Rollover Risk and Credit Risk under Time-Varying Margin

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Abstract

For a firm financed by a mixture of collateralized (short-term) debt and uncollateralized (long-term) debt, we show that fluctuations in margin requirements, reflecting funding liquidity shocks, lead to increasing firm's default risk and credit spreads. The severity with which a firm is hit by increasing margin requirements highly depends on both its financing structure and debt maturity structure. Our results imply that an additional premium should be added when evaluating debt in order to account for rollover risks especially for short-matured bonds. In terms of policy implications our results strongly indicate that regulators should intervene fast to curtail margins in crisis periods and maintain a reasonably low margin level in order to effectively prevent creditors' run on debt.

JEL Codes: G10, G32, G33

Key words: funding liquidity, margin requirements, rollover risk, structural credit risk models

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1 Introduction

Ever since the liquidity crisis in the credit turmoil 2007/2008 and the subsequent European debt crisis, the importance of an adequate measurement and management of the risk inherent in short-term financing has been recognized by both financial institutions and regulators. Crises have witnessed how funding liquidity dry-ups in the lending market lead to increasing rollover costs of collateralized short-term debt and hence trigger early bank runs and firm failures. Such bankruptcies differ considerably from those due to insolvency. When investors loose confidence in a firm's ability to repay due debt obligations, they ask for higher margin for risk sharing.[1] In normal times, margins are low and the collateral value is mainly determined by the firm's fundamental value (resp. by the firm's credit worthiness). In distressed periods, however, a change in the margin rate has a great impact on the collateral value. The latest financial crisis, for instance, has seen an abrupt upward jump in the margin rate which was mostly caused by market imperfections rather than by deriorations in firms' fundamental value directly. For example, when Lehman Brothers filed for Chapter 11 bankruptcy protection on September 15, 2008, its total assets were well above total liabilities. Negative externalities, however, caused the margin rate to soar up even though the underlying fundamental value was perfoming reasonably well. When margins increase, firms can raise less funds as they need to provide more collateral for borrowing and thus are exposed to funding liquidity problems. Thus, fluctuations in margin reflecting changes in funding liquidity play a very important role in explaining firms' default risk and credit spread during crisis periods.

In this paper we present a structural credit risk model that takes into account fluctuating margin requirements and thereby allows to study how a liquidity shock, arising from margin variations in the collateralized short-term lending market, affects debt credit spread. We consider a firm financed by a mixture of long- and short-term debt reflecting a realistic debt structure. Short-term debt is collateralized and needs to be rolled over periodically. At each rollover date, the margin rate in short-term debt contracts is adjusted to the current market conditions and to the firm fundamental value. We consider *an average margin* that is an aggregate margin rate on the whole pledged assets rather than a specific margin rate on one asset class. The margin needs to be marked to market and hence can expose a firm to high funding liquidity risk at the rollover dates of short-term debt when margins fluctuate. In the sequel, we use both terms *margin* and *average margin* interchangeably.[2] Long-term debt is locked until maturity and hence does not contribute to the firm's exposure to funding liquidity risk. We are particularly interested in how a liquidity shock, reflected by an upward jump in the initial margin requirement, as well as the length of a crisis period affect the firm's default risk and credit spread. We therefore model the time-varying margin as an exogenous mean-reverting random process which is negatively correlated with the firm's fundamental value. Thus, when the fundamental value is low corresponding to firm's low credit worthiness, the margin tends to be high which in turn leads to even worse collateral values. On top of this, the margin is also affected by exogenous market shocks which can lead to increasing credit spreads even if

[1] Margin, also called haircut, is a percentage cut from the value of assets that are used as collateral to borrow. For example, when a firm pledges assets worth 100 dollars as collateral but can only borrow 80 dollars, the margin rate is $\frac{100-80}{100} = 20\%$, meaning that 20% is sliced off from the assets' value. Margin is dependent on both quality of collateral and moral hazard of creditors.

[2] High quality collateral such as high rated bonds has small variation under normal market conditions. However, typical margins on asset-backed securities and structural products can be high as $20\% - 25\%$ even in normal times (see "The role of margin requirements and haircuts in procyclicality", CGFS Papers, No 36).

the firm's fundamental value is performing well. Thus, when either a negative systemic shock or a negative idiosyncratic shock arrives, the margin increases, reflecting creditor's concerns about the firm's ability to repay due debt, which in turn yields an increase in debt credit spread. The firm defaults due to illiquidity if margin requirements are too high for it to roll over short-term debt. Hence, our model allows for two different default scenarios. On the one hand, the firm defaults due to insolvency when the firm fundamental value falls below an exogenous default threshold which depends on the firm's total liabilities as modelled in the classical structural credit risk models. On the other hand, when the firm is unable to roll over short-term debt as margin deteriorates, this triggers the firm's default due to funding liquidity risk inherent in its financing structure.

Our paper contributes to the literature on funding liquidity risk in various aspects. First, our results show that margin requirements in firm's financing can be significant in explaining default probabilities and credit spreads. When margins increase, firm's credit spreads tend to increase and this increase is more pronounced for those firms heavily relying on short-term financing or with high rollover frequency of short-term debt. When the lending conditions deteriorate, the probability of a default due to tight margin requirements increases quickly and can eventually dominate the probability of an insolvency default. Such defaults can be understood as debt runs as discussed in Covitz et al. (2013), Acharya et al. (2013), and Schroth et al. (2014). Their papers show that soaring credit spread on debt accompanied runs on short-term financial instruments, e.g., when creditors ran on asset-backed commercial papers (ABCP) starting in August 2007, on repo in September 2007, and on money market mutual funds in September 2008. This provides evidence that margin is an important variable in firm's internal risk management, especially for firms with large fractions of short-term debt. To gauge size of secured short-term debt market, Covitz et al. (2013) documents by the end of 2006, ABCP outstanding in the United States has grown up to $1.1 trillion, larger than the amount of unsecured (non-asset-backed) commercial paper (provided to firms with high-quality debt ratings) outstanding. While the repo market operating mostly over-the-counter is short of official statistics, its volume is estimated to roughly $12 trillion (see Gorton and Metrick (2012)). Hence, our model is especially suitable for financial institution relying on short-term lending, as for instance commercial paper conduits, or more general banks in the shadow banking system.[3] Our model for quantifying firm's default risk caused by tightened margin requirements is easy to implement. It encourages firms to collect margin rate data on each class of collateral which can then be used to calibrate our model for the *average margin rate*.

Secondly, we show that the variation in margin requirements over short time periods is very important to explain firm's default probability and credit spread. Schroth et al. (2014) show that time-varying spread on debt typically makes debt runs more likely. Since determining spreads on collateralized short-term debt is equivalent to setting margin requirements as argued in Danielson et al. (2012), this indicates that time-variations in margins lead to increased firm's default risk. The existing literature on the liquidity constraint or margin constraint assumes a constant margin level. In contrast, we model the margin as a time-varying process to show the innovation to the changing

[3]Former Federal Reserve Chair Ben Bernanke provided a definition in April 2012 at the 2012 Federal Reserve Bank of Atlanta Financial Markets Conference: "Shadow banking, as usually defined, comprises a diverse set of institutions and markets that, collectively, carry out traditional banking functions–but do so outside, or in ways only loosely linked to, the traditional system of regulated depository institutions. Examples of important components of the shadow banking system include securitization vehicles, asset-backed commercial paper (ABCP) conduits, money market mutual funds, markets for repurchase agreements (repos), investment banks, and mortgage companies."

lending condition. We find that a negative liquidity shock in the initial margin, representing a sharp disruption in the credit market, can lead to an increase in credit spread compared to the case of a constant margin level.

Third, our model provides a theoretical foundation that an additional discount factor for liquidity risk should be implemented when evaluating debt in classical structural credit risk models. As pointed out by Danielson et al. (2012), slacking the margin requirement in purchasing an asset is equivalent to increasing the premium. Similarly, our result implies that a funding liquidity premium should be paid to the creditors in order to compensate them for bearing the firm's rollover risk. Such an idea has been advocated as so-called *liquidity-transfer-pricing* e.g. in Financial Stability Institute (2011) and Basel Committee on Banking Supervision (2008a). With regard to interest rate derivatives, the basis spread difference for two different maturities (also called tenor spread difference) has attained a lot of attention (see, e.g., Johannes and Sundaresan (2007) and Filipović and Trolle (2013)). However, taking into account liquidity premia in debt evaluation has been discussed much less. He and Xiong (2012) add a constant liquidity premium to the discount rate when pricing debt. Their model forecasts a severe increase in the credit spread for debt when the liquidity premium increases which is consistent with our results when the margin requirement becomes tighter. Our results, however, further indicate that the premium should be different for creditors lending at different maturities. Moreover, while He and Xiong (2012) consider market liquidity, our model investigates funding liquidity risk inherent in the short-term lending market. In this sense, our paper also bears similarity with the bank run models of Morris and Shin (2010) and Liang et al. (2014). While these models assume lending conditions to be constant, in this paper we include fluctuations in margin requirements, an essential factor in the financial crisis and the European debt crisis.

Finally, our model has some important implications for monetary policy. Our results show that the faster a central bank intervenes when market is in distress, the sooner the market will restore to the normal conditions. More explicitly, when the central bank provides funding at a generous margin such that the market induced margin can also adjust to a normal level, credit spreads on debt will drop quickly to a normal level as well. These quantitative experiments indicate that a firm should carefully maintain a reasonable level of margin on its debt in signs of soaring credit spread on debt.[4] They also support what has been suggested e.g. in Ashcraft et al. (2010), Geanakoplos (2010), Gibson and Murawski (2013), and Liang et al. (2014), that margins should be well monitored by firms and regulators. Ashcraft et al. (2010) show that the traditional monetary instrument of cutting interest rates to lower the expected return on capital, actually increases attractiveness of leveraged investments through increasing shadow costs of capital. In other words, reducing interest rates enhances short-term lending as short-term funding becomes cheaper, which however might further increase financial distress through higher rollover risk. A more effective way is to combine both tools, i.e., lower interest rates and provide lending at a more generous haircut. In fact, it has been shown in Gârleanu and Pedersen (2011) and Geanakoplos (2010) that lowering margin through lending facilities had significantly decreased the required return during the financial crisis.

