Patent Privateering, Litigation, and R&D Incentives*

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Abstract

We model "patent privateering"—whereby producing firms sell patents to patent assertion entities (PAEs) which then license them under the threat of litigation—in a bargaining game. PAEs can negotiate higher licensing fees than producing firms, because they cannot be counter-sued for infringement. Privateering produces two countervailing effects: it increases the offensive value of patents, whereas it decreases their defensive value and lowers the aggregate surplus of producing firms. Embedding the bargaining game into an R&D contest for multiple complementary technologies, we find that privateering may *increase* R&D investments, even as it induces more litigation threats and reduces industry profits.

Keywords: Patent privateering, patent assertion entities, litigation, patent licensing, R&D, innovation, counter-suing, transaction costs.
JEL: D2, K2, K4, L4, L13, O3

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1 Introduction

Patent Assertion Entities (PAEs) are companies that typically neither invest in R&D nor use their acquired patents to make new products. Over the past decade they have risen to prominence by filing complaints for patent infringement and demanding payments for patent licenses under the threat of engaging in costly litigation.[1] Whereas most of the discussion on PAEs has focused on the ability of non-producing entities to extract licensing payments from producing firms, our article explores the economic incentives behind the less understood practice of "patent privateering," whereby a producing firm sells patents to a PAE which then licenses them under the threat of litigation to other producing firms, typically rivals of the original patent owner. We present a theory to study how PAE privateers affect R&D investments and the licensing of patent portfolios under the threat of costly litigation.

Patent privateering has recently gained attention in both academic and policy debates. Ewing (2012) provides a detailed description of this phenomenon. In 2013 several large technology firms sent the Federal Trade Commission and the Department of Justice a letter asking for more scrutiny on privateers. An extract from this letter illustrates the concern for this patent monetization strategy:[2]

"PAEs impose tremendous costs on innovative industries. These costs are exacerbated by the evolving practice of operating companies employing PAE privateers as competitive weapons. The consequences of this marriage on innovation are alarming. Operating company transfers to PAEs create incentives that undermine patent peace. [...] We therefore urge the antitrust agencies to study carefully the issue of operating company patent transfers to PAEs."

To the best of our knowledge no scholar has empirically assessed the extent of patent privateering, although anecdotal evidence suggests this practice is pervasive. Examples of firms that have engaged in privateering include Alcatel-Lucent, British Telecom, Digimarc, Ericsson, Kodak, Micron Technology, Microsoft, Motorola, Nike, Nokia, Philips, Sony, Xerox, and many others. Nokia and Sony, for instance, sold some of their patent portfolios to MobileMedia, a PAE which subsequently sued Apple, HTC, and Research In Motion.[3] Micron Technology, one of the largest memory chip makers in the world, sold at least 20% of its patent portfolio to Round Rock, in multiple transactions between 2009 and 2013. Round Rock, a PAE, asserted these patents against SanDisk, a rival of Micron.[4]

[1] The U.S. Federal Trade Commission recently published "Patent Assertion Entity Activity: An FTC Study." (October, 2016). Several bills have been passed or proposed in Congress, including the SHIELD Act, the Patent Quality Improvement Act, the America Invents Act, and the End Anonymous Patents Act. President Obama has also publicly addressed the issue of PAEs in "Patent Assertion and U.S. Innovation," (June, 2013).

[2] http://patentlyo.com/media/docs/2013/06/pae-0047.pdf (Visited in October, 2016)

[3] "Patent Privateers Sail the Legal Waters Against Apple, Google," by Decker (Bloomberg; Jan. 10, 2013).

[4] "Patent 'Troll' Tactics Spread," by Ashby Jones (WSJ; July 8, 2012).

Our setting incorporates key features of the patent landscape: costly litigation is often resolved through pre-trial settlement; a firm's patent portfolio may protect some, but not all, of the components embedded in a product; a firm accused of patent infringement by a producing firm may counter-sue for patent infringement as a defensive strategy; PAEs cannot be counter-sued for infringement because they do not sell products; patents are "probabilistic property rights," as their scope and validity are uncertain. Some of these features are especially relevant to high-tech industries where final products are made up of multiple patented technologies. Holding large patent portfolios is becoming the rule rather than the exception in many industries—some examples include smartphones, wearable devices, and even financial transactions technology for banks.[5] When a firm is sued for patent infringement, there are generally two different kinds of defenses: challenging patent validity and counter-suing for infringement. Both producing and non-producing firms are exposed to validity challenges. However, only producing firms are exposed to patent infringement counter-suits. Although counter-suing for patent infringement plays a crucial role in the litigation strategy of producing firms (and is one of the reasons why they amass large portfolios in the first place), patents cannot be used defensively against PAE privateers, because they do not produce.

A novel feature of our analysis is to endogenize both the innovation and the litigation processes. Firms decide how much to invest in R&D anticipating the value of patent portfolios, which can come in the form of product sales, patent trade, licensing, or litigation revenue. This is in contrast to most existing articles that study PAEs, which take R&D investments as exogenous and look at litigation and licensing incentives in a fixed patent landscape. As the goal of the patent system is to provide incentives to generate innovation, we evaluate the effect of PAEs in the context of firms' R&D investments.

Our main contribution is to identify two effects of outsourcing patent monetization to patent privateers. First, the threat of infringement counter-suits, in conjunction with the cost of litigation, dampens ex-ante innovation incentives. PAEs help producing firms to overcome these transaction costs, enhancing the ex-ante incentive to invest in R&D. Second, PAEs reduce R&D incentives by decreasing the marginal value of patents that are used defensively, and by extracting rents from the market.

The first effect can be explained by understanding patent enforcement without PAEs. In

See also http://www.law360.com/articles/520387/sandisk-accuses-round-rock-of-patent-antitrust-plot.
In Online Appendix A we further elaborate on several high-value cases and discuss how patent privateering could be studied empirically.

[5]For example, Lloyd et al. (2011) discuss smartphone portfolios.
See https://www.chipworks.com/about-chipworks/overview/blog/assessing-the-patents-the-jawbone-vs-fitbit-lawsuit for a description of patent portfolios protecting wristbands (visited October 2016).
See http://www.law.edu/res/docs/2015-0702-AmericanBankerArticle.pdf for a description of patent portfolios of banks (visited October 2016).

their absence, competitors with similarly-sized patent portfolios will often engage in a tacit "IP truce," whereby neither firm is willing to sue its rival for infringing on its patents, as the rival's portfolio acts as a deterrent. Because going to court is costly for both parties, even if one firm has more patents than its rival the net benefit from enforcing its patents after accounting for the cost of a potential patent infringement counter-suit may be less than the expected legal costs. This "mutually assured destruction" scenario prevents the firm with the larger patent portfolio from fully realizing its value, because there is no credible litigation threat. Hence part of the value of the firm's patent portfolio is lost due to these transaction costs.

Litigation incentives, specifically the credibility of litigation threats, change in the presence of a privateer. Because PAEs cannot be counter-sued for patent infringement, their litigation threats are stronger than those of an operating firm, conditional on having the same patents. Thus a PAE privateer can extract higher licensing payments compared to a producing firm. However, selling patents also changes the bargaining position of the seller: by selling patents to the privateer the producing firm has fewer patents to use defensively in a patent infringement counter-suit, so it is more vulnerable to lawsuits. We show that when firms expect to obtain similarly-sized patent portfolios after the R&D stage, the presence of a privateer increases the value of larger portfolios. In fact, there are two effects: 1) the firm with the smaller portfolio loses more compared to the tacit "IP truce" equilibrium in the absence of privateering; and 2) the firm with the larger portfolio can capture some of the extra surplus generated by the PAE privateer, depending on its bargaining power, whereas the rest goes to the PAE as rents. Both of these effects push the R&D incentives in the same direction: they both make it more profitable to be the firm with a larger portfolio. Thus PAE privateers overcome the transaction costs created by the threat of patent infringement counter-suits and legal costs, which can lead to larger ex-ante incentives to invest in R&D.[6]

The second effect of PAEs on R&D incentives is negative and comes via two channels. First, they reduce the marginal value of *defensive portfolios*. A firm's portfolio is defensive when it is not large enough to profitably start a new lawsuit, but is valuable in a patent infringement counter-suit. When PAEs monetize patents they eliminate the value of defensive portfolios, because a PAE cannot be counter-sued for patent infringement, which lowers the ex-ante incentives to invest in R&D. Second, PAEs will generally extract rents in the process of licensing and litigation, and therefore lead to lower aggregate surplus for producing firms. Both of these effects reduce the incentives to invest in R&D.

In general, whether patent privateering increases or decreases innovation activity depends on

[6]In fact our result shows that when firms enter the market with patent portfolios of similar size, the firms' equilibrium payoffs with privateering are the same as those in an economy with no legal costs.

which of these effects dominates. Within our framework, under fairly general conditions, the effect of privateering on innovation is to increase firms' equilibrium R&D investments, even though it lowers the total surplus of producing firms.

The social benefit of larger R&D investments is to reduce the time it takes to generate innovations and to bring the final product to the market. In our setting firms' R&D investments increase the likelihood of faster arrival of the inventions. These stochastic arrivals determine the expected time to discover all technologies that are necessary for production, and also the patent portfolio of each firm. A firm that invests more in R&D compared to its rival is more likely to discover and patent more components of the final product. A larger R&D investment also speeds up innovation, which implies that both firms and consumers can capture the rewards from commercialization sooner. Unlike most of the existing literature on patent races, where firms compete for a single patent and investments are duplicative, here the complementarity of components and the existence of a "truce equilibrium" in the absence of privateers may induce firms to *under*-invest in R&D relative to the social optimum. To evaluate the welfare effects of PAEs we characterize conditions under which the firms under-invest and over-invest in equilibrium, relative to the social planner's first and second-best outcome. The R&D equilibrium features under-investment when firms are impatient, when consumer surplus in the final product market is large, when the product is more complex (i.e. involves more pieces of technology), or when patent protection is weak. Overall, for a broad range of parameter values, privateering may enhance welfare by increasing R&D investments.

