IEICE TRANSACTIONS

on Communications

VOL. E100-B NO. 6
JUNE 2017

A PUBLICATION OF THE COMMUNICATIONS SOCIETY

The Institute of Electronics, Information and Communication Engineers
Kikai-Shinko-Kaikan Bldg., 5-8, Shibakoen 3chome, Minato-ku, TOKYO, 105-0011 JAPAN

PAPER

Precoding Design for Han-Kobayashi's Signal Splitting in MIMO Interference Networks

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SUMMARY For a multiuser multi-input multi-output (MU-MIMO) multicell network, the Han-Kobayashi strategy aims to improve the achievable rate region by splitting the data information intended to a serviced user (UE) into a common message and a private message. The common message is decodable by this UE and another UE from an adjacent cell so that the corresponding intercell interference is cancelled off. This work aims to design optimal precoders for both common and private messages to maximize the network sum-rate, which is a highly nonlinear and nonsmooth function in the precoder matrix variables. Existing approaches are unable to address this difficult problem. In this paper, we develop a successive convex quadratic programming algorithm that generates a sequence of improved points. We prove that the proposed algorithm converges to at least a local optimum of the considered problem. Numerical results confirm the advantages of our proposed algorithm over conventional coordinated precoding approaches where the intercell interference is treated as noise.

key words: *interference mitigation, interference networks, nonconvex optimization, precoding design, successive convex quadratic programming*

1. Introduction

There is limited understanding of the capacity of interference networks (INs). By treating residual interference as noise, the network capacity is achieved only at low interference regime (see [1] and references therein) for a general multi-user IN or at certain sufficient conditions in terms of matrix equations for two-user INs [2]. For a two-user two-cell IN (i.e., one user per cell), the Han-Kobayashi (H-K) strategy [3] is known to give the best achievable rate region [4], [5]. With the H-K strategy, the transmitted data information of both users is split into two parts: a private message to be decoded at the intended receiver and a common message that can be decoded at both receivers. A part of the interference is thus cancelled off by decoding the common message, while the remaining private message from the other user is treated as noise. Accordingly, it is challenging to perform constructive optimization over such an achievable rate region to realize the potential of H-K strategy [4], [6].

Jointly beamforming common and private messages to maximize the achievable rate across multiuser multi-input single-output (MU-MISO) INs is first considered in [7]. At discrete points of the joint space of common and private rates, an ad-hoc intensive search is carried by rank-one constrained semi-definite programming (SDP) for the beamformer vectors. Still, the optimal rate is not achieved. Furthermore, the search proposed by [7] is not suitable for the problem of sum-rate maximization, which is a more popular metric for INs. Inspired by [7], the works of [8], [9] design covariance matrices of the common and private messages in MU-MIMO multicell INs and beamformers for such messages in MU-MISO multicell INs to maximize either the sum-rate or the achievable rate across the networks.

Our present work aims to find optimal precoder matrices for the common and private messages of independent data streams. The objective is to maximize the sum-rate of an MU-MIMO multicell network. The available solution approaches are not applicable, e.g., [10] for coordinated precoding private messages only and [8], [9] for covariance design. We propose a successive optimization algorithm in which each iteration only solves a simple convex quadratic program of low computational complexity. Once initialized from a feasible point, our algorithm generates a sequence of monotonically improved points, which eventually converge to at least a local maximum of the formulated nonconvex and nonsmooth problem.

The rest of this paper is organized as follows: Section 2 presents the system model and formulates the precoder design problem. Section 3 proposes the successive quadratic programming algorithm for solution. Section 4 verifies the advantages of our devised solution by numerical examples.

Notations. $\mathbf{I}_n$ is the identity matrix of size $n \times n$. The notation $(\cdot)^H$ stands for the Hermitian transpose. The inner product $\langle \mathbf{X}, \mathbf{Y} \rangle$ is defined as trace($\mathbf{X}^H\mathbf{Y}$). $\langle \mathbf{A} \rangle$ denotes the trace of a matrix $\mathbf{A}$, and $|\mathbf{A}|$ denotes the determinant of a square matrix $\mathbf{A}$. For Hermitian symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, the notation $\mathbf{A} \succeq \mathbf{B}$ ($\mathbf{A} \succ \mathbf{B}$, respectively) means that $\mathbf{A} - \mathbf{B}$ is a positive semidefinite (positive definite, respectively) matrix. $\mathbb{E}\{.\}$ denotes the expectation operator, $\mathbb{C}$ is the set of all complex numbers, and $\emptyset$ is an empty set. $\Re\{x\}$ denotes the real part of a complex number x.

Manuscript received May 19, 2016.
Manuscript revised October 11, 2016.
Manuscript publicized December 14, 2016.

