The volatility of Bitcoin returns and its correlation to financial markets

Nhi N.Y. Vo
Advanced Analytics Institute
University of Technology Sydney
Sydney, Australia

Guandong Xu
Advanced Analytics Institute
University of Technology Sydney
Sydney, Australia

Abstract—The 2008 financial crisis had scattered incredulity around the globe regarding traditional financial systems, which made investors and non-financial customers turn to other alternative such as digital banking systems. The existence and development of blockchain technology make cryptocurrency in recent years believably become a complete alternative to traditional ones. Bitcoin is the world's first peer-to-peer and decentralized digital cash system initiated by Nakamoto [1]. Though being the most prominent cryptocurrency, Bitcoin has not been a legal trading currency in various countries. Its exchange rate has appeared to be an exceptionally high-risk portfolio with extreme volatility, which requires a more detailed evaluation before making any decision. This paper utilizes knowledge of statistics for financial time series and machine learning to (i) fit the parametric distribution and (ii) model and forecast the volatility of Bitcoin returns, and (iii) analyze its correlation to other financial market indicators. The fitted parametric time series model significantly outperforms other standard models in explaining the stylized facts and statistical variances in the behavior of Bitcoin returns. The model forecast also outperforms some machine learning methodologies, which would benefit policy makers, banks and financial investors in trading activities for both long-term and short-term strategies.

Keywords—Bitcoin; cryptocurrency; volatility modeling; statistical finance

I. INTRODUCTION

The 2008 global financial crisis had totally transformed financial markets and restructured many big financial institutions. The necessity for unorthodox solutions has been risen, which eventually leads to the evolution of digital currencies. Since the initial inception of Bitcoin [1], more than 700 other digital currencies with similar mining methods, often referred to as Altcoins, have made their appearance into the market with a variety of adoption level. However, Bitcoin has always come first on the list with a continuously increasing market capitalization of around 29 billion USD by mid May 2017 [2]. The interest in integrating the Bitcoin into daily business operations and providing new services within the Bitcoin sphere has been growing both horizontally and vertically in recent years. Venture capitalists are likewise interest in betting their money on this rising business. Traditional financial institutions and scholars have already been involved in the daily analysis of Bitcoin returns. With much participation in such a short time, financial bubbles are prospective to happen at a more frequent rate with Bitcoin returns (symbol BTC) than any other financial indexes. Even with the recent constant rate of increase in both the number of Bitcoin and transactions, the speculative exchange rates has always been a risky investment. Besides, there have been other reported cases of defaults and attacks together with varied market information. This dynamic trading scene has both negative and positive influences on the Bitcoin volatility.

Our research purpose is to demonstrate that Bitcoin returns, despite its high volatility, can be modeled and predicted, using knowledge of statistics for financial time series and machine learning. We aim to provide better insights of the volatility of Bitcoin returns, its parametric statistical distribution and its correlation to financial markets in recent years. This paper will be a deeper extension to current literature in Bitcoin volatility modeling and forecasting with the financial time series GARCH model and different variations.

II. LITERATURE REVIEW

Despite its multiple applicable benefits, Bitcoin had not triggered any sizable attentiveness from economic and financial academics until its first bubble from early 2013 to 2014. Mainly due to the mining basic of the cryptocurrency, most of emerging literature has been focusing on finding the intrinsic value of Bitcoin. There have been disagreements among scholars and experts from related fields on whether or not Bitcoin can become a globally accepted currency.

A few researchers like Yermack [3], Hanley [4] and Ciaian [5] have been trying to investigate the price formation of Bitcoin, arguing that the currency cannot be characterized as a real currency but principally as a speculative vehicle with floating exchange rates. The foundation of the opinions mainly bases on the high volatility of Bitcoin price movements. This interpretation seems to be dominant in literature until today with papers from Fink [6] and Hur [7].

On the other hand, many scholars have taken different perspectives believing there is a fundamental value in the price of the Bitcoin. One of the most acknowledged methods is from Garcia [8] and Hayes [9], using the cost of production model through mining to calculate the price of the cryptocurrency in comparison to other cryptocurrencies. These studies are supported by the fact that there can only be roughly 21 million Bitcoins in total according to its originally defined protocol [1].

