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Performance of a multichannel active sound radiation control system near a reflecting surface

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ABSTRACT

Prior research shows that introducing a reflecting surface near an active control system can increase its noise reduction performance; however the mechanism of the performance improvement is not completely clear. This paper investigates the effects of a reflecting surface on multichannel active sound radiation control systems with a primary monopole source located on the surface. By using a genetic searching algorithm, the locations of secondary sources were optimized to maximize the noise reduction and the frequency range that can be beneficial from the reflecting surface is discussed. It is found that the performance improvement by introducing a reflecting surface is due to the increased sound pressure generated by the secondary sources at the primary source location. The beneficial frequency range extends with the number of the channels of the control system and has an upper limit frequency determined by the distance between the secondary sources and the primary source. Experiments are conducted to validate the results.

Keywords: active noise control, reflecting surface, multichannel system

1. Introduction

An active sound radiation control system employs secondary sound sources around a primary source to control its sound radiation [1]. In practice, there are usually reflecting surfaces around the system, for example, the ground and/or fire barrier walls around outdoor power transformers. This paper investigates the effects of a reflecting surface on sound radiation control performance of multichannel active noise control (ANC) systems and explores the optimal configuration of the secondary sources of the systems.

The radiation properties of sound sources near a reflecting surface are already well understood [2, 3]. An in-phase image source with an equal strength is usually introduced in the calculation when an infinitely large rigid plane presents, and the sound radiation power of a dipole source can be significantly reduced when a rigid plane is placed vertically to the dipole source axis line due to the radiation impedance reduction presented to the source by the reflecting surface [3].

The performance of a single channel ANC system in parallel and perpendicular to a rigid or soft plane has also been investigated [4]. Cunefare and Shepard found that the vertical configuration provides better noise reduction performance than the horizontal configuration for this particular application, and the influence of the plane can be neglected when the sources are placed far away from the plane (greater than one wavelength). For an ANC system with characteristic dimensions comparable to the acoustic wavelength, further study based on the boundary element models shows that the reflecting surface affects the performance significantly if the geometric center of a source is within 1/5 of a wavelength from the plane [5].

After calculating the overall power radiation from a single channel ANC system near a reflecting surface, Pan *et al.* found that the control system should be vertically placed with respect to the surface to form a longitudinal quadrupole to achieve more power reduction [6, 7]. Xue *et al.* studied the performance of a single channel ANC system near two reflecting surfaces, and proposed that the power reduction can be further improved by introducing another reflecting surface with optimized locations of the sources and surfaces [8].

When a rigid sphere is put near a single channel ANC system, the rigid sphere has scattering effects on both the primary and secondary fields and can increase global sound radiation control performance after optimizing the locations of secondary sources; however, no detailed mechanism was investigated [9]. It was also discovered that the presence of a human head in a three dimensional virtual sound barrier system can improve or decrease the system performance depending on the size of the zone surrounded by the error sensors and the noise frequency [10].

The above-mentioned papers use the total sound radiation power as the cost function, and the objective was to understand the effects of reflecting surfaces on the performance of ANC systems for global sound radiation control. There are also several papers investigating the effects of the reflecting surfaces on the performance of local ANC systems, where the main objective is to understand the variation of the quiet zone geometry rather than the total sound radiation power with control [11-15].

With all research mentioned above, no systematical research has been carried out on the location optimization of secondary sources for multichannel ANC systems with a reflecting surface [16]. This paper investigates the maximal noise reduction of a multichannel ANC system with a reflecting surface by using a genetic searching algorithm to optimize the strength and locations of the secondary sources. The formulas of the sound radiation power of a multichannel ANC system without and with a reflecting surface are given first, and then the interaction between the multichannel ANC system and a reflecting surface is analyzed to illustrate the mechanism for ANC performance improvement. Finally the effective frequency range where the noise reduction can be increased by the reflecting surface is discussed.

2. Theory

The sound radiation power of a multichannel ANC system consisting of one primary source with a constant volume velocity and N secondary sources can be formulated using the following quadratic form [17]

$$W_{\mathrm{opt}} = \mathbf{Q}^{\mathrm{H}}\mathbf{A}\mathbf{Q} + \mathbf{Q}^{\mathrm{H}}\mathbf{b} + \mathbf{b}^{\mathrm{H}}\mathbf{Q} + c\,, \tag{1}$$

where $\mathbf{Q}$ is the strength vector of the secondary sources, $\mathbf{A}$ is a $N \times N$ matrix composed by the radiation resistances between two corresponding secondary sources, $\mathbf{b}$ is a $N \times 1$ vector consisting of the mutual radiation resistances from the primary source to secondary sources, and c is the sound radiation power of the primary source without control.

Assume the distance between each secondary source and the primary source is l, the elements of the matrixes in Eq. (1) in free field are $A_{ij} = 0.5Z_0\mathrm{sinc}(kd_{ij})$, $b_i = 0.5Z_0Q_\mathrm{p}\mathrm{sinc}(kl)$, and $c = 0.5Z_0Q_\mathrm{p}^2$, where $Z_0 = \omega^2\rho_0/4\pi c_0$ is the self-radiation resistance of a monopole in free field, ω is the angular frequency, ρ_0 is the air density, c_0 is the sound speed, $k = \omega/c_0$ is the wave number, d_{ij} is the distance between the ith and the jth secondary sources, Q_p is the strength of the primary source, and the function $\mathrm{sinc}(x) = \sin(x)/x$ [4]. When the primary sound source is located at the origin of the coordinates and an infinitely large rigid plane is introduced at the plane $z = 0$ as shown in Fig. 1, the primary sound pressure in the upper half space above the reflecting surface can be obtained by adding the contributions from the primary source and its in-phase image source. The sound radiation power of the primary source W_1 turns to [17]

$$W_1 = 2W_0, \tag{2}$$

where $W_0 = 0.5Z_0Q_\mathrm{p}^2$ is the sound radiation power in the free field.

When an infinitely large rigid plane is introduced, the matrix elements in Eq. (1) are $A_{ij} = 0.5Z_0[\mathrm{sinc}(kd_{ij})+\mathrm{sinc}(kd_{i'j})]$, $b_i = Z_0Q_\mathrm{p}\mathrm{sinc}(kl)$, $c = Z_0Q_\mathrm{p}^2$, where $d_{i'j}$ is the distance between the image of the ith secondary source and the jth secondary source [4].