After reviewing the relevant literature in Section 2, in Section 3 we present the model. In Sections 4 and 5 we solve the licensing game and characterize the endogenous R&D investment equilibrium. Section 6 discusses the welfare effects of privateering. Section 7 summarizes our findings and discusses their implications. Appendix A contains the proofs of the main results. An Online Appendix includes proofs of some technical results, a broader discussion of privateering in practice, and several extensions of our model, which also serve as robustness checks for our results.

2 Literature review

We contribute to the growing literature on PAEs, R&D, and litigation. Choi and Gerlach (2017) study how the aggregation of patent portfolios affects patent litigation and product development. They consider an exogenous set of existing patents and analyze settlement as a function of the strength of firms' portfolios and whether firms produce or not. They also study the incentives of producing firms and PAEs to acquire patents from a third party, similar to Cosandier et al. (2014). Unlike Choi and Gerlach (2017), we study the endogenous formation of patent portfolios determined by firms' R&D investments, and discuss the effect of privateering on licensing, litigation, and the incentives to innovate.

Our article relates to patent thickets and cross-licensing (Shapiro, 2000; Beard and Kaserman, 2002), which are two prominent issues in markets for technology (Arora and Fosfuri, 2003). As in Choi (2010), in our model patent protection is probabilistic and producing firms cross-license their patents to settle infringement disputes. Similar to Fershtman and Kamien (1992), we study the effect of cross-licensing on producing firms' R&D incentives. Because PAEs do not produce, in contrast to producing firms they do not benefit from a royalty-free license for the other party's patents, hence PAEs do not benefit from a patent truce.

Our model contributes to the literature on patent races and contests. Models of R&D generally study contests for a single prize or patent, including the literature on patent races started by Loury (1979), Lee and Wilde (1980), and Reinganum (1982), and the literature on rent-seeking contests surveyed in Corchón (2007). We develop a novel model of "contests for bundles," where firms compete to discover and patent multiple complementary technologies. Our setting differs from Fu and Lu (2012) and Clark and Riis (1998) in that firms care (non-linearly) about the bundle of patents they obtain.[7]

Our article contributes to the strategic patent monetization literature.[8] Chien (2010) studies the proliferation of PAEs, the rise of strategic management of patents, and the importance of holding large portfolios to sustain "patent peace" among operating companies through the threat of counter-suing. Lemley and Melamed (2013), Scott Morton and Shapiro (2014), and Scott Morton and Shapiro (2016) discuss different patent assertion strategies by practicing and non-practicing firms. Scott Morton and Shapiro (2014) also present a model of PAE intermediation in which an individual inventor cannot enforce its patent, whereas the PAE can acquire the patent from the inventor and enforce it. Choi and Gerlach (2016) study the litigation incentives of a PAE that faces a sequence of potential infringers, as does Hovenkamp (2013), in a model of reputation. In Turner (2016), firms discover and patent complementary innovations and decide whether to produce or act as non-practicing entities. In Cohen et al. (2014) two firms own patents for inventions of exogenous quality and the firm that owns a low quality invention becomes a PAE. In our model the strength of the patent portfolios is endogeneous.

Bessen et al. (2011) analyze stock market events around PAE lawsuits and estimate a loss of about half a trillion dollars to defendants over the period 1990-2010. Bessen and Meurer

[7]To our knowledge, this area of research is still fairly undeveloped.

[8]Khan (2005) shows that companies whose sole business is patent monetization have existed for a long time. They were called "patent sharks" rather than "patent trolls" in the past.

(2014) estimate that the direct costs of PAE assertions was about $29 billion in 2011, although some studies (Schwartz and Kesan, 2014; Cotropia et al., 2014) have pointed out caveats in these findings. Ewing (2012), Sipe (2015), and Layne-Farrar and Schmidt (2010) study patent privateering in particular—they explore the topic from the perspective of legal scholars, providing a description of the problem and examples, but not analyzing the equilibrium consequences of privateering. To empirically study privateering, it is crucial to determine the source of patents used by PAEs in litigation. Fischer and Henkel (2012) find that PAEs acquired about 65% of their patents from operating companies with more than 100 employees. We are not aware of any other articles that study patent privateering empirically.

3 Model

Two firms (A and B) invest to discover N pieces of technology, which we call *components*, in order to produce and sell a final product that must incorporate all of them. The timing of the model, depicted in Figure 1, is as follows: first, firms invest in R&D and patent their discoveries; second, observing the realization of patent portfolios after the R&D stage, firms enter the final product market; third, once firms have entered the market, firms can engage in privateering; fourth, patent owners and producing firms engage in patent licensing under the threat of litigation.

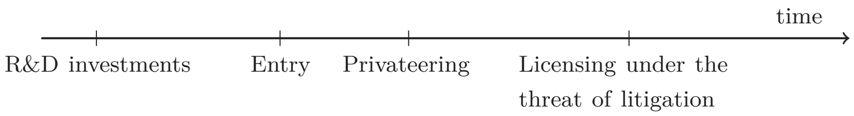

Figure 1: Timing of the events in the model.

In the first stage, firms simultaneously make sunk R&D investments to discover the N components. We assume, as in Loury (1979), that R&D investment is a one-time fixed investment.[9] The cost of investing x units of R&D is $c_I(x)$, where $c_I(\cdot)$ is increasing, convex, differentiable, and $c_I(0) = 0$. When a firm invests x units of R&D, each component $i \in \{1, .., N\}$ arrives independently at time $\tau_i(x) \sim \exp(x)$. If firm i invests x and firm j invests y, firm i discovers a component first if $\tau_i(x) < \tau_i(y)$, which occurs with probability $p(x, y) = \frac{x}{x+y}$.[10] Thus the

[9] Our R&D model is deliberately simple to preserve tractability. In Section D.2 of the Online Appendix we present a dynamic R&D model and partially characterize investment incentives. Although a full characterization of equilibrium of the dynamic R&D model is beyond the scope of this article, our simulation results suggest that similar mechanics hold in that setting.

[10] If both firms invest zero, we define $p(0, 0) = 0$.

probability that firm A discovers exactly k out of the N components is given by

$$P(k;x,y) \equiv \binom{N}{k} p(x,y)^k (1-p(x,y))^{N-k}.$$

Discoveries are publicly observable, and the firm which discovers a component immediately and costlessly obtains a patent on it.[11] At the end of the R&D stage the patent portfolio of each firm is fixed. The expected time to complete the R&D stage is endogenously determined by the level of investment of the firms. The time at which a particular component $i \in \{1, ..., N\}$ is discovered is given by $\hat{\tau}_i(x,y) = \min\{\tau_i(x), \tau_i(y)\}$. The final product can be manufactured and sold only after *every* component has been discovered by some firm. The time at which firms will enter the market and produce is therefore given by $\tau(x,y) = \max_{i=1,\ldots,N}\{\hat{\tau}_i(x,y)\}$.

Given the patent portfolios after the R&D stage, in Stage 2 firms simultaneously make entry decisions. Entry is not blocked by the lack of patents or licenses for some components, because firms can freely and immediately imitate any component discovered by any other firm. We assume that duopoly profits π are large enough, so it is strictly dominant for both firms to enter the market for any realization of patent portfolios.[12] Patent privateering becomes relevant only when both firms enter the market, so we restrict attention to this case.[13]

If PAEs are present once patent portfolios are determined, in Stage 3, firms can engage in privateering. We assume that the original inventor of a patent always retains a license for its invention, even after assigning the patent to a new firm. In consequence, the original assignor of a patent cannot infringe on that patent, even after it no longer owns it.[14]

Next, in Stage 4, firms engage in patent licensing with any patent owner (including, possibly, PAEs), and license negotiations take place under the threat of litigation. Licenses are set through symmetric Nash bargaining over the surplus that is generated by a licensing deal, relative to the firms' outside option of not licensing and potentially going to trial.[15] If a firm has entered the product market and does not have a license or patent protection for some of the N components, it may be sued for patent infringement. In this case the firm can immediately challenge the validity of the patents: we assume each patent is found valid with probability β, which is independent across patents. Notice that both producing and non-producing entities are susceptible to such patent validity challenges. If a patent is found valid and infringed, the patent owner is entitled to receive some amount $R > 0$ of damages per

[11] For simplicity, we assume away the possibility of trade secrets or strategic delay in patenting.

[12] We will assume throughout that $\pi \geq \beta RN$, defined in subsequent paragraphs.

[13] In Section D.6 of the Online Appendix we study the simultaneous entry game in the case when duopoly profits are such that only one firm has an incentive to enter.

[14] In practice this is the case in all examples of patent privateering that we are aware of.

[15] Spier (2007) discusses the role of bargaining power in settlement outcomes.

patent. Going to court is costly: we assume that each side must pay $c > 0$ in legal costs per lawsuit, and that the defendant may bring to the court counter-claims for patent infringement against the plaintiff at no additional cost, if the plaintiff is a producing firm.

Our assumptions on how the court system works are made for tractability. In Section D of the Online Appendix we explore extensions of our model in three different dimensions. First, it is not crucial for our results to have costless patent infringement counter-suing, or to have the same costs for all firms. We study extensions in which the cost structure is different from the baseline case, and find that the main result is preserved. Specifically, we explore costly counter-suing for patent infringement, different costs for producing and non-producing firms, and cost as a function of the number of patents involved in the lawsuit. Second, our model does not allow for injunctions, mostly because they are not often granted in practice, particularly to PAEs. Nonetheless, we discuss the role of injunctions, as they create different incentives for transferring patents to a privateer. Finally, in our main model we assume the market structure is invariant to privateering and we restrict damages to lump-sum rather than royalty rates. This avoids the problem of how royalties themselves affect pricing, which is interesting on its own but is not central to this article.