Other opinions on the price formation of the Bitcoin include the influence from the media and the effect of information share [10]. Bartos [11] has confirmed through empirical works that Bitcoin returns follow the hypothesis of efficient markets. All information fully and almost instantly reflects in the price, which implied that speculation was not rewarding. Regardless, speculators are still betting on Bitcoin in an increasing scale, contributing to its current instability.

One of most discussed topics within the Bitcoin financial aspects is the extreme volatility of its market prices, or also referred to as its exchange rates against other strong currencies such as USD, EUR or AUD. Buchholz et al. [12] were ones of the first efforts to depict the volatility and demand of Bitcoin. By revealing the drastically effect on price during the booming phase before reaching its first bubble burst in 2013, the paper has presented the speculative nature of the cryptocurrency. Fundamentally independent from bank and government control, Bitcoin has the most unpredictable floating exchange rates with extra noise, which makes it additionally challenging for an empirical analysis than any other financial time series.

Kroeger [13] has approached the challenges with theory from foreign exchange markets, using empirical test of purchasing power parity to show that relative purchasing power parity might hold. In contrast, there seems to be a persistent deviation from the absolute version of purchasing power parity. Since the different buying units of Bitcoin were indifferent regardless of purchasing locations, this intriguing result could hint to the correlation with other macroeconomic drivers. On the contrary, some scholars have been trying to explain the significant instability of Bitcoin returns by treating it as digital gold instead. Using the implied nominal returns, Smith [14] has demonstrated that there was a high correlation between the relative prices and other conventional exchange rates. The research has pointed towards the floating nominal exchange rate as one of the causes for Bitcoin price fluctuations.

Besides, there has been various attempts to measure the correlation between the volatility of Bitcoin returns and the market supply and demand. Approaching from the econometric point of view, Kancs [15] have captured great influences of supply and demand drivers on the exchange rates, particularly on the demand side. This is in line with other findings about the Bitcoin behaviors related to the information efficiency of the financial market [11]. Additionally, the empirical findings have contributed to current literature by showing the insignificant causal relation between its price and other macroeconomics factors, which is in contrast with some previous studies [13]. Within the financial econometrics and statistics spectrum, it is generally acknowledged that Bitcoin returns is a financial time series; hence, models such as GARCH and their variations should be considered as the foundation theory for any empirical analysis. Both Buchholz [12] and Gronwald [16] have similar conclusions on Bitcoin volatility. However, due to the time interval between their researches, comparing the two findings might not be specifically beneficial.

Last but not least, Chu [17] had tried to fit the Bitcoin returns using a variety of common parametric distributions in financial time series. The statistical analysis determined that the generalized hyperbolic distribution and its distribution family give the best fits for the highly volatile exchange rates. The findings of the literature place a strong foundation for the empirical works of this paper. The author will specifically fit the time series with generalized hyperbolic distribution family using a different dataset. Therefore, the results provide more updated and varied insights, which fill the current research gap and contribute to current literature of Bitcoin returns.

III. Data

There is a variety of Bitcoin returns datasets available online, ranging from the official Bitcoin Index on New York Stock Exchange (NYXBT) to various online trade markets such as okcoinCNY, btcnCNY, Bitfinex, BTC-e and so on. However, most of these databases are only recorded since 2014, or May 2015 for the case of official NYXBT. With such a short period or sometimes with missing values in between, a thorough time series analysis might be difficult and less meaningful. Therefore, we look to other available online exchange markets for longer time series datasets. We choose the USD/BTC hourly rates on Bitfinex, one of the largest online exchange for USD/BTC existing. The dataset starts from 2:00am on 1 April 2013 and ends at 2:00am 15 May 2017, with total 36,121 data points of hourly trade prices (Fig. 1).