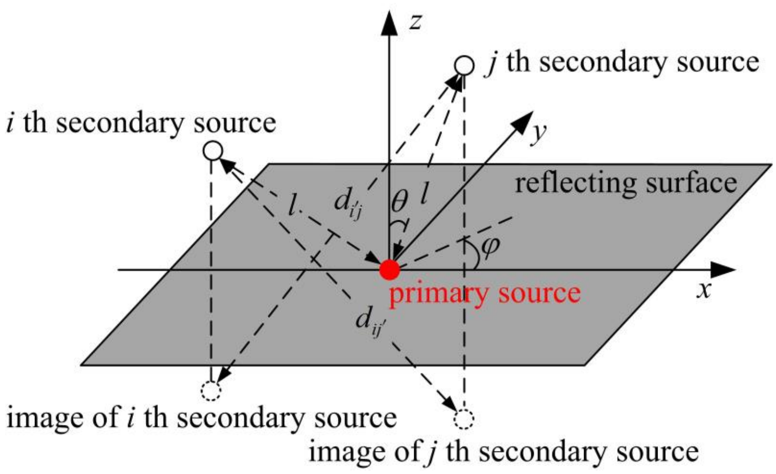


Fig. 1. A multichannel ANC system (only two secondary sources are shown in the figure) with the primary sound on an infinitely large rigid surface, the distance between each secondary source and the primary source is *l*

The optimal secondary source strength for both situations can be obtained by [17]

$$\mathbf{Q}_{\mathrm{opt}} = -\mathbf{A}^{-1}\mathbf{b}\,, \tag{3}$$

and the sound radiation power with active control is [17]

$$W_{\mathrm{opt}} = c - \mathbf{b}^{\mathrm{H}}\mathbf{A}^{-1}\mathbf{b}\,. \tag{4}$$

The noise (sound radiation power) reduction is defined as

$$NR = -10\log\left(\frac{W_{\mathrm{opt}}}{W_0}\right), \tag{5}$$

where the sound radiation power of the primary source in the free field is used as the reference so the obtained noise reduction is the net sound radiation power change to that in free field. As shown in Eq. (2), the noise reduction without active noise control is −3 dB when a reflecting surface is introduced against the primary source with a constant volume velocity.

Comparing Eq. (4) with Eq. (1) shows that the sum of the first two terms on the right hand side of Eq. (1) equals to zero with the active noise control. This indicates that the sound radiation power of the secondary sources is completely unloaded and the total sound radiation power with control is determined by the mutual unloading of the self-radiation power of the primary source [1]. It is the mutual radiation power of the primary source from the secondary sources, the $\mathbf{b}^{\mathrm{H}}\mathbf{Q}_{\mathrm{opt}}$ term, that dictates the total sound radiation power with control.

This mutual radiation power can also be formulated as $0.5\,\mathrm{Re}\left\{p_{\mathrm{S}}^{*}Q_{\mathrm{p}}\right\}$, where p_{S} is the sound pressure generated by the secondary sources at the primary source, Re{} indicates the real part of {}, and * denotes complex conjugation. Because the primary source strength Q_{p} is fixed, the increase of the mutual radiation power magnitude is proportional to the increase of the sound pressure generated by the secondary sources, and this sound pressure can be expressed as

$$p_{\mathrm{S}} = \frac{1}{2\pi l} e^{-jkl}\mathbf{I}^{\mathrm{T}}\mathbf{Q}_{\mathrm{opt}}, \tag{6}$$

where $\mathbf{I}$ is a $N \times 1$ unit vector and the superscript $^{\mathrm{T}}$ is the operator of matrix transposition. Eq. (6) shows that sound pressure produced by the secondary sources at the primary source increases with the secondary source strength and is inversely proportional to the matrix $\mathbf{A}$ according to Eq. (3). When a reflecting surface is introduced against the primary source, the total noise reduction becomes larger if the elements of $\mathbf{A}$ become smaller.

For sufficiently low frequency, A_{ij} depends only on the distance between the ith and the jth secondary sources and the distance between the image of the ith secondary source and the jth secondary source. The larger these two distances are, the smaller the value of A_{ij} will be. Therefore, the optimal secondary source locations for achieving the maximal noise reduction can be obtained by searching the minimum value of the matrix **A**.

Considering a single channel ANC system in free field, the optimal strength and the noise reduction can be obtained as [8]

$$Q_{\mathrm{opt}}^{\mathrm{f}} = -Q_{\mathrm{p}}\mathrm{sinc}(kl) \tag{7}$$

and

$$W_{\mathrm{opt}}^{\mathrm{f}} = 0.5Z_0 Q_{\mathrm{p}}^2 \left(1-\mathrm{sinc}^2(kl)\right). \tag{8}$$

Eqs. (7) and (8) indicate that the sound radiation power with control can be reduced to zero in the free field if the frequency is sufficiently low or the secondary source is sufficiently close to the primary source.

When a reflecting surface is introduced against the primary source, the optimal strength of the secondary source can be obtained by

$$Q_{\mathrm{opt}}^{\mathrm{r}} = -2Q_{\mathrm{p}}\frac{\mathrm{sinc}(kl)}{1+\mathrm{sinc}(2kl\cos\theta)}, \tag{9}$$

where θ is the zenith angle of the secondary source. Eq. (9) shows that the magnitude of $Q_{\mathrm{opt}}^{\mathrm{r}}$ increases with the decrease of the angle θ at sufficiently low frequencies. So the optimal location of the secondary source should be above the primary source to achieve the maximal noise reduction by decreasing the elements in the matrix **A** [6-8].

If the secondary source is placed at the optimal location, the sound radiation power with control is

$$W_{\mathrm{opt}}^{\mathrm{r}} = 0.5 Z_0 Q_{\mathrm{p}}^2 \left[2 - 4 \frac{\mathrm{sinc}^2(kl)}{1 + \mathrm{sinc}(2kl)} \right]. \tag{10}$$

Assuming *kl* is sufficiently small, the optimal sound radiation power with control tends to zero with the reflecting surface. Taking the term *kl* as a variable, the calculated sound radiation power curves based on Eqs. (8) and (10) show that $W_{\mathrm{opt}}^{\mathrm{r}}$ is smaller than $W_{\mathrm{opt}}^{\mathrm{f}}$ when *kl* is in the range between 0 and 0.182. This means that the optimal noise reduction is improved by introducing a reflecting surface at low frequency.

Based on the discussions above, the total sound radiation power of an ANC system with optimal control is determined completely by the mutual unloading of the self-radiation power of the primary source. When a reflecting surface is introduced against the primary source, the self-radiation power of the primary source is doubled and the variation of the mutual radiation power of the primary source is attributed to the sound pressure generated by the secondary sources at the primary source location. If the locations of the secondary sources on a given semi-sphere around the primary source can be optimized to make the mutual radiation resistance between secondary sources smaller, the optimal strength of the secondary sources increases and therefore a better noise reduction can be obtained than that in the free field.

3. Simulations and discussions

In the simulations, the primary source strength on the reflecting surface is set at 1.0 m^3/s, and the distance between the primary source and each secondary source is l = 0.124 m. A genetic searching algorithm is employed to optimize the locations of the secondary sources for maximizing the sound radiation power reduction [18]. The range of the zenith angle of the secondary sources is from 0 ° to 180 ° when the ANC system is placed in free field, and is from 0 ° to 90 ° when an the reflecting surface is introduced.

