Compensating the distortion of micro-speakers in a closed box with consideration of nonlinear mechanical resistance

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Running title: Compensating the distortion of micro-speakers

Abstract: For micro-speakers in a closed box, commonly used nonlinear compensation methods only compensate the distortion caused by the force factor and the stiffness. In this letter, a method to compensate the distortion with consideration of the nonlinear mechanical resistance is proposed based on the feedback linearization criterion. The proposed method is further improved by minimizing the variation of the output power spectrum after compensation. The simulations and experiments show that the total harmonic distortion of the sound pressure can be reduced significantly with little influence on the sound pressure level.

I. INTRODUCTION

Micro-speakers play an important role in personal audio devices such as mobile phones, laptops and tablets.[1-2] When a micro-speaker operates under high excitation, nonlinear distortions rise considerably especially in low frequency range and deteriorate sound quality significantly. Nonlinear distortions of micro-speakers are caused by many factors such as nonlinear stiffness, nonlinear force factor, nonlinear inductance, nonlinear mechanical resistance, Doppler Effect and suspension creep.[3-5] Usually, the force factor and the stiffness are the dominant factors responsible for the nonlinearities at low frequencies.[1-5] But for micro-speakers, the mechanical resistance is also an important nonlinear factor, which causes significant harmonic and intermodulation distortion near the resonance frequency.[1]

Physical modelling[2] and black-box modelling[2,6] are two commonly used approaches for modeling a loudspeaker. Physical models have been successfully used in characterizing loudspeakers and have better performance than the black-box models on nonlinear distortion reduction.[2] Based on the physical models, loudspeaker nonlinear distortions can be predicted and compensated by inverse preprocessing of the input audio signal. However, accurate estimation of the model parameters is critical to achieve effective compensation performance.[7] Several methods have been proposed to estimate the loudspeaker parameters by measuring the impedance curve of loudspeakers.[2,8,9] System identification has also been used to estimate the nonlinear parameters by measuring the input voltage and voice-coil current.[7,10,11]

Most loudspeaker nonlinearities are related directly to the geometry and the material properties of the motor, suspension, cone and enclosure.[4] However, it is hard in practice to compensate the distortions completely with only structural and material improvement. Many attempts have been made to reduce the nonlinear distortions using digital signal processing techniques.[12-17] Klippel proposed a mirror filter method, which has been applied successfully in woofer and horn loudspeaker systems.[13-14] A feedback linearization method was proposed to reduce the nonlinear distortion by pre-filtering the input audio signal based on the observed or estimated state space.[17] The total harmonic distortion (THD) can be reduced by adjusting the initial position of the voice-coil.[18]

For micro-speakers, the nonlinearity of the mechanical resistance near the resonance frequency is quite large,[1] but none of the compensators mentioned above has taken this into consideration. Based on a simplified nonlinear lumped-element model, this letter proposes a feedback linearization compensator, which takes the nonlinear mechanical resistance into account. Simulation and experiments were carried out to demonstrate the performance of the proposed compensator.

II. METHOD

A micro-speaker in a closed box can be modeled by a simplified nonlinear lumped element equivalent circuit, where the nonlinearities of the force factor $Bl(x)$, the stiffness $K_t(x)$ and the mechanical resistance $R_m(v)$ are considered.[1] The nonlinear elements depend on the voice-coil displacement x and the voice-coil velocity v. In the derivations, the nonlinearities of the voice-coil inductance $L_e(x)$ is neglected because it is very small for micro-speakers.[1] The simplified model can be described with the following two differential equations,[1,9]

$$u = R_e i + L_e \frac{di}{dt} + Bl\left(x\right)\frac{dx}{dt}, \tag{1}$$

$$Bl\left(x\right)i=m_{\mathrm{t}}\frac{d^2x}{dt^2}+R_{\mathrm{m}}\left(v\right)\frac{dx}{dt}+K_{\mathrm{t}}\left(x\right)x \tag{2}$$