[4] Our model investigates how a specific company can reduce its risk exposure to a funding liquidity shock and studies how the firm can benefit from a managed margin in distressed periods. The model, however, does not deal with the risky counterparty since we do not model the creditors that provide funding. Thus, our model cannot address quantitative easing and its implications in currency rate depreciation, inflation, and many others. Taking these systemic effects into account is beyond the scope of this paper and is left for future work.

Our model shows that a firm does benefit from the quickly restored lending condition in distress and hence enhances this argument.

The remainder of the paper is structured as follows. Section 2 presents the model and discusses the default mechanism. Section 3 examines the effects of funding liquidity on a firm's default probability and credit spread. Section 4 discusses the implications of our model while Section 5 concludes. Appendix A provides details on the numerical method used in the implementation.

2 The Model

2.1 Firm's Assets

The firm fundamental value $(V_t)_{t\geq 0}$ is assumed to follow a geometric Brownian motion

$$\frac{dV_t}{V_t} = r_f dt + \sigma dW_t^1 \tag{2.1}$$

under the risk-neutral measure $\mathbb{Q}$, where r_f is the risk-free rate, σ is the volatility to firm fundamental and $(W_t^1)_{t\geq 0}$ is a standard Brownian motion under $\mathbb{Q}$ representing shocks to the firm fundamental.

2.2 Debt Structure

Suppose the firm finances its risky assets by a mixture of collateralized short-term debt and uncollateralized long-term debt as well as equity.[5] Long-term debt has an aggregate principal of L and maturity T associated with the continuously paid coupon C_L. Short-term debt, consisting e.g. of commercial papers and repo transactions, has to be rolled over periodically until time T. At time t_0 short-term debt has an aggregate principal of S and maturity t_1. Short-term debt can successively be rolled over at each date t_n for $n = 1, \ldots, N-1$ with

$$0 = t_0 < t_1 < \ldots < t_{N-1} < t_N = T .$$

The coupon associated with short-term debt is assumed to be constant C_S over time.[6] Hence, at any rollover date t_n, $n = 1, \ldots, N-1$, the firm replaces the maturing bonds by newly issued bonds with the same face value S and coupon rate C_S maturing at the next date t_{n+1}.

The secured short-term borrowing requires the firm to pledge its assets as collateral. Therefore, the creditors impose margin requirements on the firm for risk sharing, i.e., when assets are used as collateral for borrowing a certain fraction, the margin or haircut, is cut off from the asset's

[5] Our analysis can easily be extended towards a finer financing structure where a fraction of both short- and long-term debt is secured and the rest is unsecured. A finer debt financing structure would, however, only change the payoff for both types of creditors. This would only shift the default probability as a funding liquidity default in our model is defined as an event where assets are not sufficient to collateralize short-term borrowing. Therefore, our main results, induced by an illiquidity default, would stay unchanged. Furthermore, we do not specify exactly the financing sources of the firm. Instead, we only assume that the short-term debt borrowing is colleteralized as is the case e.g. in repos and commercial papers, which expose a firm to high funding liquidity risk.

[6] The endogenized short-debt coupon incorporating liquidity risk has been studied in Lütkebohmert et al. (forthcoming) and Schroth et al. (2014).

value. Assume that the firm pledges its entire assets as collateral to raise funds[7] and denote the margin rate negotiated at time t by m_t. Note that the margin in this sequel is taken as *average margin rate* on the whole class of collateralized debts. Thus, it depends on the credit rating of all pledged assets but besides also needs to be adjust to market conditions. At each rollover date t_n, $n = 1, \ldots, N-1$, the firm can then borrow up to

$$(1 - m_{t_n})V_{t_n}$$

by pledging its risky assets as collateral. Most of the existing literature assumes the margin to be given as a constant exogenous parameter. This, however, is not able to capture the risk inherent in reliance on collateralized short-term funding. When margin requirements change, this can have a dramatic effect on the refinancing situation of a firm. Therefore, we consider time-varying margin in our setting to reflect changing market conditions. In particular, we introduce time-varying margin as a mean-reverting process

$$dm_t = \kappa(\theta - m_t)dt + \eta m_t dW_t^2 \tag{2.2}$$

under the risk neutral measure, where κ is the speed with which the margin converges to its long-run mean θ and η is the volatility of the margin. $(W_t^2)_{t \geq 0}$ is another standard Brownian motion under $\mathbb{Q}$ with $\mathrm{Cov}(W_t^1, W_t^2) = \rho t$. The correlation ρ shows the co-movement between firm fundamental and margin driven by the common market factor.[8] We assume the correlation to be negative indicating that the firm fundamental value is procyclical while the margin is countercyclical.[9] The choice of a mean-reversion process is motivated in our setting as we are particularly interested in investigating a firm's resilience to a liquidity crisis in the short end. In particular, we study how a high initial margin requirement as well as the speed of mean reversion parameter affect the firm's credit spread. Furthermore, we are also interested in the question of what monetary policy tools might help to strengthen firm's resilience to periods of financial instability. It is not the aim of this paper to study how potential future crisis periods might influence the firm's default risk. In the long-run margins will fluctuate around the mean level θ in our setting. Hence, margins are still time-varying in the long-run but liquidity shocks are rather unlikely. Thus, our model does not account for the effects of potential future crisis periods.

Remark 1. Note that margin in our model is thought of as a weighted average of margin requirements on all collateral rather than the one imposed on a certain class of securities in the market. When creditors are pessimistic about the firm's ability to repay debt, a higher margin would be asked on average although the margin for some security classes with high credit rating might stay

[7] The reason is that in our setting a default due to extremely tight margin requirements occurs when the firm's assets are not enough to sustain short-term financing profile. Hence, before defaulting, the firm would first use all available assets as collateral. If also a fraction of long-term debt is collateralized by some assets, the residual assets would be used as collateral for short-term debt resulting in a shift in the default probability.

[8] If the firm's expected mean return is a constant, denoted by μ, we can propose the market prices of risk to firm fundamental shock W^1 and margin shock W^2 as

$$\frac{\mu - r_f}{\sigma\rho} \quad \text{and} \quad \frac{\mu - r_f}{\sigma\sqrt{1-\rho^2}},$$

respectively. The particular choice of market prices of risk maintains the geometric Brownian motion dynamics of the firm fundamental and the continuous GARCH(1,1) dynamics of the margin.

[9] The countercyclical margin is motivated by the work of Adrian and Shin (2014) who show that leverage is procyclical and thus, margin as the reciprocal of leverage is countercyclical versus the firm's asset value.

unchanged. Therefore, with changes of lending conditions and firm fundamental value, firm is potentially exposed to a great of variations in margin process.

Remark 2. In our model, we take the dynamics of the margin rate as exogenously given. The margin is, of course, heavily affected by the firm's credit quality. In our setting, this is reflected through the negative correlation between margin and firm fundamental value. However, changes in margin requirements can also be caused by factors independent from firm fundamental value due to negative externality, for instance, the incentivized fraud that the excess returns that become available with extreme leverage (regardless of the fundamentals of the underlying asset) create overwhelming, short-term incentives for fraud. A negative shock such as the announcement of bankruptcy of a major bank or downgrade of government bonds potentially leads to an increase in margin. In our model, we study how such variations in margins and debt financing structure together with deviations in the firm's fundamental value determine the firm's default risk.

2.3 Default and Recovery

There are two channels triggering firm's default in our model. First, deteriorations in the firm's fundamental value can trigger an insolvency default. Following Black and Cox (1976), this happens at the first passage time

$$\tau_i = \inf\{t > 0 | V_t \leq B\},$$

where B is an exogenously given insolvency threshold depending on the firm's liabilities.

Secondly, when liquidity dries up in the collateralized short-term debt market, margin as a confidence barometer in the lending market can rocket up to a level so high that the firm is unable to roll over short-term debt, i.e., the funds the firm can raise by pledging its assets as collateral, $(1 - m_t)V_t$, are less than the short-term debt S that needs to be rolled over. The highest margin $m^*_{t_n}$ at which the firm is still able to secure funding at a rollover date t_n is determined through the relation

$$\frac{(1 - m^*_{t_n})V_{t_n}}{S} = 1.$$

Thus, if the realized margin m_{t_n} at a rollover date t_n is higher than $m^*_{t_n}$, the firm defaults due to illiquidity as it cannot roll over its short-term funding. Consequently, the firm defaults due to illiquidity at

$$\tau_m = \inf\{t_n \in \{t_1, \ldots, t_{N-1}\} | (1 - m_{t_n})V_{t_n} < S\}.$$

Hence, this type of default happens when a liquidity shock hits the firm and the firesale price $(1 - m_{\tau_m})V_{\tau_m}$ of the firm fundamental is not sufficient for the firm to maintain the short-term debt profile even though the firm fundamental value V_{τ_m} at that time can be well above the insolvency threshold. Such a default is equivalent to a default due to creditors' run on short-term debt. Absent of a funding liquidity shock, the short-term creditors renew debt every period until time T.[10]

[10]Note that if some short-term creditors were to withdraw their funding for idiosyncratic reasons, the firm will always be able to find new creditors to replace them as long as there is no systematic funding liquidity shock.

Remark 3. In principle, the firm can also decide not to roll over some short-term debt and sell some assets. In that way, short-term debt principal S becomes smaller and thus through the above mechanism the default time τ_m tends to be later. We ignore this possibility in our model as asset liquidation is usually costly in periods of market distress which tend to be accompanied by high margins.

The firm's default time is therefore given by the minimum of insolvency and illiquidity default time

$$\tau = \min\{\tau_m, \tau_i\},$$

whatever happens first, and the firm's default probability at time t can be calculated as

$$\text{PD}(t) = \mathbb{Q}\left(\tau \leq T | \mathcal{F}_t\right),$$

where $\mathbb{Q}$ denotes the risk-neutral measure and $\mathcal{F}_t$ contains the available information at time t.

