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BASIC EXISTENCE AND UNIQUENESS RESULTS FOR SOLUTIONS TO SYSTEMS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We produce new global existence and uniqueness results for solutions to systems of initial value problems involving fractional differential equations. The uniqueness results rely on differential inequalities and a comparison with monotonically converging sequences of functions. The existence results involve fixed–point theorems that rely on a strategic choice of Liapunov function and harness new *a priori* bounds on solutions. We present an example where the new results yield existence of solutions, but the classical global Cauchy–Lipschitz approach does not directly apply.

Keywords. uniqueness of solutions; nonlinear fractional differential equations of arbitrary order; initial value problem; existence of solutions; Liapunov function; fixed point theorem

AMS (MOS) subject classification: 34A08, 34A12

1 Introduction

Our discussion is centred around the following system of initial value problems (IVPs) of arbitrary order $q > 0$

$$D^q \left(\mathbf{x} - T_{\lceil q\rceil-1}[\mathbf{x}]\right) = \mathbf{f}(t, \mathbf{x}); \tag{1.1}$$

$$\mathbf{x}(0) = \mathbf{A}_0,\ \mathbf{x}'(0) = \mathbf{A}_1, \ldots,\ \mathbf{x}^{(\lceil q\rceil-1)}(0) = \mathbf{A}_{\lceil q\rceil-1}. \tag{1.2}$$

Above: $\lceil q\rceil$ is the ceiling value of q; D^q represents the Riemann–Liouville fractional differentiation operator of arbitrary order $q > 0$ (a precise definition is found in (2.3) a little later); $f : [0, a] \times \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}^n$; each component of the vector function $T_{\lceil q\rceil-1}[\mathbf{x}]$ is the Maclaurin polynomial of order $\lceil q\rceil - 1$ of