where u is the loudspeaker driving voltage, i is the voice-coil current, R_{e} is the voice-coil resistance and m_{t} is the total moving mass. Each of $Bl(x)$, $K_{\mathrm{t}}(x)$ and $R_{\mathrm{m}}(v)$ is modeled by a Nth-order polynomial approximation,

$$\begin{aligned}
Bl\left(x\right)&=b_0+\sum_{j=1}^{N}b_jx^j\\
K_{\mathrm{t}}\left(x\right)&=k_0+\sum_{j=1}^{N}k_jx^j\ ,\\
R_{\mathrm{m}}\left(v\right)&=r_0+\sum_{j=1}^{N}r_jv^j
\end{aligned} \tag{3}$$

with b_0, k_0 and r_0 representing the parameters at the neutral place (x = 0 or v = 0) and b_j, k_j and r_j are coefficients that characterize the nonlinearities.

Equations (1) and (2) can be rewritten in a general state space form as

$$\begin{aligned}
\dot{\mathbf{x}}&=\mathbf{f}\left(\mathbf{x}\right)+\mathbf{g}\left(\mathbf{x}\right)u\left(t\right)\\
y&=h\left(\mathbf{x}\right)
\end{aligned}\ , \tag{4}$$

where $\mathbf{x}$ = $[x_1, x_2, x_3]^{\mathrm{T}}$ = $[i, x, v]^{\mathrm{T}}$ is the state vector of the system, y is the output. $h(\mathbf{x})$ and the components of $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are defined by

$$h\left(\mathbf{x}\right)=x_2, \tag{5}$$

$$\mathbf{f}\left(\mathbf{x}\right)=\begin{bmatrix} -\dfrac{R_{\mathrm{e}}}{L_{\mathrm{e}}}x_1-\dfrac{Bl\left(x_2\right)}{L_{\mathrm{e}}}x_3 \\ x_3 \\ \dfrac{Bl\left(x_2\right)}{m_{\mathrm{t}}}x_1-\dfrac{K_{\mathrm{t}}\left(x_2\right)}{m_{\mathrm{t}}}x_2-\dfrac{R_{\mathrm{m}}\left(x_3\right)}{m_{\mathrm{t}}}x_3 \end{bmatrix}, \tag{6}$$

$$\mathbf{g}\left(\mathbf{x}\right)=\begin{bmatrix}\dfrac{1}{L_{\mathrm{e}}} & 0 & 0\end{bmatrix}^{\mathrm{T}}. \tag{7}$$

The basic principle of the feedback linearization method is that an expression that depends explicitly on the input u is obtained by taking sufficient number of derivatives of the output y with respect to time.[10] The lowest order n derivative that explicitly depends on input u is defined as the relative degree of the system.[17] The state-space transformation for the system can be expressed as[17]

$$\mathbf{z} = T(\mathbf{x}) = \begin{bmatrix} h(\mathbf{x}) \\ L_{\mathrm{f}} h(\mathbf{x}) + L_{\mathrm{g}} h(\mathbf{x}) u \\ L_{\mathrm{f}}^{2} h(\mathbf{x}) + L_{\mathrm{g}} L_{\mathrm{f}} h(\mathbf{x}) u \\ \vdots \\ L_{\mathrm{f}}^{n} h(\mathbf{x}) + L_{\mathrm{g}} L_{\mathrm{f}}^{n-1} h(\mathbf{x}) u \end{bmatrix}, \tag{8}$$

where $L_{\mathrm{f}}h(\mathbf{x})$ and $L_{\mathrm{g}}h(\mathbf{x})$ are the Lie derivatives of $h(\mathbf{x})$, and recursively $L_{\mathrm{f}}^{i} h(x)$ is the ith order Lie derivative.[17]

The nonlinear state feedback is then given by[17]

$$u = -\frac{L_{\mathrm{f}}^{n} h(\mathbf{x})}{L_{\mathrm{g}} L_{\mathrm{f}}^{n-1} h(\mathbf{x})} + \frac{1}{L_{\mathrm{g}} L_{\mathrm{f}}^{n-1} h(\mathbf{x})} q \quad , \tag{9}$$

where q is the new input of the system. The system is also called the inverse dynamic system because both the linear and the nonlinear dynamic of the system are cancelled by means of inversion.

