Transient Simulation of Electrical Machines using Field-Circuit Coupled Method

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In this paper, the transient simulations of electrical machines using a magnetic field-circuit coupled approach with diverse drive circuit topologies and two-dimensional finite-element analysis are described. A convenient approach to calculate the eddy loss of rotor bar in electrical machines is also presented. To illustrate the proposed methodology, this paper deals with three types of electrical machine including an interior-type permanent-magnet brushless DC motor, a single-phase induction motor and an asynchronous generator. The loss computations of these machines are carried out.

Key Words: transient simulation, field-circuit coupled approach, drive circuit topologies, eddy loss

1. Introduction

Over the past several years, field-circuit coupled method in conjunction with the time domain finite-element method (FEM) has been developed. This method may provide accurate performances of electrical machines, such as field distribution, transient currents including eddy currents, speed and torque, as well as losses.

The approaches were initially introduced to deal with voltage driving problems [1]. The circuit equations of the armature windings were coupled with the finite element equations so that the phase currents could be directly computed. Further development allowed arbitrary connected circuits to be coupled with magnetic fields. For establishing the circuit equations some authors preferred using the nodal method [2] and others preferred the loop method [3]. Many literatures are available for the handling of stranded windings [2] and solid conductors [4].

In this paper, a code using direct-coupled field-circuit method is presented to simulate the dynamics of electrical machines with diverse drive circuit topologies. The solid conductors and stranded windings coupled with different external circuits are included in this program. The loop method is applied to describe the external drive circuit. A convenient approach to calculate the loss of rotor bar in electrical machines is also presented.

The approach has been applied to the simulation of three types of electrical machine including an interior-type permanent-magnet (IPM) brushless DC (BLDC) motor, a single-phase induction motor and an asynchronous generator. The performance calculations of the three examples are proven by the comparison between computed and experimental results.

2. Fundamental Equations

2.1 Electromagnetic Field Equations

The Maxwell’s equations are applied to the air gap, iron core and permanent magnet (PM) regions by the following diffusion equation [4]:

\[ \nabla \cdot (\sigma \nabla A) - \frac{\partial A}{\partial t} = \sigma \mu_0 \left( \frac{\partial M_x}{\partial y} - \frac{\partial M_y}{\partial x} \right) \]

(1)

where \( A \) is \( z \)-axis component of magnetic vector potential, \( \sigma \) is the reluctivity of material, and \( \sigma \) is the conductivity. \( M_x \) and \( M_y \), which only exist in PM, are the \( x \)- and \( y \)-axis components of the magnetization vector, respectively.

It is assumed that the circuit branch equations of the stranded windings and the solid conductors only include the parts which are in the magnetic field region. This means that the resistances and inductances of the ending parts of the windings and conductors are all included in the external circuits. The field equation in the stranded windings can be written as:

\[ \nabla \cdot (\sigma \nabla A) + \frac{d}{\delta x} \frac{\partial N_l}{\delta x} i = 0 \]

(2)

where \( i \) is the winding current, \( S \) is the total cross-sectional area of the region occupied by this winding in the field solution domain, \( p \) is the symmetry multiplier which is defined as the ratio of the...
original full cross-sectional area to the solution area, \( d_p \) is the polarity to represent forward path or return path, \( \alpha \) is the number of parallel branches, and \( N_e \) is the total conductor number. The magnetic field equation in the region of solid conductors can be expressed as:

\[
\mathbf{V} \cdot (\sigma \mathbf{A}) - \sigma \frac{\partial A}{\partial t} + \frac{d_p \alpha \varepsilon}{N_e l} u_N = 0
\]

(3)

where \( u_N \) is the voltage difference between the terminals of solid conductor, and \( l \) is the model depth in the \( z \) axis.

2.2 Mechanical Equations

At each time step, the electromagnetic torque \( T_{em} \) is calculated by using the Maxwell stress tensor. The angular speed and the rotor displacement can be determined by the following equations:

\[
\omega_t = \omega_{t-1} + \frac{1}{J}(T_{em} - \alpha \omega_{t-1} - T_L) \Delta t
\]

(4)

\[
\theta_t = \theta_{t-1} + \omega_t \Delta t
\]

(5)

In Eq. (4), \( \omega \) is the angular speed, \( \alpha \) is the damping coefficient, \( T_{em} \) is the electromagnetic torque, \( T_L \) is the load torque, and \( J \) is the total inertia. In Eq. (5), \( \theta \) is the angular displacement of rotor.