[Figure 1 about here.]

Figure 1 illustrates the default mechanism in our model. It shows a typical simulation of firm fundamental value $(V_t)_{t\geq 0}$ (solid line) and collateral value $((1-m_t)V_t)_{t\geq 0}$ (dashed line). The dotted line represents the insolvency default barrier B while the dash-dotted line illustrates the short-term debt principal level S. The insolvency threshold B is constant over time and the firm defaults due to insolvency when the firm fundamental value drops below the threshold level B. In contrast, the margin is time-varying and the firm defaults due to illiquidity when the firesale price of assets drops below the nominal of short-term debt S. In Figure 1, the value of the collateral falls below the short-term debt principal already after about 3 months triggering an early illiquidity default of the firm, while the firm fundamental value drops below the insolvency barrier much later in time.

Remark 4. Both firm fundamental, representing the availability of collateral, and margin as an indicator of the firm's collateralized borrowing capacity, are exogenous to our model and make default modelling straightforward. On the one hand, this avoids having to consider creditors' decision problem at every rollover decision date (see Morris and Shin (2010) and Liang et al. (2014)) and thereby dramatically simplifies numerical implementation. On the other hand, however, the fact that we have two types of debt and more than one state variable prevents us from endogenizing the threshold at which an illiquidity default occurs (see Schroth et al. (2014)).

Remark 5. A more general financing structure allowing for both collateralized and uncollateralized short- and long-term debt does not change the above described default mechanism. It might expose the firm to higher funding liquidity risk dependent on the exact financing structure though. Consider e.g. the case where a fraction of short-term debt is uncollateralized and suppose margin requirements in the collateralized lending market tighten. The unsecured short-term creditors will then hoard liquidity and will likely not roll over due debt (compare Acharya and Skeie (2011)) triggering a higher illiquidity default probability. A similar phenomenon can be observed in case long-term debt is also collateralized by a fraction of the firm's assets. Suppose that margin gradually increases such that more and more assets are required as collateral until finally the entire firm assets are depleted. The scarcity of collateral then only exaggerates the likelihood of a default caused by tightened margin requirements (funding liquidity shock).

Denote by $\bar{V}_\tau$ the recovery value of the firm's assets at default time τ. In the case of a default triggered by a funding liquidity shock, the value of liquidated assets through firesales is

$$\bar{V}_\tau = \bar{V}_{\tau_m} = (1 - m_{\tau_m})V_{\tau_m}.$$

In the case of an insolvency default, the value of liquidated asset is

$$\bar{V}_\tau = \bar{V}_{\tau_i} = RB,$$

where R denotes the recovery rate which is assumed to be exogenously given. $1 - m_{\tau_m}$ can be understood as the firesale rate in a liquidity crisis. It is usually smaller than the recovery rate R because the nominal short-term debt principal is less than the insolvency default threshold. This reflects the huge loss a firm suffers in case of a market-wide liquidity crisis.

2.4 Debt Evaluation

In case of default, the collateralized debt holders receive the recovery value from the liquidated assets first and the uncollateralized debt holders obtain what is left. Equity holders get nothing since they are the residual claimants and the firm's liquidation value will not be sufficient to pay off all creditors.

The value of the long-term debt at time t can then be calculated as

$$\begin{aligned} D_L(t) &= \mathbb{E}_t\left[\int_t^{T\wedge\tau} C_L e^{-r_f(s-t)}ds\right] + \mathbb{E}_t\left[Le^{-r_f(T-t)}1_{\{T<\tau\}}\right] \\ &\quad + \mathbb{E}_t\left[e^{-r_f(\tau-t)}1_{\{\tau=\tau_i, t\le\tau\le T\}}(\bar{V}_\tau - S)^+\right], \end{aligned} \tag{2.3}$$

where expectations are calculated under the risk-neutral measure $\mathbb{Q}$ and the subscript t indicates conditioning on available information at time t. The first term in (2.3) is the present value of coupon payments before default or maturity. The second term is the present value of principal when there is no default prior to maturity. The third term is the present value of the liquidated assets distributed to the long-term creditors in an insolvency default in which the short-term creditors receive the recovered value first. In case of a default due to a funding liquidity shock, nothing is left to the long-term creditors because the liquidated assets are worth less than the outstanding short-term debt principal.

Analogously, we can derive the value of periodically rolled over short-term debt at time t as

$$\begin{aligned} D_S(t) &= \mathbb{E}_t\left[\int_t^{T\wedge\tau} C_S e^{-r_f(s-t)}ds\right] + \mathbb{E}_t\left[Se^{-r_f(T-t)}1_{\{T<\tau\}}\right] \\ &\quad + \mathbb{E}_t\left[e^{-r_f(\tau-t)}1_{\{\tau=\tau_i, t\le\tau\le T\}}\min\{\bar{V}_\tau, S\}\right] \\ &\quad + \mathbb{E}_t\left[e^{-r_f(\tau-t)}1_{\{\tau=\tau_m, t\le\tau\le T\}}\bar{V}_\tau\right]. \end{aligned} \tag{2.4}$$

The first two terms have the same interpretation as in (2.3). The last two terms are the present values of the liquidated assets distributed to the collateralized short-term creditors in case of an insolvency default and a funding liquidity default, resp.

Given the debt values in (2.3) and (2.4), the debt yield y_j, computed as the equivalent return on debt conditional on it being held to maturity without default, is determined by solving

$$D_j(t) = \frac{C_j}{y_j}(1 - e^{y_j(T-t)}) + je^{-y_j(T-t)} \quad (2.5)$$

for $j \in \{L, S\}$. The difference between debt yield and the risk-free rate gives the credit spread on the firm's debt and will be analyzed in detail in the next section.

3 Numerical Results

3.1 Model Parameters

We calibrate our model to parameters used in the related literature on structural credit risk models. We set the risk-free rate equal to 3%. For the sensitivity analysis we choose an initial asset value equal to $V_0 = 100$ monetary units. The volatility of the firm's assets is set to $\sigma = 25\%$ as in Zhang et al. (2009). We choose the recovery rate $R = 50\%$. Custódio et al. (2013) argue that the average of debt maturity is 6 years if debt expirations are uniformly distributed. Financial firms tend to have shorter debt maturities as they rely heavily on repo transactions with maturities from one day to three months and commercial papers with maturities of less than 9 months. Therefore, we assume the maturity of long-term debt to be $T = 5$ years. For short-term debt we assume a rollover frequency of 3 months.

[Table 1 about here.]

We set the long-term debt principal to 40 monetary units with a continuously paid coupon of 3.8 monetary units. Short-term debt principal is 20 monetary units with coupon of 1.8 monetary units. This implies that the coupon rate on long-term debt equals the risk-free rate plus 650 bps and the coupon rate on short-term debt equals the risk-free rate plus 600 bps. Throughout our analysis, we keep the total debt outstanding fixed as 60 monetary units and the coupon rate to every type of debt is constant. In our baseline parameter we set the default threshold $B = 44.58$ monetary units which yields a credit spread for debt of around 230 basis points based on our model. The number lies in the range on credit spread for investment grade and speculative grade. Essentially, it is chosen to have a benchmark model to compare with.

Market data on margins unfortunately is not publicly available. However, there are financial institutions collecting margin data (see e.g. Geanakoplos (2010)), and there are reports from banks revealing margin requirements for some asset classes in certain periods.[11] The Term Asset-Backed Securities Loan Facility (TALF) and the Public-Private Investment Program (PPIP), announced in early 2009, provide bond lending at exactly 50% margin. The latter is an intermediate level between the 5% margin required at the peak of the leverage bubble and the $70-90\%$ margin demanded during the crisis in 2008. Since then, the asset market enjoyed a sound rebound in prices

[11] See, e.g., "International banking and financial market developments", BIS Quarterly Review December 2011, or "The role of margin requirements and haircuts in procyclicality", CGFS Papers, No 36

and the bond market saw a solid drop. Therefore, we choose an initial margin of $m_0 = 10\%$. Analogously to Brunnermeier and Pedersen (2009), we set the long-run mean of the margin $\theta = 10\%$ which is usually understood as the normal margin level in boom times. When margin converts to its long-run mean with rate $\kappa = 1.5$, we set $\eta = 1.2$ such that the unconditional standard variation of margin is 9.6% reflecting its low variation in normal times.[12] The rate $\kappa = 1.5$ implies a relatively slow mean reverting margin process, which means a shock takes about $252 \log 2/1.5 = 116$ days to halve the deviation of margin from its long-run mean.[13] The speed of mean reversion κ indicates the duration of the liquidity crisis period in our model as it describes how quickly the margin is falling back to a normal level. The firm fundamental and its margin requirement are affected by the common market factor. The correlation between the two driving processes is assumed to be $\rho = -50\%$. [14] The sensitivity of our results with respect to the parameters of the mean-reverting margin process will be discussed in the next subsection. Table 1 summarizes these baseline parameters of our numerical analysis. In the following default probabilities and credit spreads are calculated under the risk-neutral measure according to the change of measure outlined in footnote 8. Implementation is based on Monte Carlo method with 10,000 simulations. Details on the simulation of first passage times can be found in A.