With the state-space transformation, the components of the new state vector $\mathbf{z}$ are given by

$$z_1 = h(\mathbf{x}) = x_2 , \tag{10}$$

$$z_2 = L_{\mathrm{f}} h(\mathbf{x}) + L_{\mathrm{g}} h(\mathbf{x}) u = x_3 , \tag{11}$$

$$z_3 = L_{\mathrm{f}}^{2} h(\mathbf{x}) + L_{\mathrm{g}} L_{\mathrm{f}} h(\mathbf{x}) u = \frac{Bl(x_2)}{m_{\mathrm{t}}} x_1 - \frac{K_{\mathrm{t}}(x_2)}{m_{\mathrm{t}}} x_2 - \frac{R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}} x_3 , \tag{12}$$

$$
\begin{aligned}
z_4 &= L_{\mathrm{f}}^3 h(\mathbf{x}) + L_{\mathrm{g}} L_{\mathrm{f}}^2 h(\mathbf{x}) u \\
&= -\frac{Bl(x_2)}{m_{\mathrm{t}} L_{\mathrm{e}}}\left[R_{\mathrm{e}} x_1 + Bl(x_2) x_3 - u\right] + \frac{x_3}{m_{\mathrm{t}}}\left[Bl_x(x_2) x_1 - K_{\mathrm{tx}}(x_2) x_2 - K_{\mathrm{t}}(x_2)\right] \\
&\quad - \frac{R_{\mathrm{mx}}(x_3) x_3 + R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}^2}\left[Bl(x_2) x_1 - K_{\mathrm{t}}(x_2) x_2 - R_{\mathrm{m}}(x_3) x_3\right]
\end{aligned}, \quad (13)
$$

where Bl_{x} and K_{tx} denote the first order derivatives of $Bl(x)$ and $K_{\mathrm{t}}(x)$ with respect to x, and R_{mx} is the first order derivative of $R_{\mathrm{m}}(v)$ with respect to v. Equation (13) explicitly depends on u, therefore the relative degree of the system is n = 3. According to Eq. (9), the linearizing state feedback is given by

$$
\begin{aligned}
u = \frac{L_{\mathrm{e}}}{Bl(x_2)} &\left\{ \begin{array}{l} -x_3\left[Bl_{\mathrm{x}}(x_2) x_1 - K_{\mathrm{tx}}(x_2) x_2 - K_{\mathrm{t}}(x_2)\right] + m_{\mathrm{t}} q \\ + \dfrac{R_{\mathrm{mx}}(x_3) x_3 + R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}}\left[Bl(x_2) x_1 - K_{\mathrm{t}}(x_2) x_2 - R_{\mathrm{m}}(x_3) x_3\right] \end{array} \right\}. \\
&+ R_{\mathrm{e}} x_1 + Bl(x_2) x_3
\end{aligned} \quad (14)
$$

Equation (14) cancels both the linear and the nonlinear dynamic of the system. In order to obtain the desired linear dynamic (LD) behavior, the LD needs to be reintroduced by pre-filtering the input to the inverse dynamic controller q by a linear filter with the frequency response of the LD of the system.[10] The desired LD are found from the nonlinear state-space form with all nonlinear parameter set to zero. Together with the desired linear input behavior, the linear state-space system is given by

$$
\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\dfrac{k_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} & -\left(\dfrac{r_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} + \dfrac{b_0^2}{m_{\mathrm{t}} L_{\mathrm{e}}} + \dfrac{k_0}{m_{\mathrm{t}}}\right) & -\left(\dfrac{R_{\mathrm{e}}}{L_{\mathrm{e}}} + \dfrac{r_0}{m_{\mathrm{t}}}\right) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{b_0}{m_{\mathrm{t}} L_{\mathrm{e}}} \end{bmatrix} w, \quad (15)
$$