2.3 Eddy Loss Equation

The core loss is computed by a traditional loss calculation approaches. And solid conductors are disposed by another approach. Because the current density is normally not uniform over the cross-section of rotor bar, the loss of each rotor bar is expressed as

\[
\rho = q \rho \int \left( -\sigma \frac{\partial A}{\partial t} + \sigma \frac{d_p \alpha \varepsilon}{N_e l} u_N \right)^2 dS
\]

(6)

where \( S \) is the full cross-sectional area of one rotor bar, \( \rho \) is the resistivity of the bar, and \( q \) is the ratio of the solution area to full cross-sectional area.

3. Application Examples

The approach described above has been applied to different types of electrical machines with diverse drive circuit topologies. Three application examples are presented here.

3.1 IPM BLDC Motor

Figure 1 (a) shows the cross section of the IPM BLDC motor, which is simulated. The motor is fed by a 3-phase inverter shown in Fig. 2, in which each MOSFET runs in 120 electrical degrees. Only two MOSFETs are turned on at any time under position and PWM control strategy. The working sequences of MOSFETs are \( T_1T_2, T_2T_3, T_1T_4, T_4T_5, T_3T_6, \) and \( T_6T_1 \). For example, when \( T_1 \) and \( T_2 \) are switched on, the current flows through winding A and C. In this case, the equivalent circuit with one loop is pictured in Fig. 3 (a). When \( T_1 \) is switched off and \( T_3 \) is turned on, the current flows through Winding B and C. At this time, the current flowing in Winding A is decreased to zero by passing Diode \( D_A \). In other words, the currents flow through all the winding \( A, B \) and \( C \). An equivalent circuit with two loops given in Fig. 3 (b) may describe this case.

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Fig. 1 An IPM BLDC motor

Fig. 2 Drive circuit

Fig. 3 Equivalent circuits

In Fig. 3, \( e_A, e_B, \) and \( e_C \) are the back electromotive forces (EMF) of phases \( A, B \) and \( C \) in the finite element region, respectively. \( R_y \) is the phase resistance, \( i_1 \) and \( i_2 \) are the loop currents, \( i_{ls}, i_{r}, \) and \( i_l \) are the phase currents, \( l_{ls} \) is the phase end leakage reactance, and \( U \) is the terminal voltage.

The flux distribution is shown in Fig. 1 (b). Comparisons of calculated and measured results of the phase A current are given in Fig. 4 and 5, respectively. A good agreement can be seen from the results. The electromagnetic torque is given in Fig. 6.
A single-phase two-pole induction motor (IM) for compressor is used to illustrate the approach. The motor is fed by a 50 Hz sinusoidal voltage, and motor windings are presented in each slot. Two motor windings are presented, namely the main and auxiliary windings. A capacitor is connected in the auxiliary winding, an electrical circuit is used to describe the motor as shown in Fig. 3.

In Fig. 5, $\phi_a$, and $\phi_e$ are the back EMF of the main and the auxiliary windings in the finite element region. $R_m$ and $R_e$ are resistances and $L_m$ and $L_e$ are the auxiliary windings, and $R_{a}$, $R_{e}$, $I_{a}$, and $I_{e}$ are the end leakage resistances and end leakage inductances of the main and auxiliary windings, respectively. The flux distribution at and the speed is given in Fig. 8 and 9.
3.3 Asynchronous Generator

This example is a 3-phase four-pole asynchronous generator with the ratings of 750 kW, 600 V, 50 Hz. The magnetic field distribution at startup operation and the speed are shown in Fig. 11 and 12, respectively.

The current of one rotor bar is shown in Fig. 13 and the change of slip ratio is reflected by the current in rotor bar, and when the motor is at the steady state condition, the current frequency is low. The loss density distribution at the steady operation condition is given in Fig. 14.

References


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