In the optimization, the sound radiation power reduction is chosen as the fitness function, and the initial population size and the maximal genetic term are chosen as 500 and 25. The normalized geometric distribution method is adopted in the selection process and the selecting probability is 0.08 [18]. The arithmetic crossover operator and the non-uniform mutation operator are performed twice during the crossover and mutation processes and the shape parameter in the non-uniform mutation operator is 3. To obtain the global optimal solution, the optimal solution obtained after each searching is adopted as an initial candidate in the next searching and the genetic searching algorithm is repeated 500 times.

Due to the spherical symmetrical property of the monopole sound source radiation, arranging the secondary sources around the primary source symmetrically can achieve the maximal noise reduction [17]. A 2-channel ANC system with secondary sources named as S1 and S2 is employed in the simulation first. With the spherically symmetrical characteristics of the primary field, the noise reduction performance does not change with azimuth angles. The azimuth angle of the secondary source with the smaller zenith angle, S1, is set as 0 °in the simulations.

3.1. The performance of a 2-channel ANC system

If a reflecting surface is introduced, the optimal zenith angles of two secondary sources obtained by using the genetic searching algorithm are presented in Fig. 2. Two angles are nearly the same, and their values are around 45 °at frequencies below 1240 Hz (the corresponding *kl* is around 2.8) and then increases to 90 °gradually with the frequency. This configuration is different from that in free field, where the theoretical analysis shows that the secondary sources should be placed in a line through the primary source to obtain the best noise reduction [17].

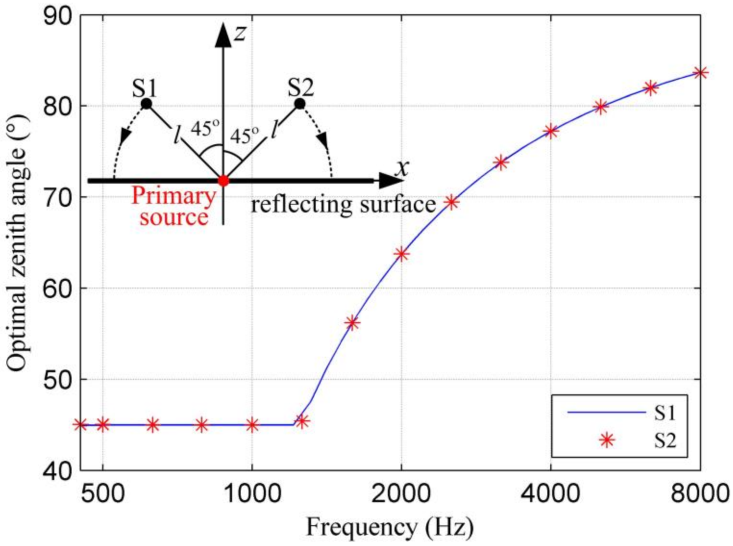


Fig. 2. Optimal zenith angles of secondary sources in a 2-channel ANC system when the primary source is on the reflecting surface

The optimal noise reduction of the 2-channel ANC system without and with a reflecting surface is shown in Fig. 3, where the optimal noise reduction in free field obtained with the genetic algorithm agrees well with the theoretical results (this confirms the validity of the genetic algorithm used in the paper). It can be observed that the optimal noise reduction decreases with the frequency first and then converges to a constant value, which is 0 dB and –3 dB respectively for configurations without and with the reflecting surface. The reason of the 3 dB difference is that the sound radiation power of the primary field is doubled when a reflecting surface is placed against the primary source. A proof is given in the Appendix to show that compared to the free field configuration, the maximal improvement of the noise reduction with a reflecting surface is 3 dB for the 2-channel ANC system when the frequency decreases to zero.

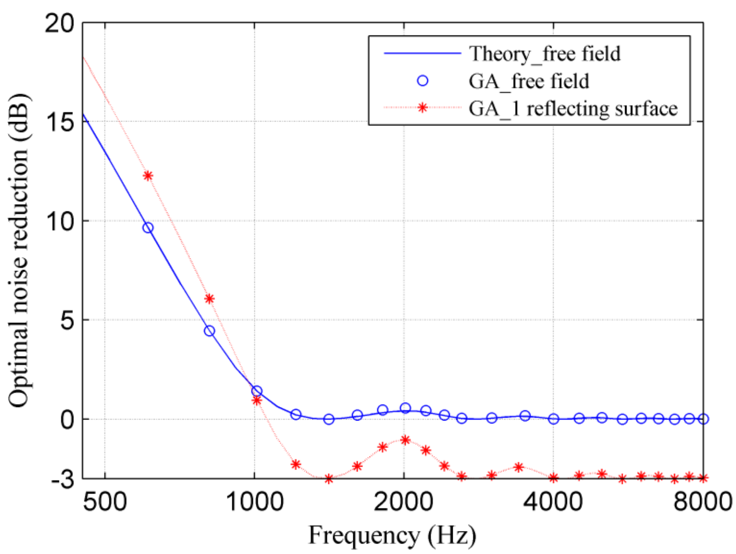
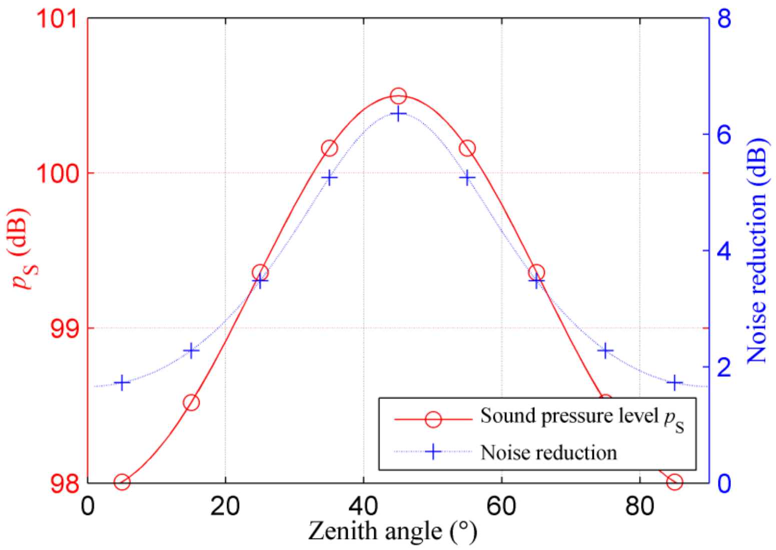