3.2 Default Probability

The margin is a bilateral agreement between a firm and its lenders asking for collateral on either commercial papers or over-the-counter repo contracts. It measures the firm's capability to borrow, which is a credential characteristic of a firm. The higher the margin is, the less confidence the lending market has in a firm's ability to repay debt principal and coupons. Panel A in Figure 2 displays that the total default probability with respect to both daily and quarterly rollover frequency increases in the initial margin but not dramatically when the initial margin is less than 25% . The total default probability, however, starts to pick up quickly for higher initial margins of 25-50%. This is caused by the fact that the default probability due to illiquidity increases very fast while the default probability due to insolvency flattens out when the initial margin is larger than 25% , as shown in Panel B. Note that the default probability due to illiquidity is defined as the probability that the firm's default time due to illiquidity occurs earlier than the one due to insolvency. In other words, the firm's bankruptcy is caused by a funding liquidity shock while its asset value is still high enough for the firm to be considered solvent. The default time due to insolvency is analogously defined. Therefore, for a firm relying on short-term borrowing, the tightened funding condition can increase the firm's default risk greatly even if it holds high quality assets. Reports have shown that during the European debt crisis the market margin on mortgage-backed, asset-backed, and structural securities was well above 50% . The abruptly soaring margin upto a level of 50% leads to a situation where the default probability due to illiquidity dominates the default probability

[12]The unconditional variance of m_1 is given by $\theta^2/(2\kappa/\eta^2 - 1) = 0.923\%$ such that the unconditional standard deviation of margin is 9.6% (compare (Barone-Adesi et al., 2005), equation (6) on p. 290, for reference).

[13]Note that the expectation of m_t is given by $\mathbb{E}_0[m_t] = \theta + (m_0 - \theta)e^{-\kappa t}$. Thus, the condition $\mathbb{E}_0[m_t] - \theta = \frac{1}{2}(m_0 - \theta)$ implies that $1/2 = e^{-\kappa t}$ or equivalently $\log 2/\kappa = t$ where t is measured in units of years. Hence, the half life can be computed as $252 \log 2/\kappa$ days.

[14]Negative correlation means the firm fundamental value tends to be low when margin is high and vice versa. Total default probability however increases slightly in negative ρ . This is because a default in our model is determined by both the firm fundamental value V and the collateral value $(1 - m)V$. Further, the correlation between an illiquidity default and an insolvency default is determined by the correlation between V and $(1 - m)V$.

due to insolvency as shown in Panel B of Figure 2.[15] The lenders' confidence collapsed and the private lending activity basically stopped before ECB stepped up to rescue. In boom times, the funding liquidity risk is small and even negligible with very loose margin requirement. However, when market switches into regime of distress with significantly high margin, our results show that the default probability of a firm can dramatically increase even if the firm fundamental performs well.

Figure 2 further demonstrates that the more frequently the short-term debt needs to be rolled over, the higher the firm's default probability is as the firm is exposed to higher rollover risk. Especially in the limiting case when debt is rolled over on a daily basis, the default probability caused by illiquidity completely dominates the one due to insolvency when the initial margin is higher than 30% . This leads to a dramatic increase in the total default probability. A similar phenomenon has been found in He and Xiong (2012). Therefore, these results imply that firm's should account for potential fluctuations in margin requirements in their internal risk management processes as tightened margins can cause huge losses for firms relying on short-term borrowing.

[Figure 2 about here.]

Figure 3 shows that the insolvency default probability is invariant against changes of short-term debt since the insolvency default barrier is chosen constant. The total default probability and illiquidity default probability, however, increase in short-term debt financing and can weigh out the insolvency risk if short-term funds are over-used. This implies that a firm's financing structure significantly affects the firm's default risk. Especially when the firm is financed mostly through short-term collateralized borrowing, its default risk due to funding liquidity can be quite substantial. This issue will be further discussed in Section 3.3.

[Figure 3 about here.]

Finally, Table 2 shows that the default probability due to a funding liquidity shock is increasing in negative correlation ρ for financing structures with different short-term debt ratios $\frac{S}{S+L}$. For a firm heavily relying on short-term debt, this even causes the total default probability to ascend with negative correlation. We have asserted earlier that higher negative correlation between firm fundamental value and margin dynamics makes defaults more likely as margins tend to be high when the firm firm fundamental value is already low. The results in the table indicate that this is true in our model, if defaults are more frequently triggered by low firm fundamental value than by low collateral value, which is the case if less short-term debt is used. Hence, negative correlation has only a minor impact on the total default probability in case of low short-term debt ratios. This produces the relatively flattened and non-monotonic dependence of the total default probability on the correlation parameter when short-term debt ratio is relatively small. In this way, Table 2 again emphasises that the firm's financing structure has a significant impact on its default risk.

[15] There is no authoritative data on the use of haircuts/initial margins in the repo market in either Europe or the US. Table 1 in the research report published by Committee on the Global Financial System Study Group shows margin data in bilateral interviews in various financial centers with various market users, including banks, prime brokers, custodians, asset managers, pension funds and hedge funds. For reference see http://www.bis.org/publ/cgfs36.pdf

[Table 2 about here.]

Remark 6. Extremely high margins ($> 50\%$) are very rare events and can be only observed in the peak of a crisis. We have calculated the probability that this scenario happens at one of the rollover dates using our baseline parameters and assuming that short-term debt is rolled over every quarter. In this settng, the likelihood that the margin hits the extreme level of 50% at a rollover date during a 5 year period is $0.009, 0.022,$ resp. 0.045 for θ taking $0.05, 0.075,$ resp. 0.1 . Note that lowering long-run mean θ is equivalent to reducing the standard deviation of the margin process. Moreover, a very high margin m does not necessarily imply a default due to illiquidity. The latter depends not only on the margin but on the collateral quality $(1-m)V$. Thus, if the margin is high but the fundamental value is performing well, a default is less likely.

3.3 Debt Structure and Credit Spread

We now analyse the effect of liquidity shocks on the firm's credit spread for different debt structures. To make our results comparable we keep the nominal of total outstanding debt constant to $S+L=60$ monetary units and vary only the ratio of short-term debt over total debt. We calculate credit spread on aggregate debt by computing the value of long-term debt and periodically rolled over short-term debt at initial time and then calculating the debt yield on total debt, i.e., in equation (2.5) we plug in the aggregated value of long-term debt and periodically rolled over short-term debt $D_L + D_S$ for debt value, the sum of principal values $L+S$ for the nominal, and the sum of coupon payments $C_L + C_S$ for coupon to determine the aggregate debt yield. Afterwards the credit spread is derived from the debt yield by subtracting the risk-free rate. The induced credit spread on this hypothetical aggregate debt reflects the total default risk of the firm and depends on its financing and maturity structure. In Table 3 we report the credit spread for this aggregate debt and investigate how it changes with varying initial margin. We first consider the base model with parameters summarized in Table 1 where $S/(S+L) = 1/3$. In case of time-varying margin with initial margin level of 10% and for maturities of long-term debt equal to 2 years, 5 years and 10 years, we calibrate credit spread of aggregate debt to 230 bps by adjusting the insolvency bankruptcy barrier B .[16] The initial margin of $m_0 = 10\%$ as a benchmark case reflecting margins in boom times is then gradually increased first to 20% and then from 20% to 30% . The increment of 10% corresponds to one standard deviation of the margin process, reflecting the tightened funding liquidity.

[Table 3 about here.]

The results in Table 3 have interesting implications in various aspects. First, when lending conditions tighten, i.e., when initial margin increases, the credit spread increases across all maturities. For a firm financed only by $1/3$ ($= S/(L+S)$) through short-term debt, funding liquidity has a minor impact on credit spreads when margin is slightly increasing. For instance in case of 5 years maturity of long-term debt, the credit spread changes from 230 bps to 254 bps when the initial margin m_0 jumps from 10% to 20% . However, when the market is in distress and margin is high at $m_0 = 30\%$, it has a significant impact on the credit spread which then ascends to 305

[16]Note that the difference in credit spreads across maturities in the baseline model is caused by simulation errors.

bps, i.e., a 33% net increase. The credit spread increases even more if a firm is financed through more collateralized short-term borrowing. The aggregate credit spread when long-term debt has 5-year maturity shoots up to 337 bps corresponding to a 40% net increase, when margin jumps from 10% to 30% and the firm is financed through 50% short-term collateralized borrowing. Since initially the firm is remote to insolvency default, it means that even a firm with good quality of fundamental could be severely hit by liquidity dry-ups in the collateralized short-term lending market. Furthermore, by comparing credit spreads when margin is constant at 10% (the long-run mean of margin process) with the corresponding spreads when margin is time-varying starting at an initial level of 10% , our results in Table 3 show that variations in the margin process have a significant impact on debt credit spreads even in normal times. We observe e.g. a 30 bps difference in credit spreads in case of $1/3$ short-term debt financing and up to 70 bps difference in case of $2/3$ short-term debt ratio. The difference is the more pronounced if the initial margin level deviates more from its long-run mean. Thus, we conclude that the time-varying average margin may well capture the dynamic nature of funding costs that influences firm's financing decisions.

Second, the credit spread increases the more, the shorter the maturity of long-term debt is. For $T = 2$ years, the credit spread for a firm with $2/3$ short-term debt ratio could rocket up to 536 bps, about a 90% net increase, if lending conditions deteriorate and margin increases from 10% to 30% . A liquidity shock can lead to a significant jump in margin which can then trigger immediate firm defaults in the early stage as collateralized short-term borrowing becomes more expensive. A debt structure with shorter long-term debt maturity asks for a higher liquidity premium on aggregate debt which in turn leads to higher credit spreads. For a debt structure with longer maturity, the risk premium is averaged out as margin gradually reverts to its long-run mean of 10% over time. The credit spread is therefore lower for longer maturity debt structures. He and Xiong (2012) obtain similar results by taking into account the rollover costs at the arrival of every liquidity shock. According to Bao et al. (2011), the trading costs of corporate bonds more than quadrupled during the recent financial crisis. Using this size of trading costs, He and Xiong (2012) derive credit spreads for debt with one year maturity in a range of 750 bps to 800 bps. Our results are consistent with these findings if the extreme margin level of more than 30% is used corresponding to a level observed during the peak of the crisis. However, the default mechanism is different in our model since tighter margin requirements lead to early defaults or runs on short-term debt in our setting, which in turn results in higher credit spreads over short time horizons.