where w is the input of the LD compensator. The resulting input-output description of the LD compensator is given by

$$
\begin{aligned}
q &= -\frac{k_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} z_1 - \left(\frac{r_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} + \frac{b_0^2}{m_{\mathrm{t}} L_{\mathrm{e}}} + \frac{k_0}{m_{\mathrm{t}}} \right) z_2 - \left(\frac{R_{\mathrm{e}}}{L_{\mathrm{e}}} + \frac{r_0}{m_{\mathrm{t}}} \right) z_3 + \frac{b_0}{m_{\mathrm{t}} L_{\mathrm{e}}} w \\
&= -p_2 \frac{Bl(x_2)}{m_{\mathrm{t}}} x_1 + \left[\frac{p_2 k(x_2)}{m_{\mathrm{t}}} - p_0 \right] x_2 + \left[\frac{p_2 R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}} - p_1 \right] x_3 + p_3 w
\end{aligned}
, \qquad (16)
$$

where p_k (k = 0, 1, 2, 3) is defined as

$$
\begin{aligned}
p_0 &= \frac{k_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} \\
p_1 &= \frac{r_0 R_{\mathrm{e}}}{m_{\mathrm{t}} L_{\mathrm{e}}} + \frac{b_0^2}{m_{\mathrm{t}} L_{\mathrm{e}}} + \frac{k_0}{m_{\mathrm{t}}} \\
p_2 &= \frac{R_{\mathrm{e}}}{L_{\mathrm{e}}} + \frac{r_0}{m_{\mathrm{t}}} \\
p_3 &= \frac{b_0}{m_{\mathrm{t}} L_{\mathrm{e}}}
\end{aligned}
. \qquad (17)
$$

Substituting Eq. (16) into Eq. (14), the total nonlinear compensator is fully described as

$$
\begin{aligned}
u = \frac{L_{\mathrm{e}}}{Bl(x_2)} & \left\{ \begin{aligned} & \left[\frac{R_{\mathrm{e}}}{L_{\mathrm{e}}} + \frac{r_0}{m_{\mathrm{t}}} - \frac{R_{\mathrm{mx}}(x_3) x_3 + R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}} \right] \left[-Bl(x_2) x_1 + K_{\mathrm{t}}(x_2) x_2 + R_{\mathrm{m}}(x_3) x_3 \right] \\ & - x_3 \left[Bl_{\mathrm{x}}(x_2) x_1 - K_{\mathrm{tx}}(x_2) x_2 - K_{\mathrm{t}}(x_2) + k_0 \right] - \frac{R_{\mathrm{e}}}{L_{\mathrm{e}}} (k_0 x_2 + r_0 x_3) + \frac{b_0}{L_{\mathrm{e}}} (w - b_0 x_3) \end{aligned} \right\} \\
& + R_{\mathrm{e}} x_1 + Bl(x_2) x_3
\end{aligned}
$$

.(18)

The proposed compensator requires measurement of the state vector **x**. However, measuring the displacement and velocity directly is either impractical or expensive, or both. One solution is to simulate the states using a state observer with the input being the output of the compensator, which is also known as a pure feedforward compensator.[10] The state vector is calculated by solving the first-order differential equation of Eq. (4) using numerical integration.