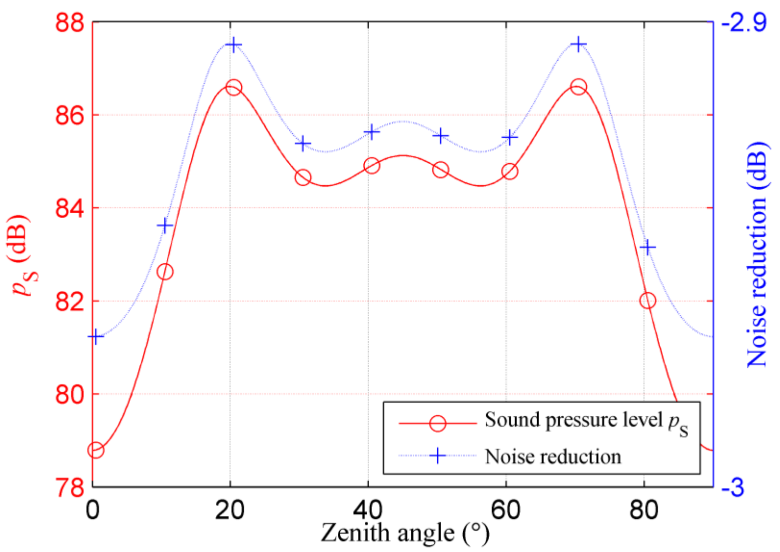
Fig. 3. Optimal noise reduction of a 2-channel ANC system without and with a

reflecting surface against the primary source

Fig. 3 also indicates that the noise reduction of the ANC system with the reflecting surface is larger at frequencies below 970 Hz but smaller at frequencies above 970 Hz. For example, the optimal noise reduction is increased from 4.7 dB to 6.4 dB at 800 Hz while it is reduced from 0 dB to −2.9 dB at 3000 Hz when the reflecting surface is introduced. The level of the sound pressure generated by the secondary sources at the primary source and the noise reduction with different zenith angles at 800 Hz and 3000 Hz are presented in Fig. 4, where it is clear that the noise reduction improvement corresponds to the increase of the sound pressure generated by the secondary sources at the primary source.



(a)



(b)

Fig. 4. Sound pressure generated by the secondary sources at the primary source and noise reduction of a 2-channel ANC system with a reflecting surface against the primary source (a) at 800 Hz (b) at 3000 Hz

In Fig. 4(a), both the optimal noise reduction and the maximal sound pressure generated by the secondary sources at the primary source location occur at the zenith angle of 45.0 °. This can be explained by analyzing the coefficient matrix **A** in Eq. (3). For the 2-channel ANC system with a reflecting surface, the matrix **A** is [17]

$$\mathbf{A} = 0.5Z_0 \begin{bmatrix} 1+\mathrm{sinc}(kd_{11'}) & \mathrm{sinc}(kd_{12})+\mathrm{sinc}(kd_{1'2}) \\ \mathrm{sinc}(kd_{21})+\mathrm{sinc}(kd_{2'1}) & 1+\mathrm{sinc}(kd_{22'}) \end{bmatrix}, \quad (11)$$

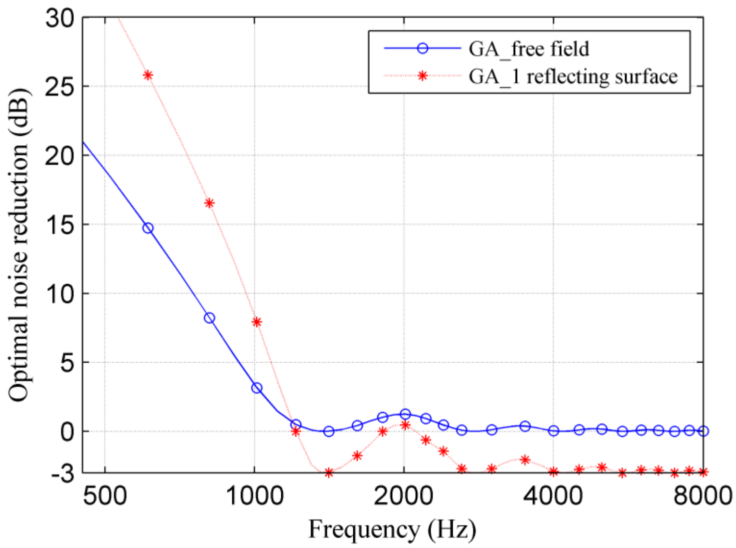
where $d_{ij'}$ is the distance between the ith secondary source and the image of the jth secondary source as shown in Fig. 1.

To maximize the sound pressure generated by the secondary sources at the primary source, the elements in the coefficient matrix **A** should be minimized, which means that the distances, $d_{11'}$, d_{12}, $d_{1'2}$ and $d_{2'2}$, should be as small as possible when the wave number k is sufficiently small. Therefore the secondary source should be placed as far as possible to the other secondary source and the reflecting surface, and this corresponds to the optimal zenith angle is 45.0 °for the 2-channel system.

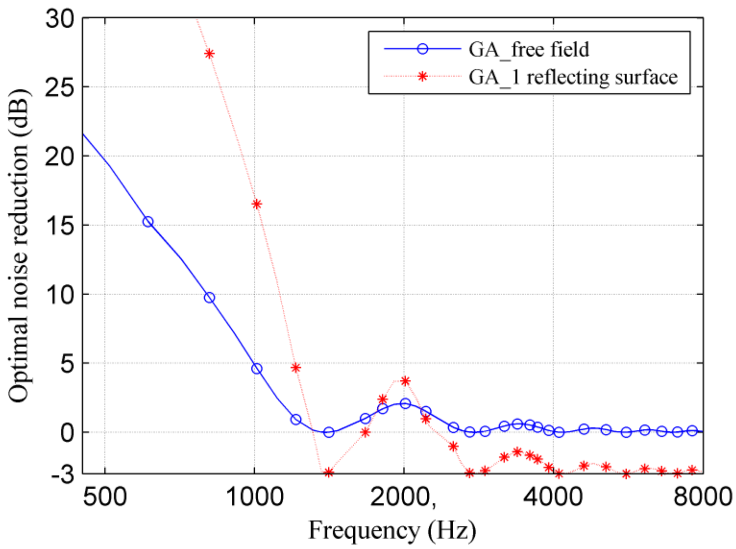
In Fig. 4(b), the maximums of both the noise reduction and the sound pressure generated by the secondary sources at the primary source appear at the zenith angle of 19.9 ° and 70.1 °. Because the whole system is symmetrical about the primary source and all the sources are in the same plane, the two configurations with a zenith angle of 19.9 °and 70.1 °are identical.

3.2. The performance of multichannel ANC systems

The optimal noise reductions of a 3-channel ANC system and a 4-channel ANC system without and with a reflecting surface are shown in Fig. 5, where the optimal noise reduction curves also decrease with the frequency first and then converges to 0 dB and −3 dB respectively without and with the reflecting surface.



(a)



(b)

Fig. 5. Optimal noise reduction of a multichannel ANC system without and with a reflecting surface against the primary source (a) a 3-channel system (b) a 4-channel system

By defining the minimal frequency that the optimal noise reduction of the system with a reflecting surface equals to that in free field as the cross frequency, the cross frequencies for the 2-channel, 3-channel and 4-channel ANC systems can be obtained from Figs. 3 and 5. The optimal noise reduction at a randomly chosen frequency 800 Hz with and without a reflecting surface together with the cross frequency are listed in Table. 1. Table. 1 shows that the cross frequency, the optimal noise reduction with and without the reflecting surface, and the improvement of the optimal noise reduction by introducing a reflecting surface increase with the number of channels of ANC systems.