Finally, the results show that the more a firm is financed through collateralized short-term borrowing, the worse the firm is hit by a funding liquidity shock. Over-reliance on short-term funding was a critical factor triggering defaults during the latest financial crisis. Gopalan et al. (2010) find that firms with more short-term financing are more likely to experience multi-notch credit rating downgrades. Similarly, our results in Table 3 show that the credit spreads of firms financed by a large proportion of collateralized short-term debt increase dramatically when margin tightens, especially when maturity is short. While credit spreads in Table 3 have been calculated for short-term debt rolling over quarterly, similar results can also be obtained when short-term debt needs to be rolled over at a different frequency. The results confirm to what has been found in Liang et al. (2014, 2015) that short-term ratio and debt tenor structure have an impact on firms' default risk. The higher the rollover frequency is and the more the firm relies on collateralized short-term funding, the more it is exposed to liquidity risk and hence the higher the firm's credit spread is. Our results also support the findings in Chen et al. (2012) who regress the credit spread of corporate bonds on the share of long-term bonds close to maturity in 2008 and find a large and positive beta. They show that a high-leverage firm with short maturity has a much higher credit spread in a recession. Our model provides some explanation for this effect. In normal times when margin is low, credit spreads are relatively flat against fluctuations in margin. However, in times of financial distress when markets experience a substantial increase in margin, our model produces a big jump in credit spreads. Hence, our model allows a firm to easily conduct stress testing on its financing structure and its exposure to liquidity shocks in the shadow banking system.

4 Model Implications

4.1 Monetary Policy Implications

Since the financial crisis of 2007/2008 academics and policy makers have started to pay more attention to the important effect of margin requirements in the lending market. Geanakoplos (2010) claims that the 2007-2008 financial crisis is the bottom of a leverage (margin) cycle. Back to the 1990s, Geanakoplos (1997) already pointed out the importance to manage margins together with the lending rate in times of market distress in order to provide liquidity. In short terms, when margin requirement is loose, asset prices go up because buyers can get easy credit and spend more; when margin requirement is highly constrained, it is difficult to obtain funding and hence prices drop and credit spreads spike. The policy implication of the leverage (margin) cycle is that the regulator could cut interest rates and reduce system-wide margins to enhance liquidity in financial markets. The regulator should seek to maintain margins within a reasonable level in normal times, stepping in to raise margins in times of distress, and curtail it as investors become pessimistic and especially during a crisis.

Traditionally, regulators and economists have regarded the interest rate cut as the most important policy tool in a crisis. Whenever the economy slows down, central banks lower interest rates. Lending with a lower margin than the one, that the market is willing to offer to borrowers who might not repay, is a huge departure from the traditional monetary policy tool. However, as shown in Geanakoplos (2010), Ashcraft et al. (2010) and Liang et al. (2014), especially in times of crisis, pushing down margin is far more effective than cutting interest rates because lowering interest rates implicitly lifts shadow prices of capital and thus its function is limited to certain asset classes. It is well understood by now that central banks could be managing margins all through the leverage cycle, and should do so especially in the booming and the recessing periods. Our analysis is based on the fact that the loose margin requirement introduced by the central bank can effectively reduce the margin level faced by a firm.

[Figure 4 about here.]

In our model, the margin slightly fluctuates around a level of 10% in normal times. The reversion parameter κ indicates how quickly a high margin level arising from a funding liquidity shock returns to its normal level. Hence, κ can represent how quickly a regulator intervenes in order to lower the margin to a reasonable level after a funding liquidity shock has hit. Figure 4 shows that the faster the regulator intervenes, the lower is the aggregate credit spread. For example for $\kappa = 1.5$ it takes 116 days to halve the deviation of margin from its long-run mean while this is reduced to only 44 days when $\kappa = 4$ and hence central bank reacts faster. The corresponding decrease in the credit spread is roughly 110 bps when $S/(L+S) = 1/2$. Thus, a fast cut in the market margin on collateral borrowing can bring down the credit spread very quickly. In addition, decreasing the volatility of the margin process, e.g., through central banks providing loans at low haircuts and thereby implicitly introducing bounds on margins, can reduce credit spreads across different maturities and different financing structures as shown in Figure 5. For example the standard deviation in the steady state decreases by roughly 80% from 51% to 9.6% when η changes from 1.7 to 1.2 (assuming $\kappa = 1.5\%$). The credit spread then decreases by about 115 bps for $S/(L+S) = 1/2$. Hence, stabilizing lending conditions such as cutting margin and reducing its variation can effectively lower credit premium. These results underline the important role that margins play as an additional monetary policy tool. Furthermore, they support the voice echoed by both academics and regulators that regulators should collect data from a broad spectrum of investors on the level of margin used to buy various classes of assets and that margin data should be made transparent. Without such data it is difficult for a firm to implement any reasonable model that can account for the risk arising from fluctuating margin requirements in the collateralized lending market.

[Figure 5 about here.]

4.2 Debt Run and Liquidity Premium Discount

The standard structural credit risk models usually assume a flat interest rate benchmark on treasury bills as our model does it too. We already showed in Section 3 that credit spreads vary with margin requirements. We have observed in Table 3 that an increase in the initial margin implies an increase in credit spreads across different maturities and financing structures. The increase in some cases is so significant, more than several hundred bps, that it cannot be explained by a bid-ask spread. This indicates that the credit spread contains a component induced by funding liquidity. Such a non-insolvency based component of the credit spread has been extensively reported (see e.g. Huang and Huang (2003) or Filipović and Trolle (2013))[17]. In our model, the funding liquidity component arises from the fact that short-term creditors have an incentive to run on debt service at every rollover date if their margin requirements cannot be fulfilled. This potentially results in a firm failure due to illiquidity. To investigate this effect further, we calculate the credit spread on long-term debt and (collateralized) short-term debt separately, i.e., we have computed the debt yield in equation (2.5) for long-term debt and periodically rolled over (collateralized) short-term debt individually and then retrieved the credit spreads for both types of debt. Results are summarized in Table 4. We compare the credit spreads of short- and long-term debt in the case of no margin requirements to the ones when the initial margin is $m_0 = 10\%$, 20%, 30% for various financing structures. Our results show that the risk premium (credit spread) paid to creditors is higher in the presence of margin requirements and can be significant when short-term debt ratio is high. Moreover, the increase in risk premium to the long-term creditors is more severe than that paid to short-term creditors when the initial margin increases. For instance, when the short-term debt ratio is $1/3$, the spread for long-term debt increases from 12 to 29 and then to 60 bps

[17]Based on TRACE bond transactions data, Bao (2009) finds evidence consistent with Huang and Huang (2003).

compared to an increase from 7 to 23 and then 33 for the short-term debt when the initial margin jumps from 0 (no margin requirement present) to $m_0 = 10\%$, 20%, and then 30% respectively. The effect is more evident when the short-term debt ratio is higher and debt maturity is shorter. Therefore, our results support the fact that funding liquidity arising from both financing structure and debt maturity structure needs to be taken into account in debt valuation.

[Table 4 about here.]

Standard credit risk models used by bond rating agencies ignore the margin requirement and hence the funding liquidity risk. As earlier mentioned, Danielson et al. (2012) argue that slacking margin requirement is the same as increasing spread. Thus, the risk premium distributed to creditors is much lower than when the premium would also compensate for this type of risk. Therefore, if credit spreads are computed in standard structural credit risk models which neglect margin requirements, a liquidity premium should be added on top of the model induced default premium in order to compensate creditors for rollover risk. This implies that the discount curve should be decomposable in three components: the risk-free rate, an insolvency/defaultable premium, and a liquidity premium. He and Xiong (2012) introduce such an idea by considering a flat market liquidity premium taken as a discount rate applying to trade debt in a secondary debt market whenever a liquidity shock arrives. Our results indicate that a non-flat discount curve should be applied: a lower liquidity premium should be given to creditors providing short-term funds and a higher liquidity premium to those creditors providing long-term funds. This idea has been used in terms of liquidity transfer pricing in practice. The latter rewards a funding liquidity premium to creditors for using capital and is supported by the Basel Committee on Banking Supervision (Principle 4 in Basel Committee on Banking Supervision (2008b)). Our model supports this idea and provides insights why such a liquidity premium is important for practical implementation. A theoretical model to back out the term structure curve of this discount factor seems highly desirable, which we leave for future research.

5 Conclusion

We propose a structural credit risk model incorporating funding liquidity risk represented by variations in margin requirements in the collateralized short-term lending market. By modelling margin as a mean-reverting process, we study firm's short-term resilience to funding liquidity shocks. In the long-run, margin fluctuates around a low mean indicating a reasonable level of margin in normal times. Defaults can be either due to insolvency or triggered by a funding liquidity shock resulting from changing lending conditions such that the firm's collateral is not sufficient to roll over short-term debt. This modelling of default due to a funding liquidity shock can be considered as a run on short-term debt. It is particularly appealing as it avoids having to deal with the creditors' binary decision problem at every rollover date and the associated optimization problems. Default probabilities and credit spreads can easily be calculated numerically in our setting. Therefore, our model is useful for financial institutions in the shadow banking system, financing medium- to long-term assets by short-term contracts such as collateralized commercial papers and repos, to quantify their exposure to funding liquidity risk. Our results show that tightened margin requirements can significantly expose a firm to rollover risk, especially for firms heavily relying on short-term financing. Both financing structure and debt maturity structure significantly affect firm's default probability and credit spread when margin is time-varying. Thus, funding liquidity risk should be taken into account in firm's internal risk management as well as in debt pricing. In terms of debt pricing, an additional premium should be added to the discount curve when evaluating debt in order to account for rollover risks. Finally, our results have some important policy implications. Our results indicate that regulators shall maintain a reasonably low margin level to help firms manage maturity risk and effectively prevent creditors' maturity rat race. Furthermore, regulators shall intervene fast to curtail margins in crisis periods, which can be seen as an equally important monetary policy tool as cutting interest rates in times of financial distress.