4 The returns to R&D investment

In this section we solve the game by backward induction. First, we assume that PAEs do not exist—so there is no scope for patent privateering—and derive the equilibrium payoffs. Then we introduce a PAE, to allow for privateering, and we solve the equilibrium of that game.

4.1 Returns to R&D, Licensing, and Litigation without Privateering

Consider the situation in which both firms have entered the final product market. Firm A's patent portfolio covers n components, whereas firm B's portfolio covers m components, with $n + m = N$. Notice that $0 < n < N$ implies that both firms have entered the product market with incomplete patent protection. Proposition 1 derives the returns to R&D when, after discovering and patenting inventions, firms license under the threat of litigation in the absence of privateering.

Proposition 1. *If firm A enters the market with a portfolio of n patents and firm B with a portfolio of m patents, the firms cross-license their portfolios for a transfer $T(n, m)$ and there is no litigation in equilibrium. The payoff of a firm that finishes the R&D stage with n patents* *in its portfolio is* $U_0(n) = \pi + T(n, N-n)$*, where*

$$T(n,m) = \begin{cases} V \cdot (n-m) & \textit{if } |n-m| > \hat{c} \\ 0 & \textit{if } |n-m| \leq \hat{c} \end{cases},$$

with $V \equiv R\beta$*,* $\hat{c} \equiv \dfrac{c}{V}$*, and* $n + m = N$*.*

Proof. Given that each infringement claim is evaluated independently, the probability that firm B's product infringes on exactly k out of the n patents owned by firm A is given by $\binom{n}{k}\beta^k(1-\beta)^{n-k}$, and the expected damage award received by A is given by

$$\sum_{k=0}^{n} \binom{n}{k} \beta^k (1-\beta)^{n-k}(Rk) = R\beta n.$$

The expected benefit of going to court is proportional to the number of patents asserted. Because counter-suing is free, firm B will counter-sue using its entire portfolio to obtain an expected payment of $R\beta m$. Thus firm A's expected payoff from going to trial is $R\beta(n-m)-c$ whereas firm B's is $R\beta(m-n) - c$. In this situation, should licensing negotiations fail, firm A is willing to initiate litigation if and only if $R\beta(n-m) > c$, whereas firm B is willing to initiate litigation if and only if $R\beta(m-n) > c$. These are the cases where one side has a positive-expected-value suit, as discussed for example in Shavell (1982) and Nalebuff (1987).

If either firm has a credible threat of litigation, the firms will Nash-bargain to avoid a reduction in joint surplus of $2c$, due to litigation costs. In this case the firms will cross-license their patent portfolios and the firm with the smallest portfolio pays $V(n-m) - c + \frac{1}{2}(2c)$ to the firm with the largest portfolio. □

Proposition 1 shows that for a fixed patent portfolio (n, m) either one of the firms has a relatively large portfolio—in which case the litigation threat is credible and there is a payment for licensing the portfolios—or firms have patent portfolios of similar sizes and litigation is not credible. In the latter case, firms produce while tacitly cross-licensing, although they may be infringing on each other's patents. The R&D payoffs for discovering n patents and licensing in the abscense of privateering is depicted in Figure 2.

When firms have portfolios of similar size, i.e. $\frac{N-\hat{c}}{2} \leq n < \frac{N+\hat{c}}{2}$, the marginal value of patents is zero. Having one more patent does not make a difference on the margin, because litigation is not credible. In this case we have an "IP truce" equilibrium, in which firms know that each firm may be infringing on the patent portfolio of the rival, but neither firm can credibly litigate, i.e., firms tacitly cross-license their portfolios.

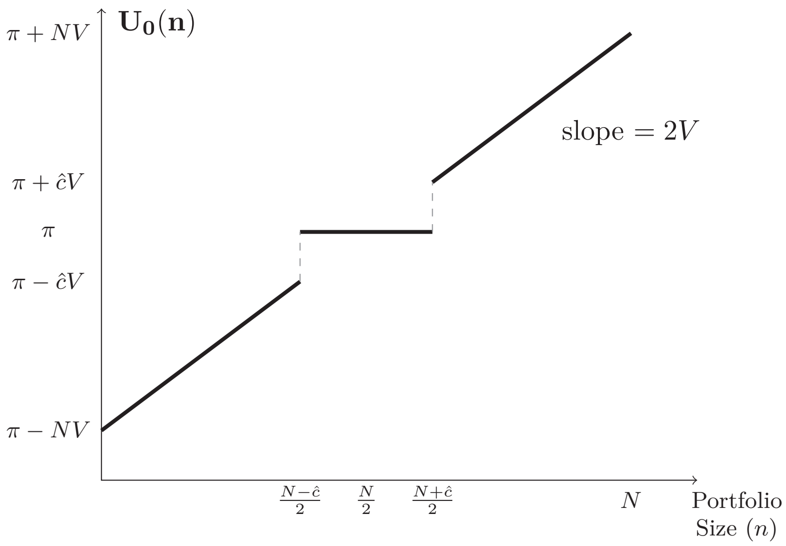

Figure 2: Payoff of a firm that finishes the R&D with n patents (its rival with $N-n$ patents). The assumption $\pi > NV$ guarantees entry for all n. The picture depicts the case $N > \hat{c}$.

When the difference in the size of the patent portfolios is relatively large, the marginal value of a patent is $2V$. If a firm has less than $\frac{N-\hat{c}}{2}$ patents, the marginal value of a patent derives from its defensive value—used in a patent infringement counter-suit, the firm expects to get V—and from its appropriation value—the rival firm has one less patent to use offensively, which saves V in licenses. Similarly, if a firm has more than $\frac{N+\hat{c}}{2}$ patents, the marginal value of a patent derives from its offensive value—one more patent in the lawsuit increases the expected payment by V—and from its appropriation value—the rival firm has one less patent to use defensively, which saves V in infringement counter-suit payments.

4.2 Licensing and Litigation under Privateering

By not manufacturing or selling products, a PAE is strategically different from producing firms because it is immune to infringement counter-suits. The PAE starts with no patents, because it does not invest in R&D, but it acquires patents from firms that invested in R&D, once the research stage is over, and after the entry decisions have been made. When the PAE acquires patents from a producing firm, that firm is granted a license for the patents it sold, so the PAE cannot sue the original inventor. A producing firm is more vulnerable to lawsuits after selling part of its patent portfolio, as those patents can no longer be used defensively; the upside of selling is the additional revenue from the price that the PAE would be willing to pay for the patents. PAEs can sue without the threat of patent infringement counter-suits, so patents have a higher monetization value for them compared to producing firms.

Suppose a PAE has acquired $n' > \hat{c}$ patents from firm A and is planning to sue firm B. The expected payoff from litigation for the PAE is $V \cdot n' - c$ and for firm B is $-V \cdot n' - c$. When the litigation threat of the PAE is credible ($n' > \hat{c}$), the parties bargain over the surplus gained by avoiding litigation. With symmetric bargaining power in the negotiation of licenses between the PAE and a producing firm, firm B is willing to pay the PAE a license fee of

$$T(n', 0) = V \cdot n' - c + \frac{1}{2}(2c) = Vn'.$$ [16]

We adopt the following notation: n are the number of patents originally invented by firm A, n' are the number of patents sold by firm A to the PAE at price $p_A(n')$; m are the number of patents originally invented by firm B and m' are the number of patents sold by firm B to the PAE at price $p_B(m')$. The values n' and m' will be endogenously determined in equilibrium, which is fully characterized in Section 4.3.

Lemma 1. *Consider the allocation of patents (n, m, n', m') when both firms enter the final product market. Then the payoffs from licensing under the threat of litigation are:*

$$\pi_A = \pi + T(n-n', m-m') - T(m', 0) + p_A(n'), \qquad \pi_B = \pi - T(n-n', m-m') - T(n', 0) + p_B(m'),$$

$$\pi_{PAE} = T(n', 0) + T(m', 0) - p_A(n') - p_B(m').$$

Proof. After selling to the PAE, firm A owns $n - n'$ patents and firm B $m - m'$. The transfer $T(n - n', m - m')$ reflects the license payment among producing firms. $T(n', 0)$ is what the PAE gets from firm B by asserting the patents acquired from A against B, and $T(m', 0)$ is what the PAE gets from firm A by asserting firm B's patents. □

In the next section we find the equilibrium values of n' and m', which are the solutions of the bilateral bargaining games between each producing firm and the PAE.

4.3 Patent acquisition by the PAE

For modeling purposes we assume there is a single PAE, which will bilaterally and simultaneously bargain with both firms for the acquisition of patents. In this negotiation we allow for arbitrary bargaining power for the PAE. We also assume that the PAE cannot commit to

[16] We study lump-sum payments. Lemley and Melamed (2013) argue that PAEs may prefer lump-sum payments over running royalties. They argue PAEs are in the business for cash, running royalties are associated with administrative burdens, and disputes can be resolved quicker and with less risk by lump-sum payment.

negotiate with one firm only. Note that under these assumptions, our results (assuming passive beliefs) are identical to a model with multiple competing PAEs offering contracts to firms to acquire patents. We adopt the simultaneous and symmetric Nash bargaining approach of Horn and Wolinsky (1988). The PAE simultaneously bargains with firms A and B over the outcomes. An outcome corresponds to the set of patents acquired by the PAE from the producing firms and the prices at which they were bought. Let $S_{PAE}(n', m')$, $S_A(n', m')$, and $S_B(n', m')$ be the payoffs from licensing in the shadow of litigation for the PAE, firm A, and firm B, respectively, after the PAE acquires n' patents from A and m' from B, at prices $p_A(n')$ and $p_B(m')$, respectively. The result of each negotiation is the solution to Nash bargaining, given the equilibrium deal reached between the PAE and the other producing firm. We allow for arbitrary bargaining power $s \in [0, 1]$ for producing firms when they negotiate patent sales with the PAE.[17]

Given a patent allocation (n, m), with $n+m = N$ and $n \geq m$, we can now find the equilibrium of this bargaining game (the case where $m \geq n$ is symmetric).

