Fuzzy Earned Value Management model for uncertain and complex projects

Leila M. Naeni
Amir Salehipour
Authors

Leila M. Naeni

- Project Management lecturer at University of Technology Sydney.
- Research on project performance evaluation and monitoring.
- Winner of the 2016 Project Management Research Award (PMAA), awarded by Australian Institute of Project Management (AIPM) – NSW.
- Hold Project Management Professional (PMP) certificate.

Amir Salehipour

- Australian Research Council DECRA Fellow
- Research on development and implementation of operations research and optimization approaches in solving real life problems.
- Member of editorial board of International Journal of Project Management (IJPM)
- Winner of the 2016 Project Management Research Award, awarded by AIPM – NSW.
Introduction to Earned Value Management (EVM)

• EVM is a well-known technique to evaluate and control the project performance

• In order to measure the project health, and predict the completion cost and time, EVM relies on three key elements:
  • Planned Value (PV)
  • Earned Value (EV)
  • Actual Cost (AC)
EVM reliability

• Reliability of EVM analysis highly depends on the correctness of its elements: PV, AC and EV.

• Different methods are recommended for measuring the EV of different project activities.

• Correct identification of the actual cost associated with the performed work requires to be differentiated from the cash outflow.
Fuzzy EVM

• Evaluating project performance when the key elements are uncertain.

  • Uncertainty in the value of performed work; e.g. see Naeni et al. 2011 for a method to represent EV by fuzzy numbers
  • Uncertainty in the cost spent in the performed work
Fuzzy numbers

\[ \tilde{A} = [60, 100, 120] \]

\[ \tilde{B} = [60, 80, 100, 120] \]
Measure uncertain percent complete

• “What fraction/percent of the activity is completed?” → Uncertain value
• Linguistic terms can be used to evaluate the percent complete of an activity or a project.

Percent Complete

<table>
<thead>
<tr>
<th>Linguistic term</th>
<th>Fuzzy number ((\tilde{P}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low</td>
<td>[0, 0, 0.1, 0.2]</td>
</tr>
<tr>
<td>Low</td>
<td>[0.1, 0.2, 0.4, 0.5]</td>
</tr>
<tr>
<td>Almost half</td>
<td>[0.4, 0.5, 0.6]</td>
</tr>
<tr>
<td>High</td>
<td>[0.5, 0.6, 0.8, 1]</td>
</tr>
<tr>
<td>Very high</td>
<td>[0.8, 0.9, 1, 1]</td>
</tr>
</tbody>
</table>
Fuzzy Earned Value

\[ \hat{EV}_i = \hat{P}_i \times BAC_i \]

\( BAC_i \): budget of activity \( i \)

\[ \hat{EV} = \sum_{i=1}^{n} \hat{EV}_i \]

E.g. EV of a work package with a total budget of $1000, which is completed by almost half, is

\[ EV = [0.4, 0.5, 0.6] \times 1000 = [400, 500, 600] \]
Measure uncertain cost

• When the actual cost spent in an activity or work item cannot be measured precisely, linguistic terms can be used to show the level of uncertainty in the measured value.

• Linguistic terms can be used to model the “Possibility of Error” in the estimated Actual Cost; e.g. if the actual cost is “about $1000”
  • a **very high** possibility of error: [800, 1000, 1200]
  • a **very low** possibility of error: [950, 1000, 1050]
Fuzzy Actual Cost

Possibility of error in AC

<table>
<thead>
<tr>
<th>Linguistic term</th>
<th>Fuzzy number $\tilde{\varepsilon}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very high</td>
<td>[-0.20, 0, 0.20]</td>
</tr>
<tr>
<td>High</td>
<td>[-0.15, 0, 0.15]</td>
</tr>
<tr>
<td>Moderate</td>
<td>[-0.10, 0, 0.10]</td>
</tr>
<tr>
<td>Low</td>
<td>[-0.05, 0, 0.05]</td>
</tr>
<tr>
<td>Very low</td>
<td>[-0.02, 0, 0.02]</td>
</tr>
</tbody>
</table>

$AC_i$: uncertain actual cost

$\tilde{AC}_i = AC_i(1 + \tilde{\varepsilon}_i)$

$\tilde{AC} = \sum_{i=1}^{n} \tilde{AC}_i$
Fuzzy Performance Index

• $\tilde{EV} = [EV_1, EV_2, EV_3, EV_4]$

• $\tilde{AC} = [AC_1, AC_2, AC_3]$

• $\tilde{SPI} = \frac{\tilde{EV}}{PV} = \left[ \frac{EV_1}{PV}, \frac{EV_2}{PV}, \frac{EV_3}{PV}, \frac{EV_4}{PV} \right]$