Tab. 1 The cross frequency and the optimal noise reduction at 800 Hz for 2-channel, 3-channel and 4 channel ANC systems

Channel number	2-channel	3-channel	4-channel
Cross frequency (Hz)	970	1180	1280
Optimal NR in free space (dB)	4.7	8.5	10
Optimal NR with a reflecting surface (dB)	6.4	17.0	28
Improvement of the optimal NR (dB)	1.7	8.5	18

The cross frequencies for multichannel ANC systems were obtained similarly by comparing the optimal noise reduction at each frequency with and without the reflecting surface. Fig. 6 shows that the cross frequency increases with the active control channel (secondary source) number and converges approximately to 1387 Hz.

The reason will be explained below by analyzing the sound pressure produced by the secondary sources at the primary source.

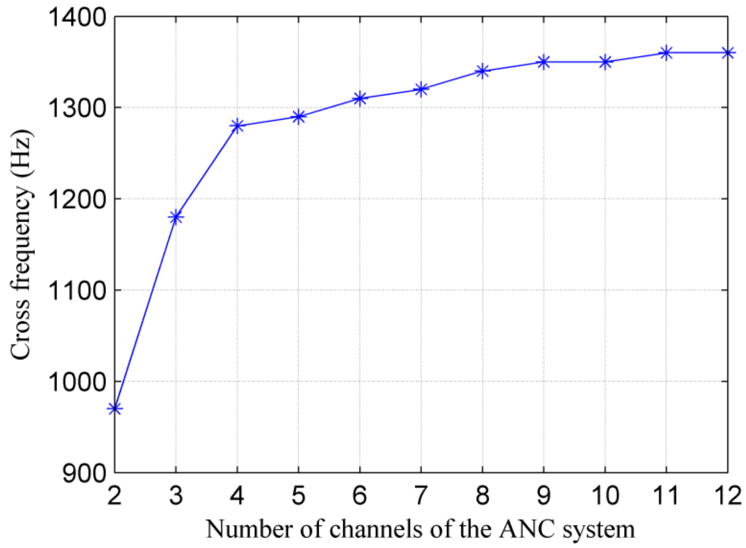


Fig. 6. The cross frequency as a function of channel (secondary source) number of the ANC system when the primary source is on a reflecting surface

The sound pressure contributed by the *i*th secondary source to the primary source is $Z_0Q_i\text{sinc}(kl)$, where Q_i is the strength of the *i*th secondary source and the value of sinc(kl) decreases with k when $kl < \pi$. The sound pressure contributed by the secondary sources equals to zero when $f = c_0/2l$, indicating that the secondary sources has no effect on the primary source radiation. The optimal noise reduction decreases to 0 dB and –3 dB when the frequency increases to $c_0/2l$ respectively for the configuration in free space and with a reflecting surface. Therefore, the cross frequency should be lower than the "half-wavelength frequency" $c_0/2l$ (1387 Hz in the simulations).

Based on the above analysis, the diagram of the optimal noise reduction of ANC systems with n and n+1 channels at frequencies below $c_0/2l$ is plotted in Fig. 7, where the optimal noise reduction decreases to 0 dB and –3 dB respectively at the frequency of $c_0/2l$ for configurations without and with the reflecting surface as shown in Figs. 3 and 5. Because the optimal noise reduction of an ANC system typically increases with the number of system channels, the noise reduction curves of the ANC system with n+1 channels should locate above the corresponding curves of the system with n channels. Therefore the intersection of the noise reduction curves with and without a reflecting surface must occur at a higher cross frequency when the number of system channels increases. For example, the intersections occur at the cross frequencies of f_n and f_{n+1} for systems with n and n+1 channels respectively where f_{n+1} is larger than f_n as shown in Fig. 7.

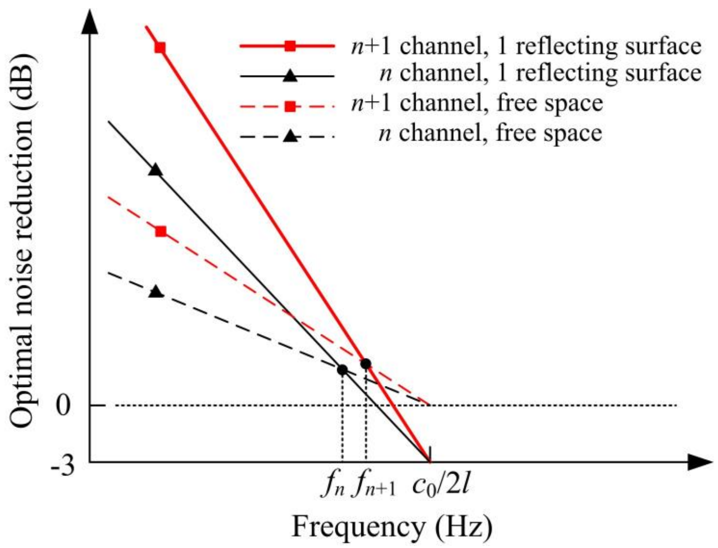


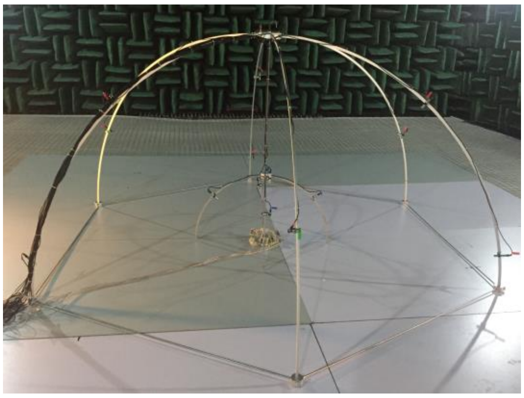
Fig. 7. Diagram of the optimal noise reduction of multichannel ANC systems at low

frequencies

This section employs a genetic algorithm to search locations for the optimal noise reduction of an ANC system with a given channel number, and the results support the analyses in Section 2. It is confirmed that the noise reduction improvement with a reflecting surface is caused by the increase of the sound pressure generated by the secondary sources at the primary source location, and each secondary source needs to be placed as far as possible to other secondary sources and the reflecting surface to increase the noise reduction improvement. The cross frequencies are identified by comparing the optimal noise reduction curves of the multi-channel ANC systems with and without a reflecting surface as shown in Figs. 3, 5 and 7. The upper limit of the cross frequency for the multichannel ANC systems under investigation is "half-wavelength frequency" $c_0/2l$, where l is the distance between the secondary source and the primary source.