IV. Empirical Analysis

A. Basic Statistics and Statistical Tests

We start with the basic statistics and Augmented Dickey-Fuller (ADF) test for the USD/BTC hourly prices. With 95% confidence, the ADF test results for USD/BTC prices point towards non-stationary, so it is essential to difference the time series and obtain the hourly log returns (Fig. 1). We have the basic statistics and ADF test for the time series as in Table I. We will continue the analysis with the USD/BTC hourly log returns. Afterwards, we use the basic t-test, test for skewness, kurtosis and Jarque-Bera test (JB test) with 95% confidence to test for normality. All test results are as in in Table II below. Based on these results, we can deduct that the distribution of the USD/BTC hourly log returns has mean around zero, is leptokurtosis and not Gaussian distributed. Therefore, we will have to try fitting the time series with other parametric distribution family. We can graphically confirm this conclusion using the histogram with the normal distribution as in Fig. 2 below.

Fig. 1. USD/BTC hourly prices and log returns on bitfinex.com

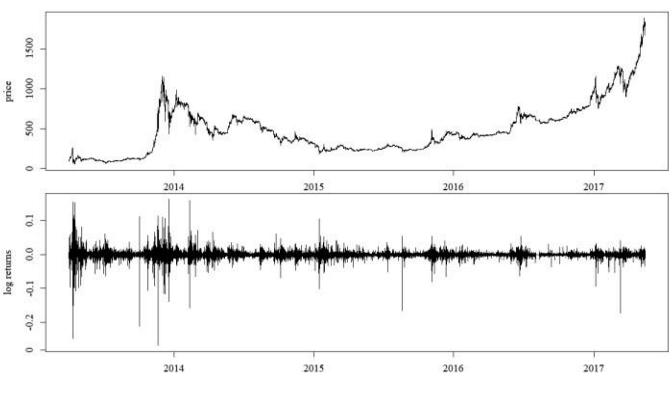


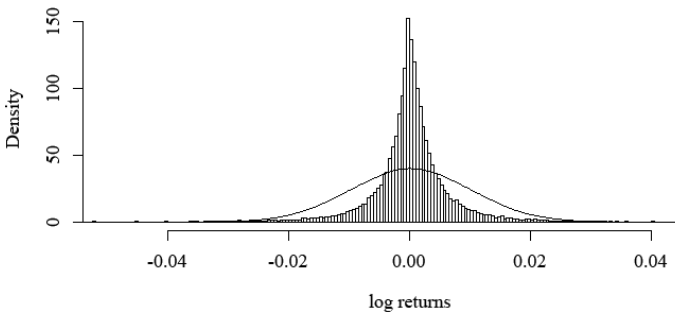
TABLE I. BASIC STATISTICS AND ADF TEST RESULT

	Prices (P_t)	Log returns ($R_t = \ln P_t - \ln P_{t-1}$)
Mean	477.804843	0.000082
Standard Deviation	301.926018	0.009789
Skewness	1.209904	- 2.416346
Kurtosis	1.903506	88.376717
ADF t-statistics	0.38697	- 32.677
ADF p-value	0.99	< 0.01

TABLE II. TWO-SIDED T-TEST, SKEWNESS TEST, KURTOSIS TEST AND JARQUE-BERA NORMALITY TEST

	Two-sided t-test	Skewness Test	Kurtosis Test	Jarque-Bera Normality Test
t-statistics	1.5961	- 0.9865	18.0398	11791216.4024
p-value	0.1105	0.3239	2	$< 2.2e^{-16}$

Fig. 2. The Histogram with Gaussian Distribution line



B. Parametric Distribution Fitting

Since the distribution of USD/BTC hourly log returns is significantly deviates from Gaussian distribution, we will try fitting with other fat-tailed parametric distributions. The generalized hyperbolic distribution family is the potential one based on previous research [17], which include the Hyperbolic (HYP), Generalized Hyperbolic (GHYP), Variance Gamma (VG), Normal Inverse Gaussian (NIG) and Student's t distributions. All fitting results for the univariate distributions, both the symmetric and asymmetric ones, and the Gaussian distribution are included for comparison. The chosen methodology of fitting distribution is the Maximum Likelihood Estimation (MLE) for its accuracy, convenience and quick convergence [18]. For model comparison and selection, we use Akaike Information Criterion (AIC) [19] given by