• $\tilde{CPI} = \frac{\tilde{EV}}{\tilde{AC}} = \left[ \frac{EV_1}{AC_3}, \frac{EV_2}{AC_2}, \frac{EV_3}{AC_2}, \frac{EV_4}{AC_1} \right]$
Interpreting a fuzzy index

- Target value of SPI and CPI is 1.
  - Above 1: the project performs better than the plan
  - Below 1: the project performs worse than the plan

Over budget

Almost on budget

Under budget

Approximately over budget

Approximately under budget
Estimating the completion

\[
\overline{EAC} = \frac{BAC}{CPI} = BAC \left[ \frac{EV_1}{AC_3}, \frac{EV_2}{AC_2}, \frac{EV_3}{AC_2}, \frac{EV_4}{AC_1} \right]
\]

\[
= \left[ \frac{BAC \times AC_1}{EV_4}, \frac{BAC \times AC_2}{EV_3}, \frac{BAC \times AC_2}{EV_2}, \frac{BAC \times AC_3}{EV_1} \right]
\]

Having a fuzzy estimate at completion (time or cost), we can compute the possibility of exceeding the project budget at completion.
# Example

<table>
<thead>
<tr>
<th>Work item</th>
<th>BAC</th>
<th>PV</th>
<th>% Complete</th>
<th>AC</th>
<th>Possibility of error in AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>700</td>
<td>High</td>
<td>~900</td>
<td>Moderate</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
<td>300</td>
<td>Less than half</td>
<td>~400</td>
<td>Very high</td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
<td>200</td>
<td>Very low</td>
<td>300</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td>300</td>
<td>20%</td>
<td>~400</td>
<td>High</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5000</td>
<td>1500</td>
<td></td>
<td>~2000</td>
<td></td>
</tr>
</tbody>
</table>
## Fuzzy EV

<table>
<thead>
<tr>
<th>Work item</th>
<th>BAC</th>
<th>% Complete</th>
<th>$\overline{EV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>High</td>
<td>[0.7, 0.8, 0.8, 0.9]</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
<td>Less than half</td>
<td>[0.2, 0.3, 0.4, 0.5]</td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
<td>Very low</td>
<td>[0, 0, 0.1, 0.2]</td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Fuzzy AC

<table>
<thead>
<tr>
<th>Work item</th>
<th>AC</th>
<th>Possibility of error</th>
<th>ε̃</th>
<th>~AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>~900</td>
<td>Moderate</td>
<td>[-0.1, 0, 0.1]</td>
<td>[810, 900, 990]</td>
</tr>
<tr>
<td>2</td>
<td>~400</td>
<td>Very high</td>
<td>[-0.2, 0, 0.2]</td>
<td>[320, 400, 480]</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>-</td>
<td>-</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>~400</td>
<td>High</td>
<td>[0.15, 0, 0.15]</td>
<td>[340, 400, 460]</td>
</tr>
<tr>
<td>Total</td>
<td>~2000</td>
<td></td>
<td></td>
<td>[1770, 2000, 2230]</td>
</tr>
</tbody>
</table>
Evaluating Project Performance

BAC = 5000 , PV = 1500

\( \widetilde{EV} = [1260, 1440, 1640, 1940] \)

\( \widetilde{AC} = [1770, 2000, 2230] \)

\[ \widetilde{CPI} = \frac{\widetilde{EV}}{\widetilde{AC}} = [0.56, 0.72, 0.82, 1.09] \]

\[ \widetilde{SPI} = \frac{\widetilde{EV}}{PV} = [0.84, 0.96, 1.09, 1.29] \]

Approximately over budget

Almost on schedule
Estimate at Completion

\[
\hat{EAC} = \frac{BAC}{CPI} = [4587, 6097, 6944, 8928]
\]

Without considering the uncertainty:
EV = 1500, AC = 2000, PV = 1500
CPI = 0.75, SPI = 1, EAC = 6667
Conclusions

• The new fuzzy EVM model is presented for complex projects, in which actual costs are inexact and uncertain.

• The developed model results in a more realistic and practical evaluation of the project performance.

• We are expanding the proposed model for more general cases.