The direct implementation of the compensator increases the signal level significantly near the resonant frequency. This will not only cause coloration of the output signal, but also lead to overflow of the system when the amplitude of the expected excitation signal is high. To solve this problem, a regularization item is introduced into Eq. (18), leading to an improved feedback linearization compensator as

$$u=\frac{L_{\mathrm{e}}}{Bl(x_2)}\left\{\begin{array}{l}\left[\frac{R_{\mathrm{e}}}{L_{\mathrm{e}}}+\frac{r_0}{m_{\mathrm{t}}}-\frac{R_{\mathrm{mx}}(x_3)x_3+R_{\mathrm{m}}(x_3)}{m_{\mathrm{t}}}\right]\left[-Bl(x_2)x_1+K_{\mathrm{t}}(x_2)x_2+R_{\mathrm{m}}(x_3)x_3\right]\\ -\alpha r_0\left(\frac{R_{\mathrm{e}}}{L_{\mathrm{e}}}+\frac{r_0}{m_{\mathrm{t}}}\right)x_3-x_3\left[Bl_{\mathrm{x}}(x_2)x_1-K_{\mathrm{tx}}(x_2)x_2-K_{\mathrm{t}}(x_2)+k_0\right]\\ -\frac{R_{\mathrm{e}}}{L_{\mathrm{e}}}(k_0x_2+r_0x_3)+\frac{b_0}{L_{\mathrm{e}}}(w-b_0x_3)\end{array}\right\}. \quad (19)$$

$$+R_e x_1+Bl(x_2)x_3$$

where $\alpha > 0$ is a regularization coefficient that minimize the signal level variation after the nonlinear compensation.

The nonlinear distortion caused by $Bl(x)$ and $K_{\mathrm{t}}(x)$ can be compensated using the compensation methods designed for the woofer and horn loudspeaker systems.[13,14,17] In this letter, an improved method is proposed by further taking the nonlinearity of the mechanical resistance into account. This is very meaningful for compensating the nonlinear distortion of the micro-speaker in a closed box since the mechanical resistance contributes considerably to the nonlinear distortion of the micro-speaker.[1]

III. SIMULATIONS AND EXPERIMENTS

Figure 1 depicts the experiment setup for measuring the THD of a micro-speaker in a closed box. A typical micro-speaker of size 0.9×1.6 cm^2 with a resonance frequency of 840 Hz is used in both simulations and experiments. The nonlinear parameters are estimated using the system identification method proposed in the Refs. 7, 10 and 11. Figure 2 depicts the THD simulated using all nonlinear parameters and the THD with each nonlinear parameter respectively. It can be observed that $Bl(x)$ and $K_t(x)$ dominate the distortion at low frequencies, while $R_m(v)$ plays an important role in the distortion near the resonance frequency.

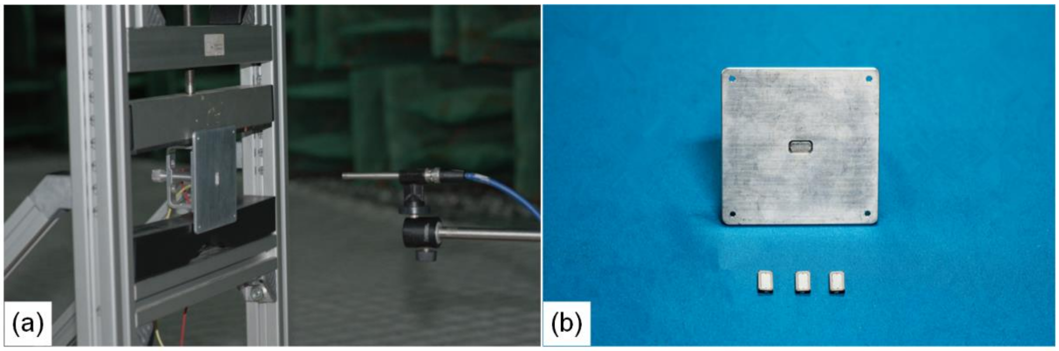

FIG. 1. Sound pressure THD measurement system. (a) a 0.9×1.6 cm^2 micro-speaker in a closed box and a measurement microphone (b) micro-speaker units and a micro-speaker in a closed box