Definition 1. (n', p_A, m', p_B) *is an equilibrium allocation of the bargaining game if*

$$(n', p_A) \in \max_{(z,p)} (S_{PAE}(z, m') - p - S_{PAE}(0, m'))^{1-s} (S_A(z, m') + p - S_A(0, m'))^s, \qquad (1)$$

$$(m', p_B) \in \max_{(z,p)} (S_{PAE}(n', z) - p - S_{PAE}(n', 0))^{1-s} (S_B(n', z) + p - S_B(n', 0))^s. \qquad (2)$$

In Definition 1, taking m' as given, firm A and the PAE bargain over their outcome à la Nash. Simultaneously, taking n' as given, firm B and the PAE bargain over their outcome. We denote by $J_{A,PAE}(z, m')$ the joint surplus of firm A and the PAE from licensing under the threat of litigation, when firm A transfers z patents to the PAE and firm B has sold m' patents to the PAE. We define $J_{B,PAE}(z, n')$ analogously for firm B and the PAE. A standard result in bargaining games with lump sum payments is the following:

Lemma 2. *The outcome of the bilateral negotiation between an operating firm and the PAE maximizes their joint surplus, for a fixed deal between the rival firm and the PAE.*

Intuitively, an operating firm and the PAE trade patents bilaterally to maximize their joint surplus. Once the allocation of patents maximizes the joint surplus, a monetary transfer splits the surplus between the parties according to their bargaining power. Thus to find the number of patents traded between producing firms and the PAE, we need to examine the allocations that simultaneously maximize the joint surplus of each pair.

[17] This parameter and the no-commitment assumption allow us to directly translate our results to an environment with multiple competing PAEs.

Proposition 2. *It is an equilibrium for each firm to sell its whole portfolio to the PAE in the bargaining game described above. In that equilibrium, the PAE extracts no rents from the producing firms. The equilibrium payoffs are*

$$\pi_A = \pi + T(n,0) - T(m,0), \quad \pi_B = \pi - T(n,0) + T(m,0), \quad \pi_{PAE} = 0. \tag{3}$$

Proposition 2 shows one equilibrium outcome of the bargaining game, in which both producing firms sell their entire portfolio to the PAE. In this equilibrium the PAE will not extract rents from the producing firms, despite the fact that it allows for more patent monetization.[18] The reason behind this result is that when firm B sells its entire portfolio to the PAE, it loses the ability to use patents defensively in a counter-suit. This implies that the maximum surplus that firm A and the PAE can achieve jointly equals what firm A can achieve on its own. The PAE does not offer any strategic advantage to firm A once its rival has sold everything to the PAE. Therefore it does not increase their joint surplus.

Depending on the size of the patent portfolios after the R&D stage, the bargaining game may have multiple equilibria. The following lemmas are used to characterize all the equilibria in this game. Lemma 3 examines the case in which firm B has enough patents to be monetized by the PAE, whereas Lemma 4 examines the case in which firm B's portfolio cannot be monetized by the PAE.

Lemma 3. *Suppose $n > m > \hat{c}$. In any equilibrium of the bargaining game firm B sells all of its patents to the PAE.*

Intuitively, when firm B holds on to some patents, firm A and the PAE have a strategy that prevents firm B from monetizing those patents. When firm A and the PAE are playing this strategy, firm B and the PAE are losing value on the patents held by firm B, because the PAE could monetize them. Thus, when $n > m > \hat{c}$ any equilibrium must have firm B selling everything to the PAE.

Lemma 4. *When $m \leq \hat{c}$, in every equilibrium of the bargaining game firm A sells all of its patents to the PAE.*

If firm B does not have enough patents to be monetized by the PAE, firm A is "safe" by selling all of its patents to the PAE. Even better, by selling, firm A avoids counterclaims that would be brought by firm B, had firm A sued directly.

Lemmas 3 and 4 characterize the unique equilibrium behavior of one of the firms in the

[18] This result arises from the assumptions of bilateral Nash bargaining and the fact that the PAE could not choose to trade only with one of the firms.

game. The multiplicity arises from the different strategies that the other firm can play. In Proposition 3 we characterize the payoffs for all the equilibria of the game.[19]

Proposition 3. *The equilibrium payoffs of the game are:*

1. *When $n = m$ there are multiple equilibria, and in all these equilibria the payoffs are:*

$$\pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0.$$

2. *When $n > m > \hat{c}$, firm B sells its entire portfolio. There are multiple equilibria, but any equilibrium has the following payoffs:*

$$\pi_A = \pi + V(n-m), \quad \pi_B = \pi - V(n-m), \quad \pi_{PAE} = 0.$$

3. *When $m \leq \hat{c}$ firm A sells its entire portfolio to the PAE and firm B is indifferent between selling any amount $m' \in [0, m]$. The equilibrium payoffs depend on how many patents firm B is selling, as they change the outside option in the bilateral bargaining of firm A and the PAE:*

$$\pi_A = \pi + T(n, m-m') + s[Vn - T(n, m-m')], \; \pi_B = \pi - Vn, \; \pi_{PAE} = (1-s)[Vn - T(n, m-m')].$$

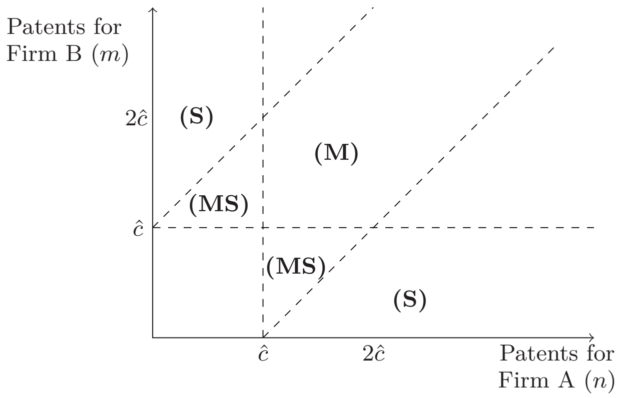

Figure 3: Effect of PAE on firms' continuation payoffs for different patent portfolio configurations. The changes in payoffs occur only in three regions.

Figure 3 summarizes the effect of the PAE on the equilibrium continuation payoffs for different patent allocations. The PAE affects the firms' continuation payoffs only in three ways: *monetization* (M), *shielding* (S), and *monetization and shielding* (MS). The monetization ef-

[19]The equilibrium allocations are also described in the Appendix, in the proof of Proposition 3.

fect is what drives the main result of the article, and is robust to a variety of alternative assumptions. We discuss each of the effects below:

1. **Monetization (Region M):** When $|n - m| \leq \hat{c}$, $n > m > \hat{c}$, without PAEs firms do not have credible litigation threats. However, when the PAE acquires all the patents it has two individually rational lawsuits, because $n > \hat{c}$ and $m > \hat{c}$. By monetizing patents, the PAE generates a positive total surplus of $V(n - m)$, which is captured by the firm with the largest portfolio. Thus, compared to the case without privateering, the PAE increases firm A's continuation payoff by $V(n-m)$, which equals the decrease in the continuation payoff for firm B. The PAE is not able to extract surplus in this case, because in equilibrium the "weak" player (the firm with fewer patents) sells everything to the PAE. This implies that firm A's outside option is to monetize all its patents without the threat of counter-suing, which is the same benefit offered by the PAE.

2. **Shielding (Region S):** When $|n - m| > \hat{c}$, $m \leq \hat{c}$, although firm A had a credible litigation without the PAE, the PAE can effectively "cancel out" firm B's portfolio, which increases firm A's surplus, and the PAE is able to extract rents. The effect of the PAE is to shield firm A from counter-suits. A's equilibrium payoff increases from $V(n - m)$ to $V(n - m) + sVm + (1 - s)Vm'$, whereas B's decreases by Vm.

3. **Monetization and Shielding (Region MS):** When $|n - m| \leq \hat{c}$, $m \leq \hat{c}$ and the PAE did not exist, neither firm had a credible threat to sue. Even if $n > \hat{c}$, firm A did not have the ability to monetize its patents, because of the fear of retaliation from firm B. With the PAE, firm A can monetize its patents (when $n > \hat{c}$) and "cancel out" firm B's portfolio. This strategic advantage offered by the PAE comes from its ability to avoid counter-suing. Thus the PAE allows for patent monetization and shields the producing firm with the largest portfolio from counter-suits.[20] The effect of the PAE on equilibrium payoffs is to increase firm A's payoff from 0 to $sVn + (1 - s)T(n, m - m')$, and decrease firm B's payoffs by Vn.

4. **Other cases:** In all of the remaining cases, either both firms are already monetizing their patents and shielding is not possible, or no firm has enough patents to start a lawsuit. In these cases, the continuation payoffs are the same in an equilibrium with or without privateering.

[20] Notice that although firm B cannot monetize its patents, its decision of how many patents to keep has an impact on the equilibrium payoffs, as they determine the outside option of firm A in the bilateral bargain with the PAE.

4.4 The returns to R&D under privateering

When firms can sell patents to a PAE, the subgame perfect equilibrium has both firms entering the final product market, trading with the PAE, and licensing patents in bilateral bargaining among all parties. Proposition 3 characterizes the payoffs of both firms as a function of the number of components they patent. Figure 4 illustrates the equilibrium payoff for the case where $N > 3\hat{c}$ for an equilibrium where a firm that discovers fewer than $\hat{c}$ components retains all of its patents. By Proposition 3, this is the equilibrium with the largest payoff for the privateer, which provides a worst-case scenario for R&D incentives.