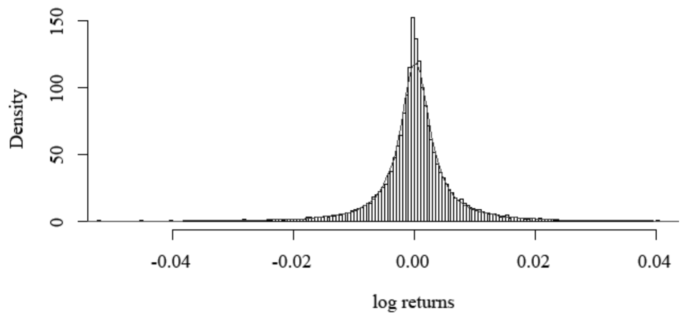
$$AIC = -2\ln L(\Theta) + 2K \quad (1)$$

where $\ln L(\Theta)$ is the maximum log-likelihood and K is the number of parameters. A lower AIC provides evidence of a better-fitted model. The log-likelihoods (under column named "LLH"), AICs and fitted parameters of these fitted distributions are as in Table III. Based on the AIC, we can say the symmetric GHYP give the best fit for the distribution of the USD/BTC hourly log returns. The histogram with GHYP distribution parameters in Fig. 3 confirm our result.

TABLE III. DISTRIBUTION FITTING RESULT

Distribution	Symmetric	AIC	LLH
GHYP	TRUE	-269586.1	134797.1
GHYP	FALSE	-269584.8	134797.4
NIG	TRUE	-269580	134793
NIG	FALSE	-269578.8	134793.4
Student's t	TRUE	-268712.4	134359.2
Student's t	FALSE	-268710.4	134359.2
VG	FALSE	-268638.6	134323.3
VG	TRUE	-268271.5	134138.7
HYP	TRUE	-263374.4	131690.2
HYP	FALSE	-263372.4	131690.2
Normal	TRUE	-231714.4	115859.2

Fig. 3. The Histogram with GHYP distribution line



C. Volatility Modeling

We then try to model the volatility of USD/BTC hourly log returns with various time series models such as Autoregressive Moving Average (ARMA). From the previous analysis, the time series distribution is not normal but characterized by fat tails. Moreover, little autocorrelation shows in the log returns time series. These stylized facts make ARMA models based on strict white noise become ineffective in capturing all volatility behaviors of the time series. Therefore, we need to combine ARMA models with variations of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models under GHYP distribution. MLE is our method for parameters estimation. We compare and select model based on AIC, Bayesian Information Criterion (BIC), Hannan-Quinn Information Criterion (HQIC) and Shibata Information Criterion (SIC) given by

$$AIC = -2\ln L(\Theta)/n + 2K/n \quad (2)$$

$$BIC = -2\ln L(\Theta)/n + K\ln(n)/n \quad (3)$$

$$SIC = -2\ln L(\Theta)/n + \ln((n+2K)/n) \quad (4)$$

$$HQIC = -2\ln L(\Theta)/n + 2K\ln(\ln(n))/n \quad (5)$$

where $\ln L(\Theta)$ is the maximum log-likelihood, K is the number of parameters and n is the total number of observations.

We fit the time series under GHYP distribution using the different ARMA mean models combined with GARCH [21] and its variations: Exponential GARCH (eGARCH) [22], Asymmetric Power ARCH (apARCH) [23], Component GARCH (cGARCH) [24], Integrated GARCH (iGARCH) [25], Glosten-Jagannathan-Runkle GARCH (gjrGARCH) [26], and Family GARCH (fGARCH) [27]. Sub-models of fGARCH include standard GARCH, APARCH, Threshold GARCH (TGARCH) [28], Absolute Value GARCH (AVGARCH) [29], Nonlinear GARCH (NGARCH) [30], Nonlinear Asymmetric GARCH (NAGARCH) [31] and Full fGARCH (ALLGARCH) [27]. These fitted GARCH models are as in the R package "rugarch" [20]. Table IV presents the result from fitting 36,120 data points with out-sample size of 100. Only the top three models are in Table IV due to space constraint, while the full one will be available upon request. According to this result, the ARMA(1,2)-fGARCH(2,2) with sub-model TGARCH is the best fitted model based on all information criteria. The standard ARMA(1,2)-GARCH(2,2) model is defined by