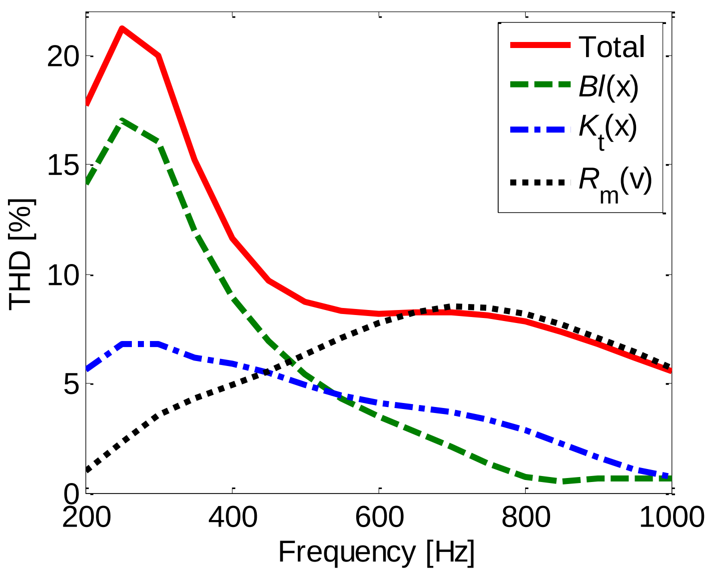

FIG. 2. Sound pressure THD obtained by simulations using all nonlinearities (solid line) and each separated nonlinear parameters $Bl(x)$ (dash line), $K_t(x)$ (dash-dot line) and $R_m(v)$

(dot line).

When measuring the THD of the sound pressure, the distance from the micro-speaker to the measurement microphone is 10 cm. To validate the efficacy of the proposed method, the nonlinear compensation of 4 different micro-speakers of the same model are conducted using the parameters identified from one of them. In the experiments, the input voltage amplitude is 1.5 V and the regularization coefficient α is 0.55. Figure 3 depicts the THD measured before and after the compensation. It is clear that the THD decreases significantly after compensation at low frequencies. However, for the compensator using only $Bl(x)$ and $K_t(x)$, the THD rises at frequencies from 500 Hz to 800 Hz. This problem is well solved when $R_m(v)$ is taken consideration in compensation. It can be clearly seen that the proposed compensator has a significant performance improvement.