4. Experiments

Experiments with a 2-channel ANC system and a 3-channel ANC system were conducted in a large anechoic chamber as shown in Fig. 8. A wooden plate with an area of 4.8 m × 4.8 m and a thickness of 1.8 mm is used as the reflecting surface. Three support frames (with three different radiuses, 1.50 m, 0.50 m and 0.088 m) centered at the primary source (also considered as the coordinate origin in this section) were used to place the monitoring microphones, the error microphones and the sound sources respectively. Ten monitoring microphones were installed on the largest semi-sphere support frame to obtain the sound radiation power in the upper half space according to ISO-3744 [19]. Five error microphones were placed on a semi-sphere support frame in the configuration with the wooden plate against the primary source, as shown in Fig. 8(a). One error microphone was placed on the top of the semi-sphere and other 4 error microphones were placed with the zenith angle of 45^o and an azimuth angle difference of 45^o. In the configuration without the plate, another symmetrical semi-sphere support frame was added and 10 error microphones were used in total as shown in Fig. 8(b). The smallest support frame for sound source placement has two types, one for the 2-channel ANC system and one for the 3-channel system, as shown in Fig. 8(c) and Fig. 8(d), where the zenith angle of secondary sources can be adjusted manually.



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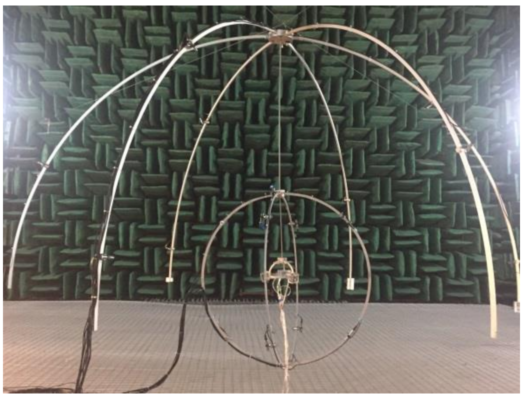

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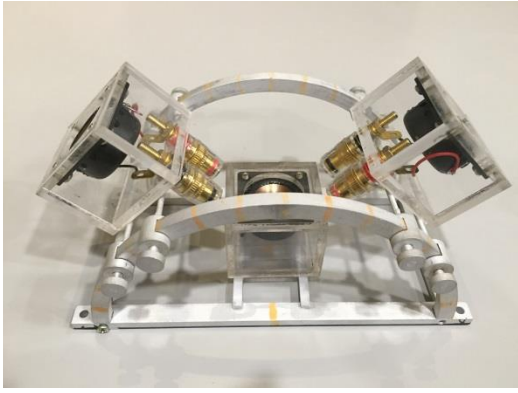

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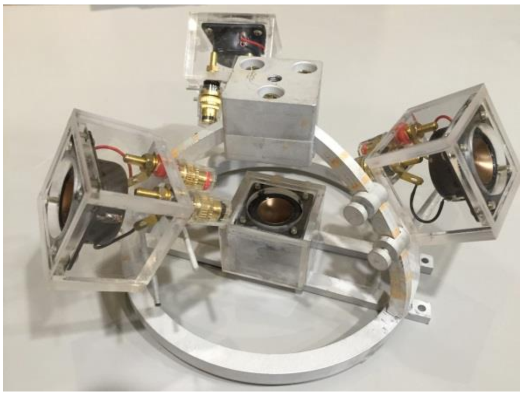
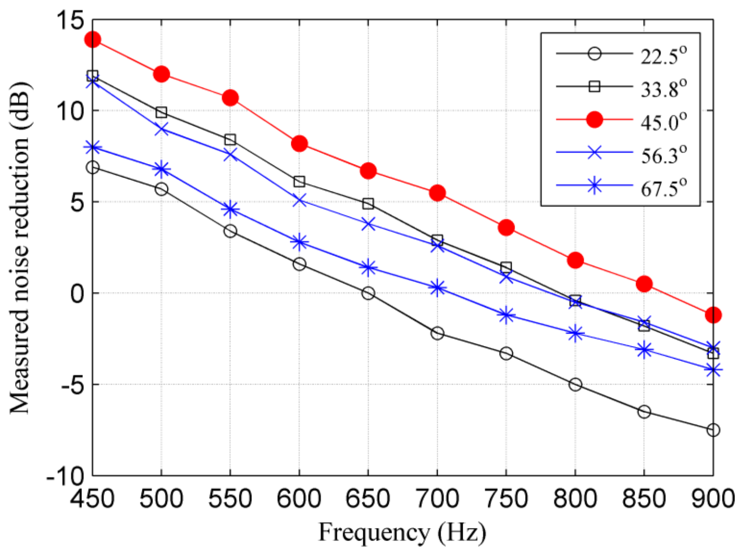


(d)

FIG. 8. Experimental setup (a) a 3-channel ANC system setup with a wooden plate as the reflecting surface (b) a 3-channel ANC system setup in free field (c) a configuration of a 2-channel ANC system (d) a configuration of a 3-channel ANC system

All the primary source and secondary sources are custom made loudspeakers, and each source was made by assembling a 1 inch loudspeaker unit in a 48 mm × 48 mm × 36 mm plexiglass box. A commercial active noise controller (Antysound Tiger ANC-II) embedded with the Filtered-x LMS algorithm was used for control and a multi-channel analyzer (B&K PULSE 3560D) was used for data analysis. The electrical signal driving the primary source was also fed to the controller as the reference signal. Considering the frequency response of the loudspeaker and the computation capability of the controller, the experiments were conducted at a number of single frequencies from 450 Hz to 900 Hz with an interval of 50 Hz.

In the experiments, the zenith angle of all the secondary sources were kept the same and adjusted synchronously. The measured noise reduction with the wooden plate as the reflecting surface is shown in Fig. 9, where the noise reduction increases first and then decreases with the zenith angle of the secondary sources. The simulated noise reduction with a reflecting surface is shown in Fig. 10, assuming that the zenith angles of secondary sources are the same. Both measurement and simulation results show that the maximal noise reduction occurs when the zenith angle is around 45° for the 2-channel ANC system and 54° for the 3-channel ANC system.