$$R_{\{t\}} = c + aR_{\{t-1\}} + \varepsilon_{\{t\}} + b_1\varepsilon_{\{t-1\}} + b_2\varepsilon_{\{t-2\}} \quad (6)$$

$$\varepsilon_{\{t\}} = \sigma_{\{t\}}\eta_{\{t\}} \quad (7)$$

$$\sigma_{\{t\}}^2 = w + d_1(\varepsilon_{\{t-1\}})^2 + d_2(\varepsilon_{\{t-2\}})^2 + e_1(\sigma_{\{t-1\}})^2 + e_2(\sigma_{\{t-2\}})^2 \quad (8)$$

where $R_{\{t\}}$ is the USD/BTC hourly log returns, σ_t^2 is the conditional variance, $\varepsilon_{\{t\}}$ is the residuals over time, intercept c and $w > 0$, $a \geq 0$, b_1 and $b_2 \geq 0$, d_1 and $d_2 \in R$, e_1 and $e_2 \in R$, and $\eta_t \in N(0,1)$. The standard sub-model TGARCH of the fGarch has some small modifications on Eq. (8) as

$$\sigma_{\{t\}} = w + x(1-c)\sigma_{\{t-1\}}\varepsilon_{\{t\}}^+ - x(1+c)\sigma_{\{t-1\}}\varepsilon_{\{t\}}^- + y\sigma_{\{t-1\}} \quad (9)$$

where x and y $\in$ R, $\varepsilon_{\{t\}}^+$ is max{$\varepsilon_{\{t\}}$,0}, and $\varepsilon_{\{t\}}^-$ is min{$\varepsilon_{\{t\}}$,0}.

D. Forecast with Fitted Model

Before forecasting, we are also interested in the difference between the influences of positive and negative shocks. The Sign Bias test [31] captures the leverage effects in the standardized residuals by regressing the standardized squared residuals on lagged negative and positive shocks as given by

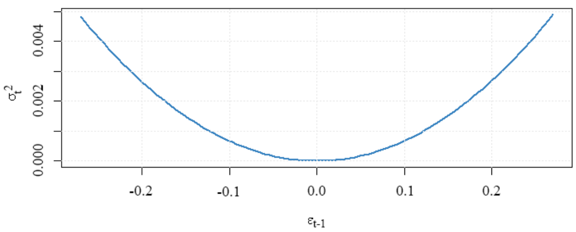
$$\varepsilon_{\{t\}}^2 = c_0 + c_1 I_{\varepsilon\{t-1\}<0} + c_2 I_{\varepsilon\{t-1\}<0}\,\varepsilon_{\{t-1\}} + c_3 I_{\varepsilon\{t-1\}\geq 0}\,\varepsilon_{\{t-1\}} + \sigma_{\{t\}} \quad (10)$$

where I is the indicator function, $\sigma_{\{t\}}$ is noise effects and $\varepsilon_{\{t\}}$ is estimated standardized residuals from fitted GARCH process. From the test result, we can deduct that there is no sign bias in the volatility of the fitted model. This finding is conversely not in line with other financial time series that generally tend to have a sign bias towards negative shocks. The News-Impact Curve (Fig. 4) also shows no asymmetry, which is a valid point for our conclusion that the USD/BTC hourly log returns reflects both positive and negative shocks equally.

TABLE IV. TIME SERIES MODEL FITTING RESULT

Time Series Model	AIC	BIC	SIC	HQIC
ARMA(1,2)-fGARCH(2,2)/TGARCH	-7.9324	-7.9291	-7.9324	-7.9313
ARMA(2,2)-fGARCH(2,2)/TGARCH	-7.9324	-7.9288	-7.9324	-7.9312
ARMA(1,2)-fGARCH(2,2)/NGARCH	-7.9296	-7.9265	-7.9296	-7.9286

Fig. 4. The News-Impact Curve



To get more insights regarding the predictability of the USD/BTC hourly log returns using our fitted model, we make use of the Variance Ratio (VR) test [32] for the Random Walk Hypotheses 1 and 2 (RWH1 and RWH2 accordingly). From Table V, we can see that the test statistics for both RWH1 and RWH2 are statistically deviate from one for 95% confidence level. Both the null hypotheses RWH1 and RWH2 are rejected. This provides evidence against the financial time series being a random walk, and it is predictable with our fitted model.