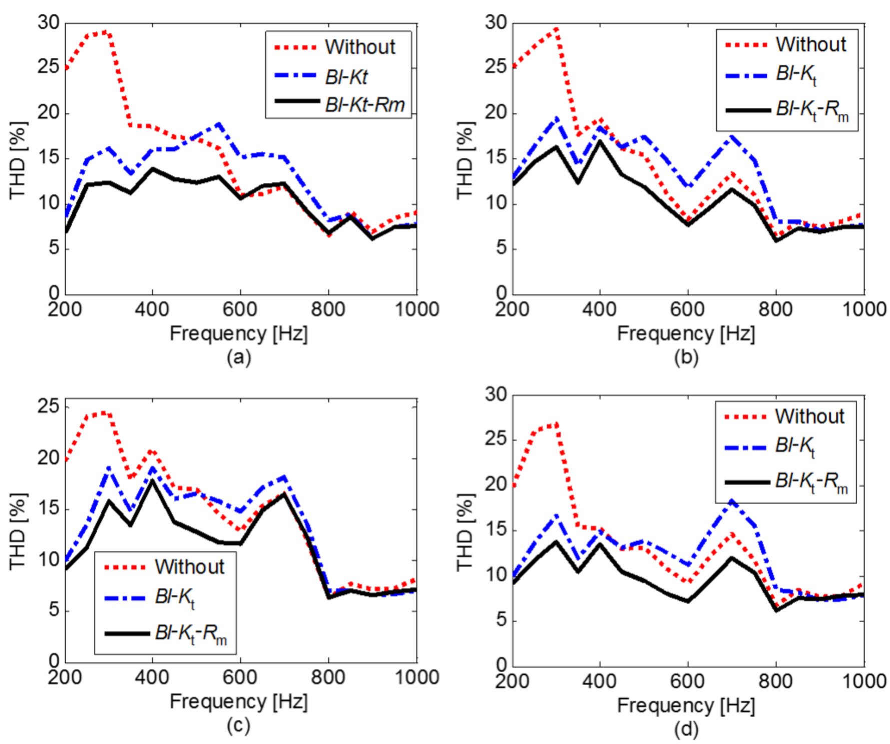

FIG. 3. Sound pressure THD measured for 4 different micro-speakers without compensation (Without, dot line), with $Bl(x)$ and $K_t(x)$ being compensated (Bl-K_t, dash-dot line), and with Bl(x), $K_t(x)$ and $R_m(v)$ being compensated (Bl-K_t-R_m, solid line). The nonlinear parameters are estimated from micro-speaker 1. (a) micro-speaker 1 (b) micro-speaker 2 (c) micro-speaker 3 (d) micro-speaker 4.

It should be noted that apart from micro-speaker 1, micro-speakers 2 - 4 are also compensated using the parameters identified from micro-speaker 1. Considering that the parameters of the micro-speakers vary from unit to unit, it is well demonstrated that the proposed method is insensitive to the variation of nonlinear parameters.

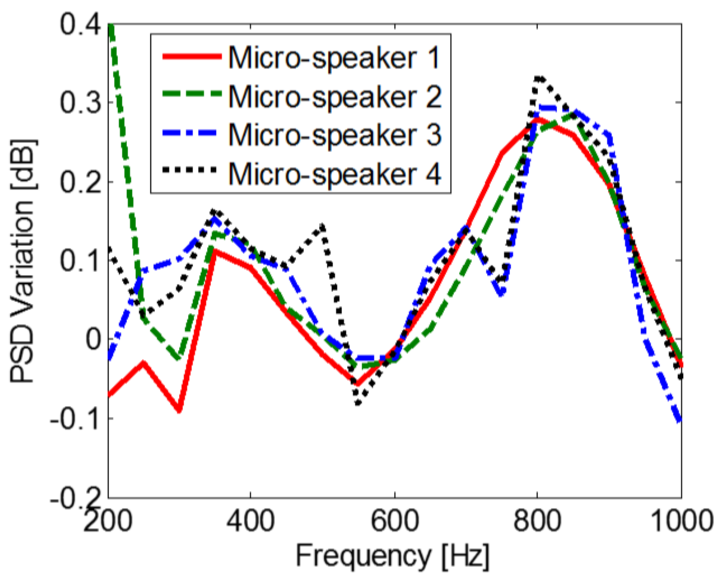

FIG. 4. Power spectral density (PSD) variation of the output measured for 4 different micro-speakers.

The power spectral density (PSD) variation of the output is measured as the difference between the power spectral density with and without compensation. Figure 4 depicts the power spectral density variation of the 4 micro-speakers using the proposed compensator. It can be seen that the variation is kept at a very low level between -0.1 dB and 0.4 dB. Therefore the proposed compensator can significantly decrease the THD but without large influence on the original linear frequency response.

In the experiments, the nonlinear distortion cannot be compensated completely. This is mainly caused by the non-ideal identified parameters of the target micro-speaker system. Furthermore, the loudspeaker system is also known to be time-varying, when it is working with different amplitude signal. A better nonlinear parameter estimation method should lead to better compensation result. The proposed nonlinear compensator can also be combined with an online parameter identification algorithm for tracking changing parameters.

IV. CONCLUSIONS

This letter proposes a feedback linearization compensator for a micro-loudspeaker in a closed box by taking the nonlinearity of the mechanical resistance into consideration because it is the dominant factor for the THD near the resonance frequency. The compensator is further improved by minimizing the variation of the output power spectral density after compensation. The experiments on 4 different micro-speakers of the same model demonstrate that the proposed method can effectively reduce the THD without large influence on the original linear frequency response.

ACKNOWLEDGMENTS

This work was supported by National Natural Science Foundation of China (Grants No. 11374156 and No. 11474163) and the Fundamental Research Funds for the Central Universities.

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