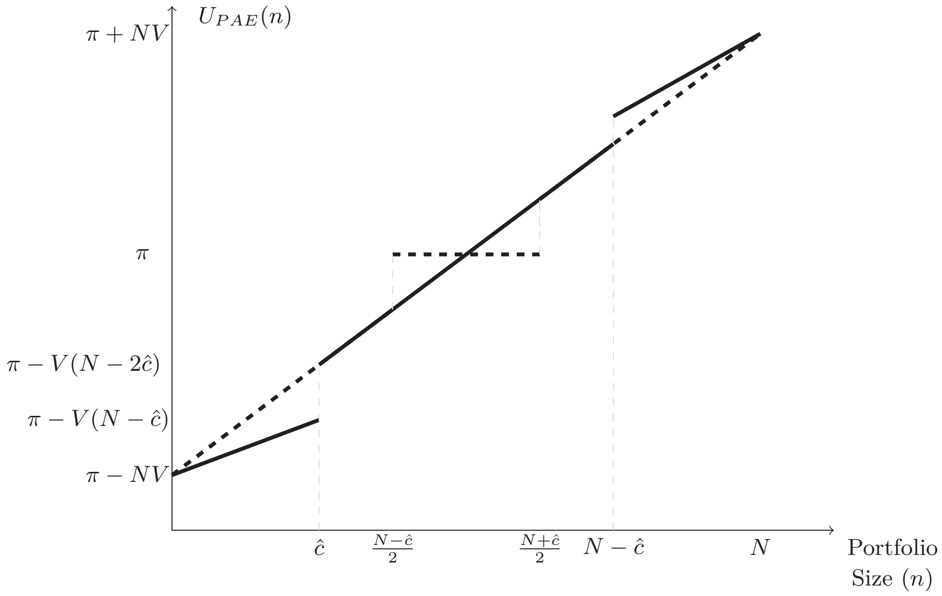

Figure 4: Continuation payoff for a firm that enters with n patents in the game with a PAE (solid line). Superimposed is the payoff of the game without privateering (dashed line). The figure depicts the case $N > 3\hat{c}$ for the equilibrium where any firm which discovers fewer than $\hat{c}$ components retains all of its patents.

The PAE has three effects on the continuation payoffs of the producing firms. First, when firms enter with roughly half of the patents—to be precise, $\frac{N-\hat{c}}{2} \leq n \leq \frac{N+\hat{c}}{2}$—the payoff has positive slope due to the *monetization* effect of a PAE. In other words, each patent has positive marginal value, whereas in the absence of PAEs the marginal value of a patent in this region is zero. Second, the payoff function is flatter in the two extreme regions, implying that the marginal value of a patent in those regions is lower, compared to what it would be without privateering. Absent PAEs, a firm with fewer than $\hat{c}$ patents can only use its portfolio defensively, but when its rival's portfolio is monetized by a PAE the patents have no defensive value. This explains why in Figure 4 the payoff is flatter on the left side of the graph. The slope is also flatter on the right side, compared to the payoff without PAEs where the marginal value of each extra patent is $2V$, as explained in Section 4.1. Because the defensive value of patents is destroyed, the *appropriation* value of a patent to the firm with the larger portfolio is also destroyed by PAEs. Thus, the marginal value for the firm with the larger number of patents decreases, although the level of payoff increases. The *shielding* effect increases the payoff for the firm with the largest portfolio and decreases it for the firm with fewer patents, but the marginal value of a patent is decreased for both firms. Finally, the PAE reduces the total expected profit of a firm by extracting some of the total surplus as bargaining rents, characterized in Proposition 3.

It is important to note that these differences in payoffs are driven by the commitment to litigate which privateering provides to firms, and not by the assumption that counter-suing is free. Section D.1.1 of the Online Appendix shows that the same effects persist when counter-suing is costly, and in fact in that setting the effect of privateering is amplified. Instead, the differences in payoffs with and without privateering arise because outsourcing the licensing and litigation of a patent essentially allows a firm to commit to litigate in certain cases where, absent privateering, litigation would not be a credible threat.[21]

The case $N \in [2\hat{c}, 3\hat{c})$ is qualitatively similar to $N > 3\hat{c}$. When $N \in [\hat{c}, 2\hat{c})$, the PAE is ineffective to monetize patents when both firms individually have fewer than $\hat{c}$ patents, but it can still help monetization when one firm has more than $\hat{c}$ patents. The details for these two cases are in Section D.5 of the Online Appendix. For the remainder of the article we focus on the neatest case, $N > 3\hat{c}$.

In the next section we study the firms' R&D investment problem, taking as given the equilibrium continuation payoffs above.

5 Endogenous R&D investments

We now turn to the optimal R&D investments when firms anticipate the equilibrium continuation payoffs.[22] We denote by $U_0(k)$ the continuation payoff of a firm that discovers and patents k out of the N components and enters the market in the absence of PAEs (see Figure 2), and we denote by $U_{PAE}(k)$ the continuation payoff of a firm that ends the R&D stage with

[21] Without privateering, litigation threats are not credible when $|n - m| < \hat{c}$, so the licensing game in this case has a Nash equilbrium where firms are able to monetize their patents, but this equilibrium is not subgame perfect. Privateering enables us to sustain this outcome as a subgame perfect equilibrium.

[22] Throughout this section we maintain the assumption $\pi > NV > 3c$, which guarantees entry and the existence of the monetization effect. In the Online Appendix we also present a dynamic extension of our R&D model and show that the main qualitative result is preserved.

k patents and can engage in privateering (see Figure 4). Notice that firms' payoffs depend on the *bundle* of objects they discover and also on how soon they discover them. Although competition over multiple prizes has been studied before (e.g., Clark and Riis (1998)), our setting differs from these results in that firms care non-linearly for the *bundle* of components they obtain. Figures (2) and (4) show that the continuation payoff is weakly increasing, non-linear, and not concave. Besides this difficulty, the expected discount factor for $N > 1$ is not a concave function of the R&D investment (see Section C of the Online Appendix). As a consequence the R&D decision problem is not generally well-behaved,[23] which does not allow us to apply standard results of existence and comparative statics for symmetric aggregate games.

Consider a game without privateering. Each firm chooses its R&D investment to maximize its expected payoff, given the investment level chosen by its rival. Because firms are ex-ante symmetric, they solve:

$$\max_{x \geq 0} \ \mathbf{E}_{\tau,k}[e^{-r\tau}U_0(k)|x,y] - c_I(x). \tag{4}$$

From the properties of the exponential distribution we can show the following result.

Lemma 5. *The random variables k and τ are independent for all x and y.*

By Lemma 5, equation (4) can be written as

$$\max_{x \geq 0} \ \underbrace{\mathbf{E}_{\tau}[e^{-r\tau}|x,y]}_{\equiv G(x,y)} \cdot \underbrace{\mathbf{E}_{k}[U_0(k)|x,y]}_{\equiv \Pi(x,y)} - c_I(x).$$

$G(x,y)$ is the expected discount rate at the time of production and $\Pi(x,y)$ the expected continuation payoff. In Online Appendix B we derive explicit formulas for $G(x,y)$ and $\Pi(x,y)$, and discuss their properties. Note that $\Pi(x,x) = \pi$ because, given symmetric R&D investments, firms symmetrically expect each portfolio allocation, expected licensing transfers for each firm net to zero, and the expected reward of entering the market is π. Under a stability condition, similar in spirit to the one in Lee and Wilde (1980), we can show that the problem has a unique interior solution $x^* > 0$ that solves[24]

$$G_x(x^*,x^*)\pi + G(x^*,x^*)\Pi_x(x^*,x^*) = c_I'(x^*). \tag{5}$$

Intuitively, in equation (5) investing one more unit of R&D has two effects. First, the expected continuation payoff arrives earlier. This effect is given by the term $G_x(x^*,x^*)\pi$. An earlier

[23] In particular, the objective function is not pseudo-concave.

[24] Section C of the Online Appendix studies the existence and uniqueness of a symmetric equilibrium under the condition $x^* > \frac{r\ln(N)}{2}$, which is likely to be satisfied when π is relatively large.

arrival of the expected continuation payoff is discounted at a smaller rate, captured by the marginal change in the expected discount rate, G_x. Second, as the firm with the largest portfolio captures weakly more rents than π through licenses, there is a rent-seeking incentive to discover more components. This is represented by the second term $G(x^*, x^*)\Pi_x(x^*, x^*)$. The marginal gain $\Pi_x(x, y)$ is positive for all x and y, by first order stochastic dominance.[25]

Consider now the case of privateering. In this case, a firm that finishes the R&D stage with k patents expects a continuation payoff of $U_{PAE}(k)$ instead of $U_0(k)$.

Lemma 6. *The problem with a PAE is equivalent to*

$$\max_{x \geq 0} \; G(x, y)\Pi(x, y) - c_I(x) + \Delta(x, y),$$

where $\Delta(x, y) = G(x, y)D(x, y)$ *and* $D(x, y) = \mathbf{E}_k[U_{PAE}(k) - U_0(k)|x, y]$.

Privateering changes the marginal incentives to invest in R&D by

$$\Delta_x(x, y) = \underbrace{G_x(x, y)D(x, y)}_{\text{Rent Extraction}} + \underbrace{G(x, y)D_x(x, y)}_{\text{R\&D premium}}.$$

The first term is a *rent extraction* effect. A PAE privateer changes the *level* of the payoff expected by the firms. When $x > y$, firm B expects a lower payoff under privateering. When $x = y$, because the PAE may extract rent during the bargaining stage, the expected equilibrium payoff for both firms is weakly lower compared to the case of no privateering. In a symmetric equilibrium, the rent extraction corresponds to reduction in the area under the equilibrium payoff in Figure 4 relative to the case of no privateering (Figure 2).