While our results show that on individual firm level firm's can benefit from low margins resulting from liquidity injection by the government, caution is needed to analyse the effectiveness of low margin policy of a government bailout in crisis periods in a systematic way. Our approach fails to capture firm-sovereign contagion risk and moral hazard. Concerning the former, low margin requirements in the financial sector as a consequence of government bailouts is costly and increases sovereign credit risk. The deterioration of sovereign creditworthiness in turn negatively impacts the valuation of firms' bond portfolios and hence their ability to obtain funding. Such a firm-sovereign contagion is studied in Acharya et al. (2014). Concerning the latter, bailing firms out in crisis periods can lead to higher leverage (lower margin) in the next cycle due to moral hazard problems as argued in Geanakoplos (2010). The author suggests that moral hazard can be controlled and eliminated if a systemic policy of low margin gives prudent firms a better chance to survive compared to imprudent firms. Hence, if regulators use margins as additional monitary tool besides interest rates as strongly advocated by Geanakoplos (2010), it can be highly beneficial for prudent firms to collect margin data on both securities' level and investors' level in order to monitor and manage margins according to the reasonable level set by regulators.

References

Acharya, V., Drechsler, I., Schnabl, P., 2014. A pyrrhic victory? Bank bailouts and sovereign credit risk. Journal of Finance 69, 2689–2739.

Acharya, V., Schnabl, P., Suarez, G., 2013. Securitization without risk transfer. Journal of Financial Economics 107, 515–536.

Acharya, V., Skeie, D., 2011. A model of liquidity hoarding and term premia in inter-bank markets. Journal of Monetary Economics 58, 436–447.

Adrian, T., Shin, H., 2014. Procyclical leverage and value-at-risk. Review of Financial Studies 27, 373–403.

Ashcraft, A., Gârleanu, N., Pedersen, L., 2010. Two monetary tools: Interest rates and haircuts, in: NBER Macroeconomics Annual 25, pp. 143–180.

Bao, J., 2009. Structural models of default and the cross section of corporate bond yield spreads. Working paper, MIT.

Bao, J., Pan, J., Wang, J., 2011. The illiquidity of corporate bonds. The Journal of Finance 66, 911–946.

Barone-Adesi, G., Rasmussen, H., Ravanelli, C., 2005. An option pricing formula for the GARCH diffusion model. Computational Statistics and Data Analysis 49, 287–310.

Basel Committee on Banking Supervision, 2008a. Liquidity Risk: Management and Supervisory Challenges. Bank for International Settlements.

Basel Committee on Banking Supervision, 2008b. Principles for Sound Liquidity Risk Management and Supervision. Bank for International Settlements.

Black, F., Cox, J., 1976. Valuing corporate securities: Some effects of bond indenture provisions. Journal of Finance 31, 351–367.

Brunnermeier, M., Pedersen, L., 2009. Market liquidity and funding liquidity. Review of Financial Studies 22, 2201–2238.

Chen, H., Xu, Y., Yang, J., 2012. Systematic risk, debt maturity, and the term structure of credit spreads. NBER Working Paper No. w18367.

Covitz, D., Liang, N., Suarez, G., 2013. The evolution of a financial crisis: collapse of the asset-backed commercial paper market. The Journal of Finance 68, 815–848.

Custódio, C., Ferreira, M., Laureano, L., 2013. Why are US firms using more short-term debt? Journal of Financial Economics 108, 182–212.

Danielson, J., Shin, H., Zigrand, J., 2012. Procyclical leverage and endogenous risk. Working paper, available at: http://ssrn. com/abstract.

Filipović, D., Trolle, A., 2013. The term structure of interbank risk. Journal of Financial Economics 109, 707–733.

Financial Stability Institute, 2011. Liquidity transfer pricing: a guide to better practice. Occasional Paper No 10, Bank for International Settlements.

Gârleanu, M., Pedersen, L., 2011. Margin-based asset pricing and the law of one price. Review of Financial Studies 24, 1980–2022.

Geanakoplos, J., 1997. Promises, promises, in: W. B. Arthur, S.D., Lane, D. (Eds.), The Economy as an Evolving Complex System II, Addison-Wesley, Reading, MA. pp. 285–320.

Geanakoplos, J., 2010. Solving the present crisis and managing the leverage cycle. Federal Reserve Bank of New York Economic Policy Review , 101–131.

Gibson, R., Murawski, C., 2013. Margining in derivatives markets and the stability of the banking sector. Journal of Banking & Finance 37, 1119–1132.

Gopalan, R., Song, F., Yerramilli, V., 2010. Debt maturity structure and credit quality. Working paper, Washington University St. Louis.

Gorton, G., Metrick, A., 2012. Securitized banking and the run on repo. Journal of Financial Economics 104, 425–451.

He, Z., Xiong, W., 2012. Rollover risk and credit risk. Journal of Finance 67, 391–430.

Huang, J., Huang, M., 2003. How much of the corporate-treasury yield spread is due to credit risk? Review of Asset Pricing Studies 2, 153–202.

Johannes, M., Sundaresan, S., 2007. The impact of collateralization on swap rates. The Journal of Finance 62, 383–410.

Karatzas, I., Shreve, S., 1998. Brownian motion and stochastic calculus. volume 113 of *Graduate Texts in Mathematics*. 2nd ed., Springer Verlag.

Liang, G., Lütkebohmert, E., Wei, W., 2015. Funding liquidity, debt tenor structure, and creditor's belief: An exogenous dynamic debt run model. Mathematics and Financial Economics 9, 271–302.

Liang, G., Lütkebohmert, E., Xiao, Y., 2014. A multi-period bank run model for liquidity risk. Review of Finance 18, 803–842.

Lütkebohmert, E., Oeltz, D., Xiao, Y., forthcoming. Endogenous credit spreads and optimal debt financing structure in the presence of liquidity risk. European Financial Management .

Morris, S., Shin, H., 2010. Illiquidity component of credit risk. Princeton University.

Schroth, E., Suarez, G., Taylor, L., 2014. Dynamic debt runs and financial fragility: Evidence from the 2007 ABCP crisis. Journal of Financial Economics 112, 164–189.

Zhang, B., Zhou, H., Zhu, H., 2009. Explaining credit default swap spreads with the equity volatility and jump risks of individual firms. Review of Financial Studies 22, 5099–5131.

A First Passage Time Simulations

Debt evaluation and credit spread calculations are based on Monte Carlo simulation in our model. To simulate first passage times to default, we choose a partition $\{s_0, s_1, \dots, s_K\} \in [0, T]$ such that $0 = s_0 < s_1 < \cdots < s_{K-1} < s_K = T$ and the rollover dates $\{t_1, \dots, t_{N-1}\} \subset \{s_0, s_1, \dots, s_K\}$. For such a partition, we sample M paths of (V_{s_k}, m_{s_k}) , $k = 0, 1, \dots, K$, according to the dynamics of the firm fundamental (2.1) and the margin (2.2), resp. To determine the default times τ_i and τ_m for each simulated path $(V^i_{s_k}, m^i_{s_k})_{k=0,1,\dots,K}$ for $i = 1, \dots, M$, for the insolvency default time, we need to account for a potential over-shooting problem that the firm fundamental may drop below the barrier between two discretization points s_k and s_{k+1} . To address this, we tie to a Brownian bridge (see Karatzas and Shreve (1998)). Note that

$$d \ln V_t = (r_f - 0.5\sigma^2)dt + \sigma dW^1_t \, .$$

The probability that the minimum of a Brownian motion $(B_s)_{s \geq 0}$ with $B_{s_k} = \ln V_{s_k}$ and $B_{s_{k+1}} = \ln V_{s_{k+1}}$ is always above the barrier $\ln B$ in the interval $[s_k, s_{k+1}]$ can be calculated as

$$\begin{aligned} P_k &= \mathbb{Q}\left(\inf_{s_k \leq s \leq s_{k+1}} > \ln B \middle| B_{s_k} = \ln V_{s_k}, B_{s_{k+1}} = \ln V_{s_{k+1}} \right) \\ &= \begin{cases} 1 - \exp\{-2\frac{(\ln V_{s_k} - \ln B)(\ln V_{s_{k+1}} - \ln B)}{\sigma^2 (s_{k+1} - s_k)}\} & \ln V_{s_{k+1}} > \ln B \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

We make use of this to decide whether a default event has happened between s_k and s_{k+1} . The following scheme provides the first passage time to the insolvency barrier B for path i .

(a) Provided that there is no default due to insolvency prior to s_k , we set the default time due to insolvency as

$$\tau_i = s_k$$

if $\ln V^i_{s_k} \leq \ln B$.

(b) If $\ln V^i_{s_k} > \ln B$ and $\ln V^i_{s_{k+1}} > \ln B$, we set $b = \frac{s_{k+1}-s_k}{1-P^i_k}$ and generate u from a uniform distribution in the interval $[s_k, s_{k+1}+b]$. If $u \in [s_k, s_{k+1}]$, then the first passage time to the barrier occurs in this interval $[s_k, s_{k+1}]$ and we set

$$\tau_i = u.$$

(c) If u is not in the interval $[s_k, s_{k+1}]$, it implies that there is no insolvency default prior to time s_{k+1} . We then move to the next time point s_{k+1} and repeat the same process starting from (a) .

The default time due to funding liquidity for path i can be computed as follows. Since a run can only occur at the rollover dates t_n we only need to consider the discretization points s_k that coincide with a rollover date.

(a) Provided that there is no default due to funding liquidity prior to s_k , we set the default time due to funding liquidity as

$$\tau_m = s_k$$

if $(1 - m^i_{s_k}) \ln V^i_{s_k} < S$.

(b) If $(1 - m^i_{s_k}) \ln V^i_{s_k} \geq S$, it implies that there is no default at time s_k . Thus, we move to the next time point s_{k+1} and repeat the same process starting from (a) .

The default time for path i is the minimum of τ_i and τ_m . Once the default times have been determined, we can evaluate debt and calculate default probabilities and credit spreads.