We now examine the rolling forecasts for the USD/BTC hourly log returns and its sigma with forecast length of 100 values using the fitted model from previous section (Fig. 5). The forecasted series shows mean reversion process similar to the actual time series. The forecast for the sigma, which is the standard deviation of the time series, is more precise, but it still shows some deviations from the empirical data. This deviation might point towards high investment risk and arbitrage opportunities involving the USD/BTC hourly returns.

The rolling forecast with a recursive refit window of 10 values also presents a good 1% Value-at-Risk (VaR) forecast. To confirm this result, we also check the VaR Backtests at 95% and 99% confidence levels. The unconditional coverage [33] are conducted as likelihood ratio tests with result as in Table VI. The null hypotheses of correct exceedances cannot be rejected at both 95% and 99% confidence level. Our fitted model is a good model for VaR forecast.

Last but not least, we want to check the credibility of the model forecast. Three different models are under comparison. The first one is our previously fitted model, the ARMA(1,2)-fGARCH(2,2)/TGARCH model under the GHYP distribution. The other two models are using machine learning techniques, particularly the Support Vector Machine (SVM) and Neural Networks (NN). We use the R package "e1071" and "nnet" to build SVM and NN models accordingly. The SVM model has gamma = 10^(-2) and cost = 10^(4.5) using radial kernel, while the NN model has 6 hidden layers with skip layer, max iterations = 10^(4) and decay = 10^(-2). We compare the Residual Sum of Squares (RSS) and Normalize Residual Mean Square Error (NRMSE) as measures for the evaluation in Table VII below, in which RSS closer to zero and NRMSE closer to 1 point toward a higher prediction accuracy.

TABLE V. LO-MACKINLAY VARIANCE RATIO TEST RESULT

q period	RWH1 Test Statistics	RWH2 Test Statistics
k=2	13.321281	2.4491239
k=3	7.772083	1.4313921
k=4	5.736661	1.0726703
k=5	4.786785	0.9106368

Fig. 5. The Rolling Forecast of USD/BTC Hourly Log Returns and Sigma

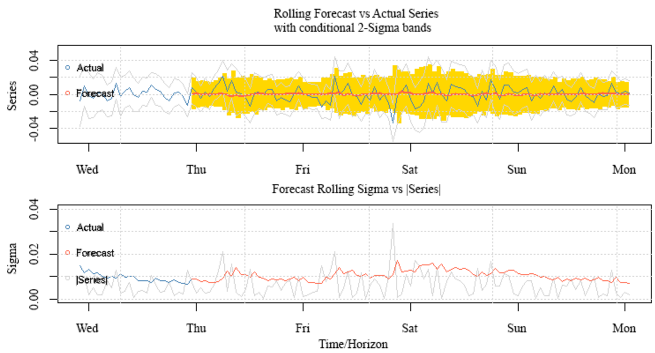


TABLE VI. VAR BACKTEST RESULT

	Unconditional Coverage	
	99% Confidence	95% Confidence
Statistic	0	0.977
Critical	6.635	3.841
p-value	1	0.323

TABLE VII. COMPARING THE FORECAST OF THE FITTED MODEL, SUPPORT VECTOR MACHINE AND NEURAL NETWORKS

Measure	Fitted Model	SVM	NN
RSS	0.00733	0.00806	0.00733
NRMSE	0.99817	1.04618	0.99765

According to the result in Table VII, our fitted model has been proven to outperform both SVM and NN in forecasting the USD/BTC hourly log returns. It has already been proven to marginally perform better than other standard ARMA or GARCH models. Cautiously, other advanced models of forecasting and back testing should be used for cross-checking this result in any later research.