The second term, the *R&D premium* effect, corresponds to the change in the discounted marginal benefit from increasing the likelihood of inventing more components. As we can see from the *slope* of the equilibrium payoff in Figure 4, relative to Figure 2, this effect is positive when firms end up with similar numbers of patents after the R&D stage (steeper slope with a PAE) and negative if their portfolios are very asymmetric (flatter slope with a PAE).

Intuitively, in a symmetric equilibrium the rent extraction effect weakly reduces the incentive to invest, whereas the R&D premium effect increases it. We show that in a symmetric equilibrium—where firms expect to finish the R&D stage with patent portfolios of similar size—the R&D premium effect dominates.

Proposition 4. *When* $N > 3\hat{c}$*, for symmetric R&D investments,* $x = y = x^*_{PAE}$*, we have:*

[25] The proof follows from the properties of the binomial distribution and the weak monotonicity of $U_0(\cdot)$.

(a) The rent extraction effect (RE) is weakly negative *and equal to*

$$RE(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{1}{(x^*_{PAE})^2} \frac{r \ln(N)}{2^{N+2}} \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \eta(k; s),$$

where $\eta(k; s) = -(1-s)V\ell_k$, ℓ_k *is the amount of patents retained in equilibrium by the firm with the smaller portfolio, and* $(1-s)$ *is the PAE's bargaining power.*

(b) The R&D premium (RP) effect is strictly positive *and equal to*

$$RP(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{1}{2^{N+1} x^*_{PAE}} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)[U_{PAE}(k) - U_0(k)].$$

(c) The R&D premium effect is larger than the rent extraction effect if

$$x^*_{PAE} > \frac{(1-s) r \ln(N)}{2}. \tag{6}$$

Proposition 4(a) shows that the rent extraction effect is weakly negative in a symmetric equilibrium, because the PAE may extract some of the total industry surplus as bargaining rents, reducing the payoff expected by the firms. Proposition 4(b) shows that, in a symmetric equilibrium, privateering has a positive impact on each firm's expected marginal payoff through the R&D premium, because by investing the same amount firms expect to obtain patent portfolios of similar size with a high probability. Finally, Proposition 4(c) provides a sufficient condition under which the overall effect of privateering on the marginal payoff is positive, i.e., the R&D premium dominates the rent extraction effect.[26]

Condition (6) characterizes the environments in which R&D incentives increase under privateering, so it is important to discuss when it is likely to hold and to interpret its empirical content. First, the condition is more likely to hold in product markets where π is large, because equilibrium R&D investments, x^*_{PAE}, are larger when the downstream product market is more profitable; on the other hand, the right-hand side of (6) is independent of π. Thus privateering may increase R&D incentives when downstream profits are relatively large, and vice versa it may decrease them when downstream profits are relatively small.

Second, condition (6) is more likely to hold when producing firms have more bargaining power when they negotiate with a privateer, i.e., when s is larger. Producing firms would have more bargaining power at the patent trade stage in markets and industries where there is intense competition among PAE privateers. We would expect privateers to extract larger rents and

[26] The proof of the proposition characterizes a necessary and sufficient condition for the overall effect to be positive. The sufficient condition (6) in Proposition 4(c) is intuitive and easier to interpret.

reduce R&D investments if they are more specialized and focused on sourcing patents from specific firms, but to increase R&D incentives if they compete more broadly.

Third, condition (6) will tend to be satisfied when firms negotiate licenses with their competitors under the *threat* of privateering, i.e., before patents are actually sold to a privateer. Using privateering as a threat rather than engaging in privateering would not reduce industry surplus—it is equivalent to trading with a PAE with zero bargaining power—although it would still increase the monetization and the marginal value of patents among firms with similar portfolios. Intuitively, this is more likely to be the cases in more mature industries and markets, where producing firms have well-established relationships with their rivals, and possibly already have negotiated other licensing deals in the past. In contrast, in relatively younger industries a producing firm may not be aware of all potential licensing targets, so a privateer may be able to capture larger surplus and thus buy patents before the producing firms negotiate licenses among themselves. Section D.6.3 of the Online Appendix formally studies the setting where firms can negotiate licenses before trading with a privateer, and shows that the rent extraction effect is indeed zero in this case, so condition (6) is satisfied.

Finally, from a technical perspective, condition (6) is implied by the equilibrium existence and uniqueness conditions $x^* > \frac{r\ln(N)}{2}$ and $x^*_{PAE} > \frac{r\ln(N)}{2}$. When these two inequalities hold and the R&D contest has a unique equilibrium, with privateering the equilibrium level of R&D investments is larger than in the game without privateering. A detailed discussion and simulations regarding these conditions are found in Section C.3 of the Online Appendix.

Having understood the marginal effect of privateering on payoffs, we can now study how the equilibrium R&D investments compare with and without privateering.

Proposition 5. *Suppose symmetric equilibria exist in the games with and without the PAE, and the equilibrium investments are such that $x^* > \frac{r\ln N}{2}$ in both games. Then the equilibrium R&D investments are larger under privateering.*

Proposition 5 establishes that privateering can increase the equilibrium level of R&D investment, even when the PAE extracts rents from producing firms. Privateering increases the marginal value of patents when firms have patent portfolios of similar size. This monetization effect translates into a stronger incentive to capture more rents at the R&D stage, which increases the equilibrium level of investment. For technical reasons we need to impose the condition $x^* > \frac{r\ln(N)}{2}$, because of the characteristics of the contest for multiple components: this condition guarantees the existence and uniqueness of an equilibrium, because under this condition investments are on the quasi-concave region of the payoff function. Under this condition, we can apply the results of Proposition 4 to show that the equilibrium R&D investment is larger under privateering.

The results that firms invest *more* in R&D under privateering may also arise in a dynamic model in which firms discover the components sequentially. In this environment, R&D investment are not symmetric over time, and firms do not invest the same amount to discover each component. In fact, in some cases we observe an "encouragement" effect, meaning that the firm that lags behind invests more than the firm that is ahead. In this dynamic setting, privateering generates an equilibrium distribution of patents that is less centered around the equal allocation region, where the payoff function is flat without privateering. Our partial results in this dynamic setting suggest that, overall, privateering tends to increase the expected aggregate equilibrium R&D investments. Details of this analysis are discussed in Section D.2 of the Online Appendix.

6 Welfare

Welfare can be evaluated ex-post (in terms of the efficiency of the downstream market) or ex-ante (in terms of R&D investment). Productive efficiency can be achieved by a monopolist that perfectly price discriminates, or by perfect competition. Unless the return to R&D is the full total surplus, ex-ante investments will generally be distorted away from the efficient level. As the trade-off between ex-ante efficiency and downstream consumer surplus is well understood, we focus on a second-best analysis: we take the market structure after entry as given (duopoly) and evaluate ex-ante R&D investments. In our model consumers and firms discount future payoffs, so faster product introduction (implied by larger R&D investments) is welfare enhancing.

Consider the problem of a social planner who controls each firm's R&D investment. When both firms enter the final product market after the R&D stage, the firms and the PAE play a zero-sum game with aggregate payoff 2π, because there is always settlement in equilibrium.[27] Hence, as long as both firms enter the downstream market, the planner is indifferent among any allocation of patents that can be achieved via transfers. The planner's payoff differs from that of a joint research venture in that it incorporates the consumer surplus associated with the final product, which we denote by W.[28] Thus, the social planner solves

$$\max_{x\geq 0,\ y\geq 0} (2\pi + W)G(x,y) - c_I(x) - c_I(y).$$

Lemma 7. *The unique planner solution is symmetric and characterized by*

$$(2\pi + W)G_x(x_P^*, x_P^*) = c_I'(x_P^*)$$

[27] As in the rest of the article, we restrict attention to the case where both firms to enter.

[28] We assume the market outcome is unaffected by the allocation of patents, as is the consumer surplus.

Compare the condition in Lemma 7 with the equilibrium condition without privateering, $G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) = c_I'(x^*)$, in equation (5). The term $G_x(x, x)(\pi + W)$, which we call the *planner incentive*, represents the marginal benefit of higher investment that is internalized by the planner but not the firms—the marginal expected discounted payoff of the rival firm plus the consumer surplus. The term $G(x, x)\Pi_x(x, x)$, which we call the *competition incentive*, represents the R&D premium which is taken into account by firms, but not by the planner. These two effects determine whether firms over- or under-invest in equilibrium. Lemma 8 shows that which effect dominates depends on how equilibrium investments compare against a particular threshold.

Lemma 8. *For N sufficiently large, the planner incentive is larger than the competition incentive if and only if $x < x_M$, where x_M is defined by:*

$$x_M = \frac{2^{N-1}(\pi + W)r\ln(N)}{V \cdot \sum\limits_{\{k:\,|2k-N|\geq \hat{c}\}} \binom{N}{k}(2k-N)^2}$$

Lemma 8 implies that, without privateering, firms *under-invest* in R&D relative to the planner if and only if $x^* < x_M$, where x^* is the symmetric equilibrium investment without privateering. There are some natural intuitions and comparative statics for the threshold x_M. First, it is increasing in W; when firms do not internalize a large consumer surplus, the private return to R&D is much smaller than the social return, so firms under-invest. Second, x_M is increasing in π; when downstream profits are large, firms fail to internalize the profit captured by their rivals, and hence are likely to invest less than what is socially optimal. Third, x_M is increasing in the discount rate r; when firms are less patient, they are more likely to invest less than the social optimum. Fourth, x_M is decreasing in $V \equiv \beta R$; when patents are weak (i.e., β is low), or when patent protection is weak (i.e., the damages payment R is low), it is more likely that firms will invest less than the social optimum. Finally, the condition in Lemma 8 also relates to our equilibrium existence and uniqueness condition. It can be shown that an equilibrium with underinvestment can exist and be unique for a non-trivial set of parameters.[29]

If without privateering firms under-invest in R&D relative to the efficient level, privateering may bring the level of R&D closer to the efficient level by enhancing the rent-seeking incentive, by giving larger rents to firms with larger patent portfolios. On the other hand, when firms over-invest without privateering, the rent-seeking incentive created by privateering is detrimental to welfare, as firms invest even more. One trivial example where privateering induces over-investment is when $r = 0$. In that extreme case, it is easy to see that the optimal level

[29] For instance, without transaction costs ($\hat{c} = 0$), it can be shown that $x_M = \bar{x}_M \equiv \frac{1}{N}\left(\frac{\pi+W}{V}\right)\frac{r\ln(N)}{2}$, so $\frac{r\ln(N)}{2} < x^* < x_M$ for example when W is sufficiently large. With transaction costs, the condition becomes easier to satisfy, because $x_M \geq \bar{x}_M$.

of R&D is zero,[30] whereas the equilibrium level of R&D with privateering is strictly larger than without privateering. Although measuring the private and social returns of R&D is difficult, Jones and Williams (1998) and Hall (1996) find evidence that private R&D investment is lower than the socially optimal level of investment. Hence in practice it is likely that in some industries and markets firms under-invest relative to the social optimum. In such cases privateering may attenuate the under-investment problem.