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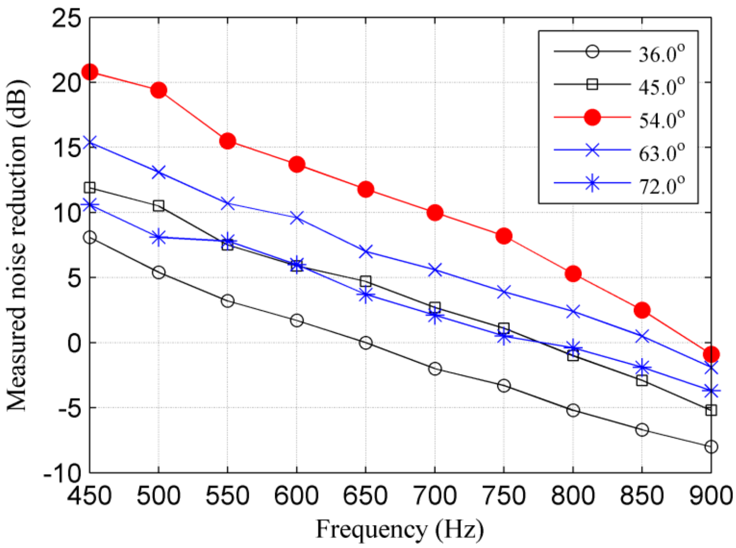
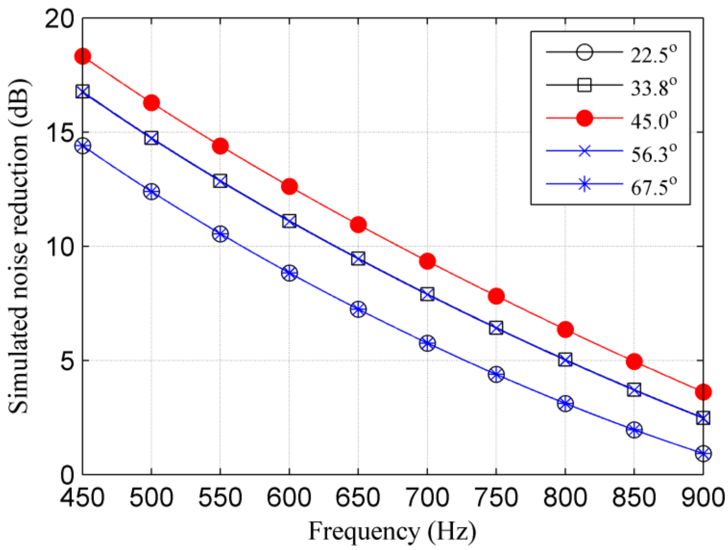
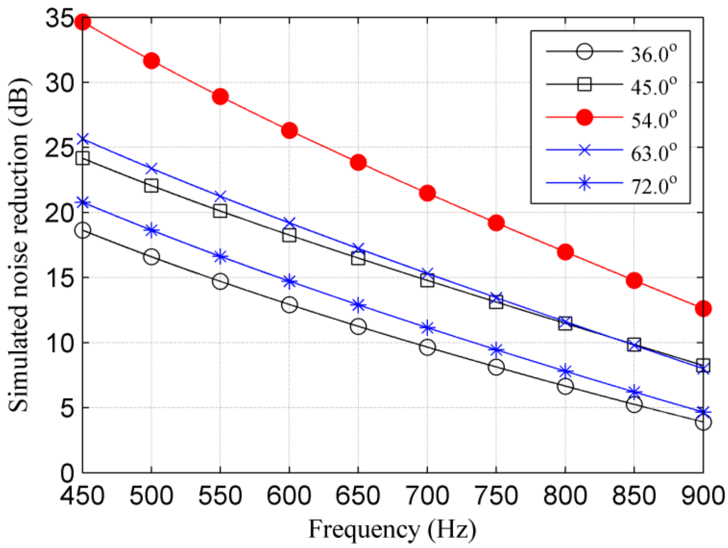


(b)

Fig. 9. Measured noise reduction with a wooden plate as the reflecting surface at different zenith angle of the secondary sources (a) of the 2-channel system (b) of the 3-channel system



(a)



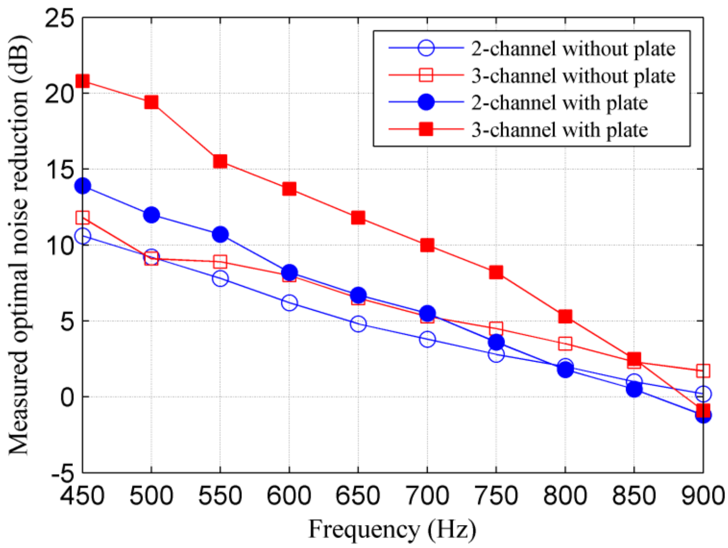
(b)

Fig. 10. Simulated noise reduction with a reflecting surface at different zenith angle of the secondary sources (a) of the 2-channel system (b) of the 3-channel system

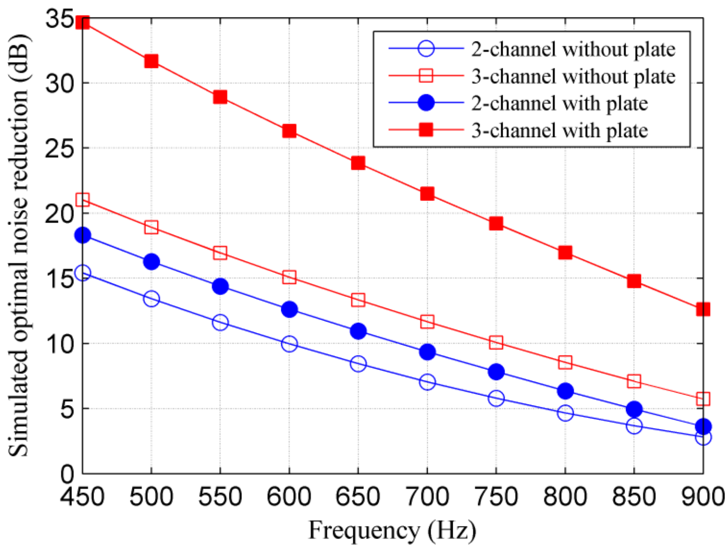
If the 2-channel system or 3-channel ANC system is arranged in its optimal configuration, the distance between the secondary sources and the distance between the secondary source and its corresponding image are smaller in the optimal 3-channel system, indicating that the optimal secondary source strength of the optimal 3-channel system is smaller. However, the sound pressure contributed by the 3 secondary sources at the primary source location of the optimal 3-channel system is larger, so the noise reduction of the optimal 3-channel system is larger than that of the 2-channel system.

The measured and simulated optimal noise reduction with and without the wooden plate is compared in Fig. 11. It is clear that the noise reduction can be improved by introducing the wooden plate and this improvement is more significant at lower frequency. For example, the maximal noise reduction improvement after introducing the wooden plate occurs at 450 Hz in both Figs. 11(a) and 11(b). The maximal noise reduction improvement from simulations is 2.9 dB for the 2-channel system and 13.6 dB for the 3-channel system while the measured improvement is 3.3 dB and 9.0 dB respectively.

The reasons that the observed maximal improvement is larger than 3 dB for the 2-channel ANC system is caused by the cost function used in the experiments, where the sum of the squared sound pressure at the error microphones rather than the total sound radiation power was adopted as the cost function, so the measured optimal noise reduction differs a little from the theoretical value. For example, the optimal noise reduction of the 3-channel system without the reflecting plate is larger than that of the 2-channel system with the plate in the theoretical simulation; however, the measured optimal noise reduction of the 3-channel system is lower at low frequency in the experiments.



