* described the random field models by tools from geostatistics, that is Kriging, and proposed to utilize either a known [68] or an unknown [69] covariance function. The sensor network includes static computing nodes and mobile sensing agents taking measurements of a random process. The static nodes compute the gradient of variance and send control commands to robotic sensors. Nevertheless, in [70], maximizing joint entropy of measurements in a distributed fashion was investigated to consider the adaptive sampling paths, where motion coordination was designed based upon Voronoi partitions.

Popa *et al.* proposed extended Kalman filter [71] and non-linear extended Kalman filter [72] based adaptive sampling approaches to optimally estimate the parameters of distributed variable field models. These schemes also aim to decline the uncertainty in the knowledge of a linear-in-parameters field distribution (linear model). Choi *et al.* [73] introduced a Kalman filter based technique to learn the parameters of a physical spatio-temporal process model and then presented criteria to navigate mobile sensors throughout an environment in order to maximize a specified performance. In [74], the authors delineated an objective function of a trajectory optimization problem for robotic wireless sensors as a deterministic optimal control problem. In other words, the objective function is imposed on minimizing the variance of the estimate of the environment, which is eventually resolved by a dynamic programming algorithm. Euler *et al.* in their work [75] introduced a sampling navigation scheme for a group of unmanned aerial vehicles to simultaneously observe multiple concentration levels of an atmospheric plume. The authors incorporated the estimations of the concentration into the uncertainty at these levels to find out the optimal sampling locations. Wu *et al.* [76] proposed a switching scheme for a team of mobile sensors to switch between individual exploration and cooperative exploration as they were exploring an unknown environment. It is proposed that the density field is defined by mapping uncertainty, it changes with every measurement taken; a symmetry-preserving coordinated motion strategy for sensing agents was delineated in [77] to provide optimal measurements.

Cortés in [78] developed a distributed Kriged Kalman filter for robotic wireless sensors to predict the field of interest. A consensus algorithm is implemented on new measurements to calculate state predictions of the field. A gradient based controller was designed to drive the mobile wireless sensors to take optimal samples so that the variance of the estimate error is decreased. In [79], Oh *et al.* proposed a distributed learning algorithm for robotic sensing systems, called cross validation. In this proposition, each mobile sensor learns model parameters using its own measurements and sends the learned parameters to other sensors to validate until all mobile sensors share the best fitness results.

Xu *et al.* primarily used the GP regression for estimating and predicting the generally scalar field and designed optimality criteria based on the Fisher information matrix [80] and the average of the prediction error variances [81], [82] for the optimal sampling paths of the MRWSNs. A maximum likelihood recursive filter is proposed to learn unknown model parameters for the covariance function as well as the basis functions. More specially, in [82], a theoretical foundation of the GP regression with a subset of measurements is derived for the MRWSNs. Based on this proposition, a gradient descent based algorithm is delineated to manage the sensing robot coordination. The authors in [83] introduced the Bayesian optimization based technique for the purpose of choosing the much more relevant informative locations for the MRWSs in the GP modelled field. Moreover, a new utility function based on travelled distances of the MRWSs was also proposed to be used indirectly in trade-off between the exploration and the exploitation of the mobile sensors. In our previous works [84], [85], efficient approaches based on the conditional entropy and posterior variances were proposed to design near-optimal sampling paths for the mobile robotic sensors, where it is more important that the solutions were proved to be bound.

IV. Gaussian Markov random field for Sensor Placement

With respect to the GP model, the computational issues have always been a bottleneck, since the computational complexity of factorizing dense covariance matrices is cubic in dimension, which is known as "the big n problem" [86]. In the context of statistics, this challenge has been dealt with by a reduced-rank approximation of the Gram matrix [87], a sparse greedy approach [88], and a sparse GP [89]. Recently, [82] proposed an approach to diminish the computational complexity of a large dense covariance matrix by shortening mobile sensor observations. The disadvantage of this method is that the model parameters need to be known in advance. Other efforts are to represent a continuously spatial process by a discretely indexed Gaussian field. In other words, [90]–[93] have attempted to enhance the computational complexity in modelling the spatial field by replacing the GP by a computationally efficient Gaussian Markov random field (GMRF) [94]. The GMRF is considerably specified by a sparse precision matrix that makes it substantially advantageous to effective computation. The sparsity property of the precision matrix is constituted by a conditional independence concept in which conditional distribution of every random variable only depends on its neighbours. As a consequence, the sparsity of the precision matrix allows the GMRF to have received more and more attention for resource-constrained MRWSNs as compared to the standard GP [74], [95]–[99]. In their work [74], Ny *et al.* introduced an estimator by the use of the Kalman filter, where the sensor trajectory optimization problem based upon an information criterion is in turn a deterministic optimal control problem. In addition, [95] represented an approach in which the physical phenomenon of interest is regularly discretized and modelled by a GMRF. In this proposition, the hyperparameters of the GMRF model are supposed to be known and chosen with a support. The authors in [98] introduced a new class of a GP built on a GMRF for modelling the spatial process. Nonetheless, this technique proposes a model with a known precision matrix. More interestingly, Jadaliha *et al.* [96] investigated the GMRF to tackle the simultaneous localization and spatial prediction problem in a fully Bayesian fashion. These authors also considered localization uncertainty of the mobile sensing agent in the spatial prediction utilizing the GMRF in another work [97]. However, the approaches proposed by the works [95]–[98] are limited to a representation of a regular lattice, which requires the model parameters to be known a priori.

V. Conclusions

In this paper, a detailed literature review on the spatial prediction by the utilization of the WSNs has been given. Various criteria for the sensor selection problem in the SWSNs have been proposed, which have been then addressed by different approaches, in the literature. Many other methods to resolve centralized and distributed spatial prediction issues and to find the sampling paths for the MRWSs in the sensor placement problems have been introduced. Some disadvantages of the existing techniques have been also analysed.

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