Our welfare conclusions must be interpreted with caution, subject to the assumptions in the framework we work with. For example, we explicitly incorporate weak patents through the parameter β, and very low quality patents (i.e., when $\beta R < c$) are not enforced. In practice, producing firms hold beliefs about the parameter β which can be influenced by past behavior. Thus an entity may try to build a reputation for litigiousness and be able to monetize weak patents by threatening with frivolous litigation. Such enforcement strategies are beyond the scope of our article and could lead to different welfare conclusions, especially if PAEs are more likely to carry out such litigation.

Finally, it is important to point out that these welfare results are most relevant to patent privateering, rather than to PAE licensing and litigation in general. Indeed, PAEs employ a variety of business strategies, and it is possible that many of these strategies, which are not studied in this article, could have negative welfare implications and reduce the overall level of R&D in the economy. Our welfare result is on the specific practice of privateering, and so it must be interpreted as such. Sections 5 and 6 highlight a new insights for why patent truces may dampen R&D incentives and sometimes decrease welfare, and how PAE privateers may enhance rent-seeking incentives and increase equilibrium R&D levels. However, we do not claim that PAE practices in general are welfare enhancing, and it should be clear from our discussion above that there are factors beyond our model which can lead to negative welfare conclusions.

7 Conclusions and Remarks

We identify different channels through which PAE privateers can affect the incentives to innovate when firms choose their level of R&D investment in anticipation of licensing under the threat of litigation. Privateering changes the returns to R&D by increasing patent monetization, destroying the value of defensive patent portfolios, and extracting rents. Without privateering, the fear of retaliation and the cost of litigation may preclude producing firms with similarly-sized patent portfolios from negotiating for licenses. The incentive to discover

[30] Similar to Bertrand competition, it is optimal to choose an investment as close to zero as possible.

and patent decreases when firms remain in a tacit “IP truce” equilibrium, meaning that they cannot credibly enforce their patents even when their rival infringes on them.

Under privateering, firms anticipate better continuation payoffs if they have a stronger patent portfolio. The strategic advantage of PAE privateers is that they cannot be counter-sued for patent infringement, giving them stronger litigation incentives than those of a producing firm holding the same patents. PAEs disrupt the IP truce equilibrium and create incentives on the margin to invest more in R&D, by allowing firms to capture value from their patents which would otherwise not have been realized. In fact, when firms have patent portfolios of similar size, the equilibrium payoffs they receive under privateering are identical to the payoffs they would receive in a hypothetical setting where litigation costs are zero.

Privateering also affects firms’ continuation payoffs by reducing the value of defensive patent portfolios and by potentially extracting rents from producing firms. These effects hinder the ex-ante incentive to innovate. By avoiding counter-suits, PAEs destroy the value of patents that otherwise would be used defensively in counter-suing a producing firm. This reduces the incentives to obtain those patents in the first place. PAE privateers may also extract rents from the market, reducing the firms’ incentives to invest in R&D in order to discover quickly and commercialize the final product earlier.

Our main result shows that even when PAE privateers extract rents from producing firms, do not invest in R&D, do not use the patents to make or sell products, and do not have any cost advantage in litigation with respect to the producing firms, they can increase equilibrium R&D investments. By creating credible litigation threats and taking advantage of their ability to avoid patent infringement counter-suits, PAE privateers can increase the incentive to invest in R&D. This is most likely to be the case when a large fraction of the firms’ patent portfolios cannot be monetized—e.g. when legal costs are significant and the number of components involved in the final product is large.

In Section D of the Online Appendix we discuss several variations of our model including a dynamic R&D model, asymmetric litigation costs and costly counter-suing, the role of injunctions, and a setting with more than two producing firms. Future research building on our work could incorporate a detailed downstream pricing model, multiple final products, and private information, among others. Finally, as discussed in Online Appendix A, more empirical work is required to assess the extent of patent privateering, to determine which industries are more prone to use this practice, and to determine the characteristics of the firms engaging in privateering.

8 References

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A Appendix: Proofs of the Main Results

In this Appendix we provide the proofs of the main propositions of the article. We refer the reader to Section B of the Online Appendix for proofs of the remaining results.

Proof of Proposition 2.

Proof. When a producing firm sells its entire portfolio to the PAE, the other producing firm and the PAE achieve the same joint surplus at any allocation that monetizes all patents (when possible). Suppose without loss of generality that firm A sold its entire portfolio to the PAE in their bilateral negotiation (that is, $n' = n$), and consider the negotiation between firm B and the PAE.

Firm B, on its own, can use its entire portfolio and get $T(m, 0)$ from firm A, because firm A has no patents to use defensively. The PAE will use the patents acquired from firm A against firm B, to obtain $T(n, 0)$ in licenses. Hence firm B's outside option is $S_B(n, 0) = T(m, 0) - T(n, 0)$ and the PAE's outside option (from the bilateral negotiation with firm B) is $S_{PAE}(n, 0) = T(n, 0)$.

The joint surplus between firm B and the PAE without agreement is $J_{B,PAE}(n, 0) = T(m, 0)$. Any number $z > 0$ of patents allocated from firm B to the PAE must generate a weakly lower surplus $J_{B,PAE}(n, z) \leq T(m, 0)$, and it attains this level when $z = m$.[31] Therefore, an outcome of the bargaining process between firm B and the PAE is $z^* = m$ and $p(m) = T(m, 0)$.

Thus, whenever firm A has sold everything to the PAE, the joint surplus of firm B and the PAE is maximized by selling all of B's patents to the PAE. Analogously, when firm B is selling all of its patents to the PAE, an outcome of the bilateral negotiation between firm A and the PAE is to have firm A sell all its portfolio to the PAE at price Vn. In each case, the PAE pays each firm an amount exactly equal to its licensing revenue from the acquired patents, and so earns no profit. □

Proof of Lemma 3.

Proof. Suppose by contradiction there is an equilibrium in which firm B sells $m' < m$ and retains $k = m - m' > 0$. Given m', the strategy that maximizes the joint surplus between firm A and the PAE is for firm A to retain $\ell = \max\{0, k - \hat{c}\}$ and for the PAE to acquire $n - \ell$ patents from firm A. To show this claim, we analyze two cases: $k \leq \hat{c}$ and $k > \hat{c}$.

[31] When $z > 0$, the joint surplus between firm B and the PAE is strictly less than $T(m, 0)$ when $T(m, 0) = Vm$, and $m - z < \hat{c}$ or $z < \hat{c}$, which means, respectively, that firm B or the PAE cannot credibly sue firm A.

When $k \leq \hat{c}$, firm A does not face a direct litigation threat from firm B. But if firm A were to sue firm B, those k patents would be used defensively. We distinguish the cases $m' > \hat{c}$ and $m' \leq \hat{c}$. When $m' > \hat{c}$, without an agreement between firm A and the PAE, the PAE gets Vm' from threatening to sue firm A with the patents acquired from firm B, and firm A gets $T(n, k)$ from firm B. Thus, the bilateral joint surplus without agreement is $J_{A,PAE}(0, m') = Vm' + T(n, k) - Vm'$. When $m' \leq \hat{c}$, without an agreement between firm A and the PAE, firm A gets $T(n, k)$ by threatening to sue firm B and the PAE gets 0. Thus, for any value of m', the bilateral joint surplus between firm A and the PAE without agreement is $J_{A,PAE}(0, m') = T(n, k)$. By selling everything to the PAE, firm A and the PAE generate their maximal joint surplus of $J_{A,PAE}(n, m') = Vn$, which is strictly larger than $J_{A,PAE}(0, m')$ for $m' < m$.

When $k > \hat{c}$ and firm A sells everything to the PAE, firm A leaves itself vulnerable to firm B's litigation threat. However, firm A can "cancel out" this litigation threat by holding onto some patents. The minimum number of retained patents that are sufficient to deter firm B from litigation is $\ell = k - \hat{c}$. Again, we distinguish the cases $m' > \hat{c}$ and $m' \leq \hat{c}$. When $m' > \hat{c}$, without an agreement between firm A and the PAE, the PAE gets Vm' from firm A, using the patents acquired from firm B, and firm A gets $T(n, k)$ from firm B. The bilateral joint surplus without agreement is $J_{A,PAE}(0, m') = T(n, k)$. When $m' \leq \hat{c}$, without an agreement the PAE gets 0 and firm A gets $T(n, k)$. Therefore, for any value of m', firm A and the PAE get $T(n, k)$ as joint surplus without an agreement. Consider firm A keeping $k - \hat{c}$ patents and selling $n' = n - k + \hat{c}$ to the PAE. Notice that $n > m \geq k$ implies that $n' > \hat{c}$, so the PAE can credibly monetize the patents acquired from firm A. By keeping $k - \hat{c}$ patents, firm A effectively deters firm B from starting a lawsuit. Thus, the joint surplus between firm A and the PAE in this case is $J_{A,PAE}(n', m') = Vn'$ which is strictly larger than $J_{A,PAE}(0, m')$. In fact, this is the largest joint surplus that firm A and the PAE can achieve, because selling more than n' would imply that firm B has a credible threat against firm A (which lowers the bilateral joint surplus), and selling less than n' would imply that the PAE extracts less surplus from firm B.