E. Correlation to other Financial Market Indicators

Economists and investors are also interested in the relation between Bitcoin returns and other financial market indicators. Nominally, we analyze the correlation between daily prices of Bitcoin and other financial indices: Dow Jones Industrial Average (DJIA), Australian Securities Exchange all ordinaries (ASX) and the exchange rates of AUD/USD as in Table VIII.

TABLE VIII. CORRELATION ANALYSIS RESULT

Coefficient	DJIA	ASX	AUD/USD
Pearson's	0.4327586	0.2657405	-0.1432585
Kendall's	0.2474947	0.2019860	-0.1042584
Spearman's	0.3939350	0.3064450	-0.1693724

The daily prices is obtained from Investing.com for the time period between 1st April 2013 and 30th December 2016. Since these traditional financial markets are closed on weekends and holidays while Bitcoin trade market is still opened, we will exclude all the values of Bitcoin prices on weekends and holidays for consistent combined datasets. To statistically check our hypothesis point that Bitcoin prices is still not regulated and affected by traditional financial markets, we calculate the correlations between Bitcoin prices and these market indices using three different correlation coefficients: Pearson's correlation coefficient [34], Kendall's Tau coefficient [35] and Spearman's rank correlation [36]. The result from Table VIII with all correlation coefficients have smaller values than 0.5 confirms that Bitcoin prices has little influence from these financial indices.

The uncorrelated characteristic of Bitcoin makes it an ideal reserve currency against currency inflation and other potential global financial crisis, which is the initial idea of its inception [1]. However, it has become a risky investment with extreme fluctuated prices due to increasing mining effort, speculative trading and even some money laundering activities. This requires financial investors to look further beyond traditional market news for more indications of price movement. One latest example is with the ransomeware "Wannacry" which causes the upsurge of Bitcoin prices in 2017. This exponential rise of Bitcoin returns leads to the suitable use of the eGARCH model for volatility prediction at the time of this paper. Carefully, researchers, investors and financial institutions should monitor the Bitcoin returns using suitable financial time series models in order to understand and predict its movements in the near future.

V. CONCLUSION

The Bitcoin prices have been exceptionally fluctuated since its first booming phase in 2013. With anticipated bubbles, the risk is high on these investments. It is essential to find the suitable parameter distribution and the appropriate time series model to explain some stylized facts and some statistical differences in the behavior of the financial time series.

With the empirical analysis in this paper, the USD/BTC hourly log returns have been proven to be not normally distributed and be characterized by leptokurtosis. Through the empirical works presented in this paper, the author suggests the Generalized Hyperbolic distribution is the best fits for the time series. After evaluation with multiple information criteria, the best time series model is ARMA(1,2)-fGARCH(2,2)/TGARCH model under the GHYP distribution. This model is a suitable choice with little to no autocorrelations in the standardized residuals and the standardized squared residuals series. It also shows that the time series have no bias under the influences of positive or negative shocks. The fitted model of the USD/BTC hourly log returns is also an accurate model for VaR forecast, which can outperform some standard machine learning models such as Support Vector Machines and Neural Networks.

The correlation analysis of Bitcoin returns and the financial indices, particularly DJIA for US market and ASX for Australian market and the exchange rate AUD/USD, has showed the Bitcoin price movement does not follow any of these instruments. Being unaffected by tradition financial markets should enable Bitcoin to play its initial role as defined by [1]. The possible explanation for its high volatility in recent years might be due to the speculative trading by investors and money laundering by hackers. Therefore, even with a high accurate prediction model, financial investors must be careful when forecasting the Bitcoin returns.

Due to the limit of this paper, the author cannot cover all aspects related to this financial time series. Some discussions on the correlation between Bitcoin prices and other financial market indices have been initiated, even though more empirical analysis should be carried out to gain a deeper and broader view. Future researchers might consider fitting with non-parametric and semi-parametric models, fitting different time series models such as regime-switching models, using different types of tests and other advanced forecasting methodologies. Moreover, different financial indices and instruments should be included in the correlation test to build an even better prediction model for Bitcoin returns.

In summary, Bitcoin returns has various intriguing statistical facts and is expected to continuously be unpredictable, which might require deep analytics to further explain its extreme volatility behavior.

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