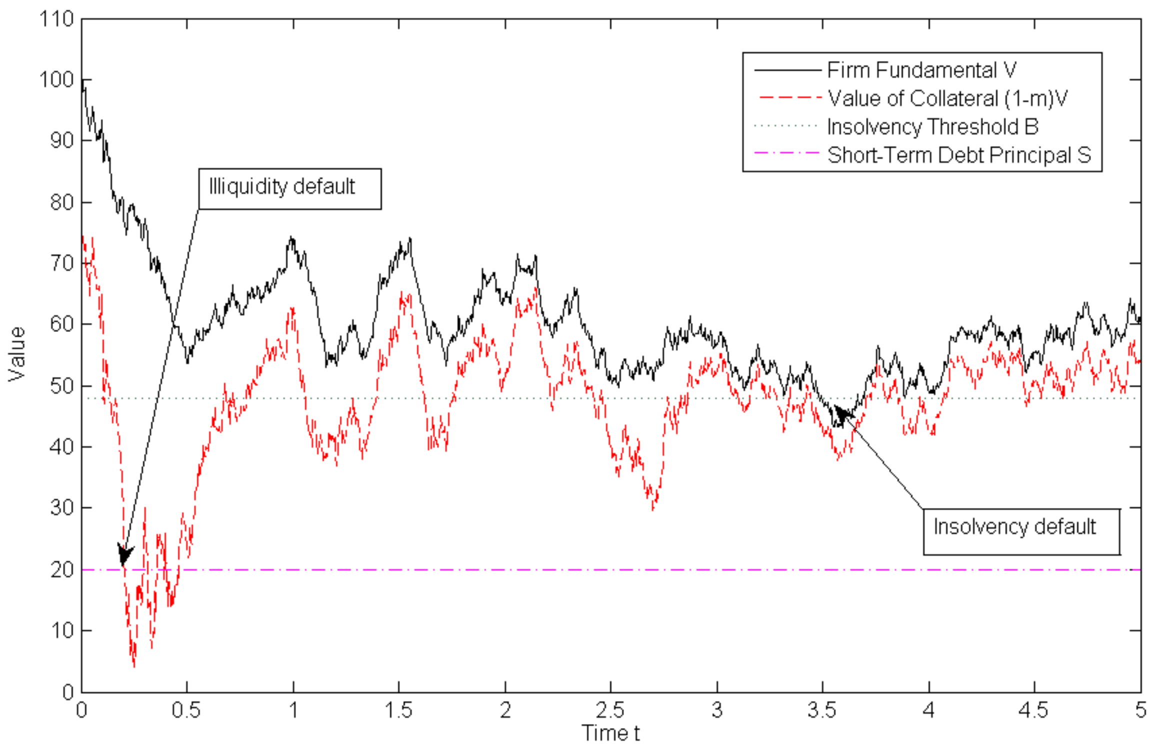

Figure 1: Simulation of Default Scenarios
The figure shows a simulation of firm fundamental value $(V_t)_{t\geq 0}$ (solid) and collateral value $((1-m_t)V_t)_{t\geq 0}$ (dashed). The time points when the firm defaults due to illiquidity and due to insolvency are marked.

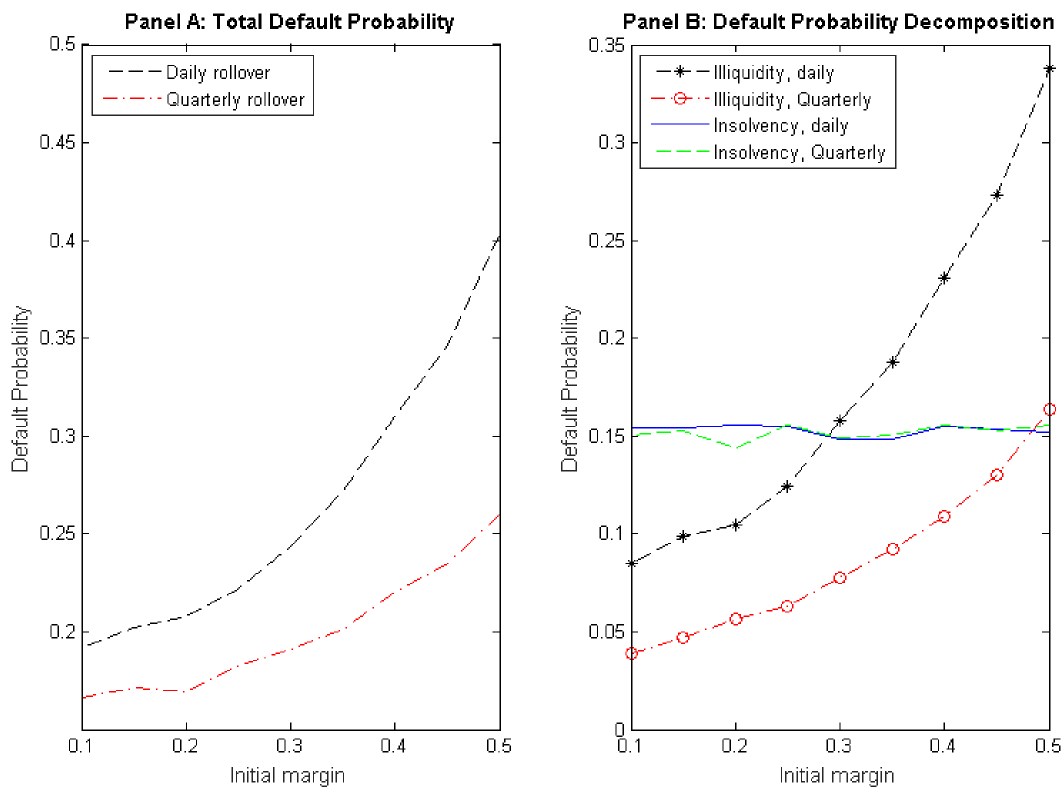

Figure 2: Dependence of Default Probability on Initial Margin.
Panel A shows the firm's total default probability while Panel B illustrates its individual components separately, the default probability due to funding liquidity $\mathbb{Q}(\tau_i \leq T, \tau_i < \tau_m)$, and the default probability due to insolvency $\mathbb{Q}(\tau_m \leq T, \tau_m < \tau_i)$, for daily and quarterly rollover frequency. The baseline parameters listed in Table 1 are used, apart from the proportion of short-term debt to total debt which is set to $1/3$ here. Calculations are performed under the risk-neutral measure.

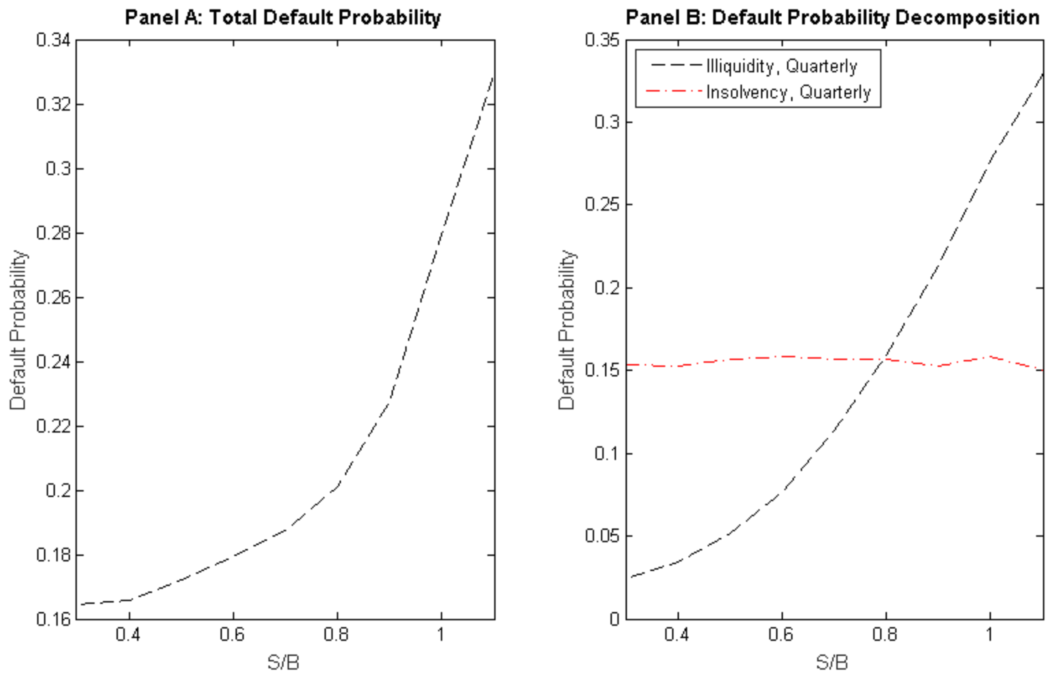

Figure 3: Dependence of Default Probability on Initial Margin.
Panel A shows the firm's total default probability versus the ratio of short-term debt to insolvency default barrier S/B while Panel B illustrates the individual components separately, i.e., the default probability due to funding liquidity $\mathbb{Q}(\tau_i \leq T, \tau_i < \tau_m)$, and the default probability due to insolvency $\mathbb{Q}(\tau_m \leq T, \tau_m < \tau_i)$, for quarterly rollover frequency. The baseline parameters listed in Table 1 are used, apart from the short-term debt S which varies such that S/B changes while B is kept constant. Calculations are performed under the risk-neutral measure.