We have shown that firm A and the PAE best respond to $m' < m$ by playing the strategy $n'(m') = n - \max\{0, k - \hat{c}\}$. But in this case, firm B and the PAE do not maximize their joint surplus at $m' < m$, because $J_{B,PAE}(n'(m'), m') = T(m', 0) < Vm$. By selling everything to the PAE, firm B and the PAE can guarantee a larger joint surplus of Vm. Therefore, selling $m' < m$ cannot be an equilibrium. □

Proof of Lemma 4.

Proof. Suppose there is an equilibrium in which firm A sells $n' < n$ and retains $\ell = n - n' > 0$. If $\ell \leq \hat{c}$ firm A is not monetizing those patents (firm B cannot sue firm A) so firm A and the PAE could increase their joint surplus by transferring all of those patents to the PAE. Suppose, instead, that $\ell > \hat{c}$. Because the PAE cannot credibly sue firm A using firm B's patents, in this case, firm B and the PAE best respond by having firm B retain all of its patents and counter-suing firm A. But this cannot be an equilibrium, because when firm B retains all of its patents, firm A and the PAE maximize their joint surplus by allocating all the patents to the PAE and letting it monetize them. □

Proof of Proposition 3.

Proof. In this proof we find all the equilibria of the game.

1. When $n = m \leq \hat{c}$, any allocation is an equilibrium, because patents cannot be monetized. When $n = m > \hat{c}$, any equilibrium features firms either holding onto all of their patents or selling all of them. Suppose in equilibrium firm A kept $0 < \ell < n$ patents. Firm B can "cancel out" this threat by keeping $k = \ell - \hat{c}$ patents, and the PAE will be able to monetize the rest as long as $n > \ell$. Thus, firm A will always lose the value of those ℓ patents, unless $\ell = n$. Therefore, in equilibrium, firms either sell all or keep all.

$$\pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0.$$

2. When $m > \hat{c}$, by lemma 3, any equilibrium features firm B selling everything to the PAE.

   Consider an equilibrium where firm A keeps $\ell > 0$ patents. Because this is an equilibrium, firm A and the PAE must get the highest joint surplus with this allocation of patents. If firm A kept all of its patents, the joint surplus from the threat of litigation of firm A and the PAE would equal Vn. Any other equilibrium allocation must have all the patents being monetized to achieve this maximal joint surplus Vn. This is the maximal joint surplus because firm B has sold all its patents to the PAE. Therefore, for any equilibrium with $\ell > 0$ we must have $\ell \in (\hat{c}, n - \hat{c})$. This implies that when firm A deals with the PAE there is no increase in joint surplus between firm A and the PAE. Thus, the PAE extracts no rents from firm A.

   In addition, firm B and the PAE must maximize their joint surplus when firm B sells all of its patents to the PAE. Firm B and the PAE could increase their joint surplus by "cancelling out" firm A's litigation threat. The minimum number of patents that firm B must keep to prevent firm A from starting a lawsuit is $k = \ell - \hat{c}$. If the PAE can still monetize the patents bought from firm B, that is $m - k > \hat{c}$, and $k < m$, then firm B and the PAE would have a profitable deviation because by canceling out firm A's threat, firm B and the PAE can increase their joint surplus by $V\hat{c}$.[32] But this cannot be an equilibrium, because by lemma (3) firm B sells all its patents in equilibrium. Hence, either firm B does not have enough patents to cancel firm A's lawsuit ($k > m$) or, by keeping some patents, the remaining patents cannot be monetized by the PAE or firm B ($m - k < \hat{c}$). Suppose firm B has enough to "cancel out" firm A's litigation threat, but when doing so the remaining patents cannot be monetized by the PAE, which is equivalent to $m < \ell < m + \hat{c}$. In this case, by keeping all of its portfolio, firm B and the PAE have a joint bilateral surplus of $-V(n - \ell)$ which is a profitable deviation. Thus, in equilibrium it must be the case that firm B does not have enough patents to "cancel out" firm A's litigation threat. That is, $\ell > m + \hat{c}$. When firm A keeps this large amount of patents, firm B cannot avoid the litigation threat by holding onto some patents, and therefore firm B's outside option is the same as if firm A sold all of its patents to the PAE.

Therefore, for an equilibrium with $\ell > 0$ to exist, it must be the case that $m + \hat{c} < \ell < n - \hat{c}$. In this equilibrium, firm B and the PAE's joint surplus is $-V(n - m)$, and the PAE does not increase their joint surplus either, so extracts no rents from firm B.

This implies that the PAE extracts no surplus in this case.

3. When $m \leq \hat{c}$ and firm B keeps $k = m - m'$ the outside option of firm A is to use its portfolio to litigate when possible, which only happens when $n - k > \hat{c}$. In this case, firm A obtains $T(n, k)$. By selling all its patents to the PAE, firm A avoids the counterclaims brought by firm B using the portfolio it withheld. Thus, the PAE monetizes all firm A's patents and firm A does not face a threat of litigation from firm B. Thus, the joint surplus between firm A and the PAE is in this case Vn. The extra surplus from selling everything to the PAE is given by $Vn - T(n, k)$, which is split according to firm A's bargain power s.

□

Proof of Proposition 4.

Proof. We define $\delta(k) \equiv U_{PAE}(k) - U_0(k)$.

[32] This is because by holding on to k patents firm B has to pay $V(n - \ell)$ to the PAE, as the PAE monetizes the patents bought from firm A, and also the PAE gets $V(m - k)$ from monetizing the patents bought from firm B. When firm B sells everything to the PAE, the joint surplus is $-V(n - m)$ as firm B gets $-Vn$ from the litigation against firm A and the PAE, and the PAE monetizes firm B patents getting Vm.

(a) For $\hat{c} \leq k \leq N - \hat{c}$, $U_{PAE}(k) = -U_{PAE}(N-k)$. For $k \leq \hat{c}$, $U_{PAE}(k) = -V(N-k)$. And for $k > N - \hat{c}$, $U_{PAE}(k) = Vk - (1-s)V\ell_k$, where ℓ_k is the number of patents retained by the firm with the smaller portfolio in equilibrium. Thus, for $k \geq N - \hat{c}$ we have that $U_{PAE}(k) + (1-s)V\ell_k = -U_{PAE}(N-k)$. In a symmetric equilibrium $p = \frac{1}{2}$ and by using the symmetry of the binomial coefficients we have:

$$RE\left(x^*_{PAE}\right) = \frac{1}{2^N}\sum_{k=0}^{N/2}\binom{N}{k}U_{PAE}(k) + \frac{1}{2^N}\sum_{k=N/2}^{N}\binom{N}{k}U_{PAE}(N-k).$$

Using the relation between $U_{PAE}(k)$ and $U_{PAE}(N-k)$ (for all k) we have:

$$\sum_{k=0}^{N}\binom{N}{k}U_{PAE}(k) = -(1-s)V\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\ell_k.$$

Defining $\eta(k;s) = -(1-s)V\binom{N}{k}\ell_k$ and noticing that $\eta \leq 0$, we have the result.

(b) We borrow algebra from the derivation of $\Pi_x(x,x)$ in Appendix B. Because $\delta(k) \geq 0$ for $k > \frac{N}{2}$ and non-positive otherwise, we have

$$RP(x^*_{PAE}) = G(x^*_{PAE}, x^*_{PAE})\frac{V}{2^{N+2}x^*_{PAE}} \cdot \sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k) > 0.$$

(c) To show the last part of the proposition, define

$$\kappa(x^*_{PAE}) \equiv \left[RP(x^*_{PAE}) + RE(x^*_{PAE})\right] \cdot \left[G(x^*_{PAE}, x^*_{PAE})\frac{V}{2^{N+1}x^*_{PAE}}\right]^{-1}.$$

We have that $\kappa(x^*_{PAE}) > 0$ if and only if

$$\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k) + \frac{r\ln(N)}{2x^*_{PAE}}\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\eta(k;s) > 0,$$

which implies that $\kappa(x^*_{PAE}) > 0$ if and only if

$$x^*_{PAE} > \frac{r\ln(N)(1-s)V\sum_{k=N-\hat{c}}^{N}\binom{N}{k}\ell_k}{2\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)}.$$

It is easy to see that

$$0 \leq \frac{\sum_{k=N-\hat{c}}^{N}\binom{N}{k}V\ell_k}{\sum_{k=0}^{N}\binom{N}{k}(2k-N)\delta(k)} \leq \frac{\sum_{k=0}^{\hat{c}}\binom{N}{k}Vk}{\sum_{k=0}^{N}\binom{N}{k}(N-2k)(-\delta(k))} \leq 1.$$

This implies the sufficient condition: $x^*_{PAE} > \frac{(1-s)r\ln(N)}{2}$.

□

Proof of Proposition 5.

Proof. Let $foc(x)$ be the FOC without privateering, and let x^* be the equilibrium R&D level. By Proposition 4, we know that $\Delta_x(x) > 0$ for all $x > \frac{(1-s)r\ln N}{2}$. Therefore, for any x^*_{PAE} satisfying the first order condition with privateering, we have that $foc(x^*_{PAE}) < 0 = foc(x^*)$. But in the region where $x > \frac{r\ln N}{2}$, section C.2 proves that $foc(x)$ is strictly decreasing, and therefore we must have $x^* < x^*_{PAE}$. □