(a)



(b)

Fig. 11. Optimal noise reduction with and without the wooden plate (a) measured results (b) simulation results

The cross frequencies of the 2-channel and 3-channel ANC systems from simulations are around 970 Hz and 1180 Hz respectively, while the measured values are around 800 Hz and 850 Hz in Fig. 11(a). This difference is partially caused by the location precision of the acoustic centers of the secondary sources in the experiments. Assuming that the distance between the primary source and the secondary sources increases from 0.124 m (the actual value considering the size of the loudspeaker and the radius of the support frame in the experiment) to 0.18 m linearly when the frequency increases from 450 Hz to 900 Hz, the obtained optimal noise reduction in the simulation is presented in Fig. 12, where the cross frequency is around 750 Hz and 850 Hz for the 2-channel and 3-channel systems, respectively. Nevertheless, both simulation and measurement results validate that the cross frequency increases with the number of system channels and converges to the half wavelength frequency (1387 Hz).

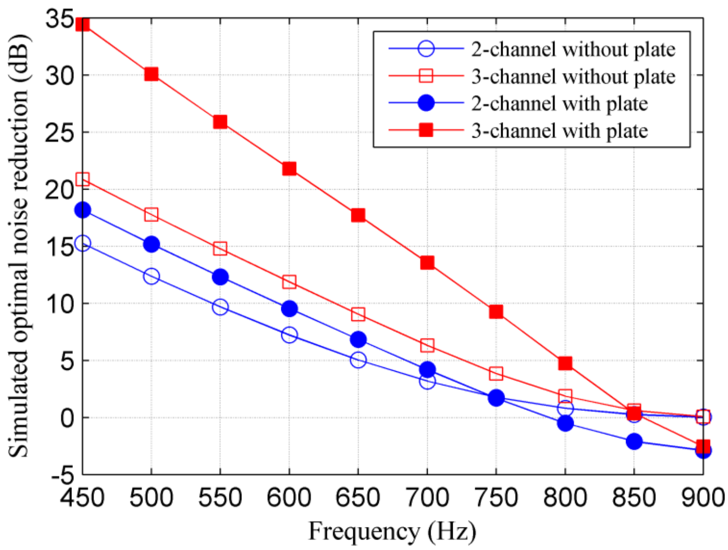


Fig. 12. Optimal noise reduction obtained by simulation when considering the variation of acoustic centers of the loudspeakers in the experiment

5. Conclusions

This paper demonstrates that the noise reduction of a multichannel active sound radiation control system at low frequencies can be improved by introducing a reflecting surface against the primary source after optimizing the location of the secondary sources. The mechanism for the performance improvement is due to the increased sound pressure generated by the secondary sources at the primary source location. To maximize the sound pressure generated by secondary sources at the primary source, the secondary sources should be placed as far apart as possible to each other and to the reflecting surface. If more secondary sources are adopted, the noise reduction improvement by introducing a reflecting surface will be larger, and the beneficial frequency range will extend to the half-wavelength frequency, which is determined by the distance between the secondary sources and the primary source.

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Appendix

If a 2-channel ANC system is adopted in free field, the optimal locations of the secondary sources should lie in a line cross the primary source[17]. The optimal sound radiation power with control can be formulated as

$$W_{\mathrm{opt0}} = 0.5 Z_0 Q_{\mathrm{p}}^2 - \mathbf{b}^{\mathrm{H}} \mathbf{A}^{-1} \mathbf{b}, \tag{A1}$$

where $Z_0 = \omega^2\rho_0/4\pi c_0$, $\mathbf{b} = 0.5Q_{\mathrm{p}}Z_0[\mathrm{sinc}(kl)\ \mathrm{sinc}(kl)]^{\mathrm{H}}$, $\mathbf{A} = 0.5Z_0[1\ \mathrm{sinc}(2kl);\ \mathrm{sinc}(2kl)\ 1]$, ω is the angular frequency, ρ_0 is the air density, c_0 is the sound speed, k is the wave number and l is the distance between the secondary sources and the primary source.

If a reflecting surface is placed against the primary source, it has been obtained from the genetic searching results that the zenith angle of the secondary sources is 45$^{\mathrm{o}}$ when the frequency decreases to zero. Therefore, the optimal sound radiation power turns to

$$W_{\mathrm{opt1}} = Z_0 Q_{\mathrm{p}}^2 - \mathbf{b}^{\mathrm{H}} \mathbf{A}^{-1} \mathbf{b}, \tag{A2}$$

where $\mathbf{b} = Q_{\mathrm{p}}Z_0[\mathrm{sinc}(kl)\quad \mathrm{sinc}(kl)]^{\mathrm{H}}$ and

$$\mathbf{A} = 0.5 Z_0 \begin{bmatrix} 1+\mathrm{sinc}\left(\sqrt{2}kl\right) & \mathrm{sinc}\left(\sqrt{2}kl\right)+\mathrm{sinc}(2kl) \\ \mathrm{sinc}\left(\sqrt{2}kl\right)+\mathrm{sinc}(2kl) & 1+\mathrm{sinc}\left(\sqrt{2}kl\right) \end{bmatrix}. \tag{A3}$$

The ratio between the optimal sound radiation power with and without the reflecting surface can be calculated as

$$\frac{W_{\mathrm{opt1}}}{W_{\mathrm{opt0}}} = \frac{2-y}{1-x}, \tag{A4}$$

where

$$x = \frac{2\mathrm{sinc}^2 kl(1-\mathrm{sinc}2kl)}{1-\mathrm{sinc}^2 2kl} \quad \text{and} \tag{A5}$$

$$y = \frac{8\mathrm{sinc}^2 kl(1-\mathrm{sinc}2kl)}{\left(1+\mathrm{sinc}\sqrt{2}kl\right)^2 - \left(\mathrm{sinc}\sqrt{2}kl+\mathrm{sinc}2kl\right)^2}. \tag{A6}$$

Both sinc($2kl$) and sinc($\sqrt{2}kl$) trend to 1 when the wave number k decreases to zero, so it can be obtained that

$$\frac{y}{x} = \frac{4}{1+\frac{\mathrm{sinc}\sqrt{2}kl}{1+\mathrm{sinc}2kl}} = \frac{8}{3}. \tag{A7}$$

Assuming $m = 10\log_{10}(2-y)$ and $n = 10\log_{10}(1-x)$, it can be obtained that

$$\frac{8}{3}10^{\frac{n}{10}} - 10^{\frac{m}{10}} = \frac{2}{3}. \tag{A8}$$

Finally, it can be obtained that

$$10\log_{10}\frac{W_{\mathrm{opt1}}}{W_{\mathrm{opt0}}} = m-n = 10\log_{10}2 = 3 \ \text{ dB} \tag{A9}$$