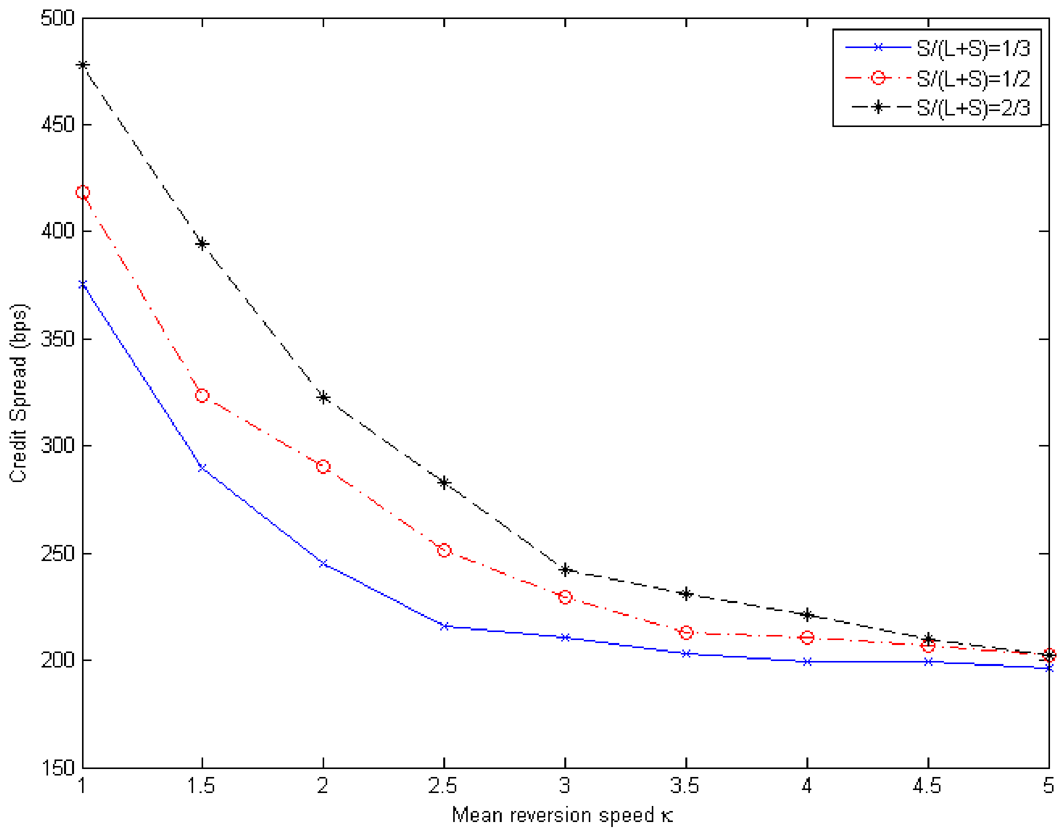

Figure 4: Impact of Speed of Mean Reversion on Credit Spread.
Results are based on the baseline parameters listed in Table I. The initial margin level is chosen at 30% . The short-term debt ratio is fixed as 1/3 (solid), 1/2 (dash-dotted), and 2/3 (dashed), resp.

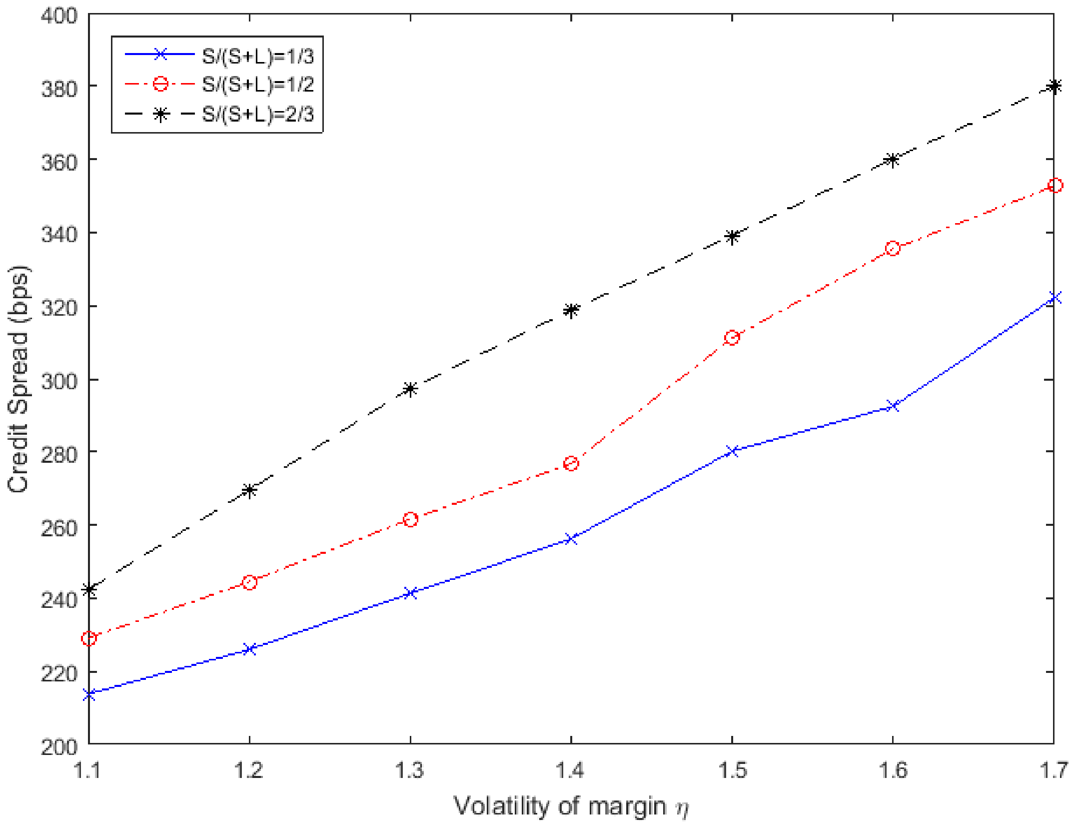

Figure 5: Impact of Volatility of Margin on Credit Spread.
Results are based on the baseline parameters listed in Table I. The short-term debt ratio is fixed as 1/3 (solid), 1/2 (dash-dotted), and 2/3 (dashed), resp.

Table 1: Baseline Parameters

Firm Characteristic			
Initial firm fundamental	V_0	=	100
Volatility of firm fundamental	σ	=	25%
Bankruptcy recovery rate	R	=	50%
Insolvency threshold	B	=	44.58
Debt Structure			
Uncollateralized long-term debt principal	L	=	40
Long-term debt coupon	C_L	=	3.8
Maturity of long-term debt	T	=	5 years
Collateralized short-term debt principal	S	=	20
Short-term debt coupon	C_S	=	1.8
Short-term debt rollover frequency	Δt	=	3 months
Margin			
Initial margin	m_0	=	10%
Speed of mean-reversion of margin	κ	=	1.5
Long-run mean of margin	θ	=	10%
Volatility of margin	η	=	120%
Correlation parameter	ρ	=	-0.5
Macro Variables			
Risk-free interest rate	r_f	=	3%

Table 2: Default probability and correlation

The table reports the total default probability $\mathbb{Q}(\tau \leq T)$ and default probability due to funding liquidity $\mathbb{Q}(\tau_i \leq T, \tau_i < \tau_m)$ for different correlations ρ between the firm fundamental value and the margin process for quarterly rollover frequency. Parameters are chosen according to Table 1.

ρ	-1	-0.8	-0.6	-0.4	-0.2	0
			$\frac{S}{S+L} = \frac{1}{3}$			
Total PD	0.159	0.166	0.165	0.167	0.166	0.173
Illiquidity PD	0.059	0.049	0.042	0.039	0.032	0.030
			$\frac{S}{S+L} = \frac{1}{2}$			
Total PD	0.172	0.178	0.175	0.190	0.183	0.172
Illiquidity PD	0.126	0.117	0.101	0.100	0.092	0.076
			$\frac{S}{S+L} = \frac{2}{3}$			
Total PD	0.232	0.232	0.227	0.216	0.207	0.198
Illiquidity PD	0.230	0.224	0.213	0.198	0.182	0.166

Table 3: Credit spreads for different debt financing structures. The parameters are based on Table 1. The ratio of short-term debt increases from 1/3 to 1/2 and to 2/3. Maturity (in years) refers to the maturity of long-term debt. Short-term debt is rolled over every 3 months. Both constant margin $m = 10\%$ and time-varying margin with initial margins $m_0 = 10\%$, 20% , and 30% are considered. The cases $m_0 = 20\%$ and $m_0 = 30\%$ describe two shocks of sizes 1 and 2 times standard deviations, respectively, to the initial margin level. Δ spread gives the difference in credit spreads after the margin shock and before the shock, i.e., for initial margin of $m_0 = 10\%$ while fraction reports the percentage increase in credit spread.

	const. $m = 10\%$	time-varying, $m_0 = 10\%$	time-varying, $m_0 = 20\%$			time-varying, $m_0 = 30\%$		
Maturity (yrs)	Spread (bps)	Spread (bps)	Spread (bps)	ΔSpread (bps)	Fraction %	Spread (bps)	ΔSpread (bps)	Fraction %
$\frac{S}{S+L} = \frac{1}{3}$								
$T = 2$	201	229	273	44	19 %	390	161	70 %
$T = 5$	202	230	254	24	10%	305	75	33 %
$T = 10$	203	230	253	23	10%	274	44	19 %
$\frac{S}{S+L} = \frac{1}{2}$								
$T = 2$	208	253	328	75	30%	437	184	72%
$T = 5$	200	241	280	39	16%	337	96	40%
$T = 10$	204	254	274	20	8%	302	48	19%
$\frac{S}{S+L} = \frac{2}{3}$								
$T = 2$	213	285	368	83	29%	536	251	88%
$T = 5$	211	275	318	43	16%	394	119	43%
$T = 10$	194	262	296	34	13%	348	86	33%

Table 4: Credit Spreads for Long- and Short-Term Debt
The table reports credit spreads for both long-term (uncollateralized) debt and periodically rolled over short-term (collateralized) debt for different levels of initial margin and different financing structures. The parameters are chosen according to Table 1.

Debt	no margin	$m_0 = 10\%$	$m_0 = 20\%$	$m_0 = 30\%$
		$\frac{S}{S+L} = \frac{1}{3}$		
Long-term	302	314	343	403
Short-term	43	50	73	106
		$\frac{S}{S+L} = \frac{1}{2}$		
Long-term	351	365	422	879
Short-term	114	125	150	269
		$\frac{S}{S+L} = \frac{2}{3}$		
Long-term	447	461	566	696
Short-term	160	162	200	257