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DOI: 10.1587/transcom.2016EBP3209

2. System Model and Problem Formulation

Consider the downlink transmissions in a network consisting of N cells, where the base station (BS) of each cell is equipped with N_t antennas and it serves K UEs within its cell. Each UE is equipped with N_r antennas. Upon denoting $\mathcal{I} \triangleq \{1, 2, \ldots, N\}$ and $\mathcal{J} \triangleq \{1, 2, \ldots, K\}$, the j-th UE in the i-th cell is referred as UE $(i, j) \in \mathcal{S} \triangleq \mathcal{I} \times \mathcal{J}$. Signals are precoded at the BSs prior to transmitting to the UEs. To implement the H-K strategy, where each user decodes the common message of at most one other user, we follow [7], [8] and introduce the pairing operator $a(i, j)$ to specify which other UE, beside UE (i, j) itself, decodes the common message of UE (i, j). When UE (i, j) has no common message, we let $a(i, j)$ be an empty set. Formally, it is a mapping $a : \mathcal{I} \times \mathcal{J} \to (\mathcal{I} \times \mathcal{J}) \cup \{\emptyset\}$ with the restriction that $a(i, j) = (\tilde{i}, \tilde{j})$ always has $\tilde{i} \neq i$ and $a^{-1}(\tilde{i}, \tilde{j})$ has cardinality of no more than one. With $\emptyset \neq a(i, j) = (\tilde{i}, \tilde{j})$, $\tilde{i} \neq i$, UE (i, j) may split its $L\,(\leq N_t)$ data streams into two parts: the private message $\mathbf{s}^p_{i,j} \in \mathbb{C}^L$ with $\mathbb{E}\{\mathbf{s}^p_{i,j}(\mathbf{s}^p_{i,j})^H\} = \mathbf{I}_L$, and the common message $\mathbf{s}^c_{i,j} \in \mathbb{C}^L$ with $\mathbb{E}\{\mathbf{s}^c_{i,j}(\mathbf{s}^c_{i,j})^H\} = \mathbf{I}_L$. The private and common messages are precoded by matrices $\mathbf{V}^p_{i,j} \in \mathbb{C}^{N_t \times L}$ and $\mathbf{V}^c_{i,j} \in \mathbb{C}^{N_t \times L}$, respectively. The common message $\mathbf{s}^c_{i,j}$ of UE (i, j) is to be decoded by UE (i, j)'s receiver and also by UE $(\tilde{i}, \tilde{j})$'s receiver in a different cell $\tilde{i}$. On the other hand, if $(i, j) = a(\hat{i}, \hat{j})$ for some $\hat{i} \neq i$, the receiver of UE (i, j) also decodes the common message $\mathbf{s}^c_{\hat{i},\hat{j}}$ from UE $(\hat{i}, \hat{j})$ in a different cell $\hat{i} \neq i$.

As in [7], [8], each UE (i, j) successively decodes the following messages (in the following strict order): (a) its common message $\mathbf{s}^c_{i,j}$ from its own transmitter; (b) the common message $\mathbf{s}^c_{\hat{i},\hat{j}}$ from UE $(\hat{i}, \hat{j})$'s transmitter in the different cell $\hat{i} \neq i$ for which $a(\hat{i}, \hat{j}) = (i, j)$; (c) the private message $\mathbf{s}^p_{i,j}$ from its own transmitter. Note that the decoded messages are also successively subtracted from the received signal for interference mitigation. Intuitively, one's own common message is decoded first to help the decoding of the common information from the other transmitter, while its own private message is decoded last to take advantage of the reduced interference due to common message decoding.

For notational convenience, let us define $\mathbf{V}_{i,j} \triangleq [\mathbf{V}^p_{i,j} \quad \mathbf{V}^c_{i,j}]$, $\mathbf{V} \triangleq [\mathbf{V}_{i,j}]_{(i,j)\in\mathcal{S}}$, $\mathbf{s}_{i,j} \triangleq \begin{bmatrix} \mathbf{s}^p_{i,j} \\ \mathbf{s}^c_{i,j} \end{bmatrix}$. The received signal at UE $(i, j) \in \mathcal{S}$ is expressed as:

$$\mathbf{y}_{i,j} = \sum_{(s,l)\in\mathcal{S}} \mathbf{H}_{s,i,j}\mathbf{V}_{s,l}\mathbf{s}_{s,l} + \mathbf{n}_{i,j},$$

where $\mathbf{H}_{s,i,j} \in \mathbb{C}^{N_r \times N_t}$ is the matrix of channel coefficients from BS $s \in \mathcal{I}$ to UE $(i, j) \in \mathcal{S}$. The entries of the additive noise $\mathbf{n}_{i,j} \in \mathbb{C}^{N_r}$ are independent and identically distributed (i.i.d) noise samples with zero mean and variance σ^2. The covariance of $\mathbf{y}_{i,j}$ is thus

$$\mathbf{M}_{i,j}(\mathbf{V}) = \sum_{(s,l)\in\mathcal{S}} \mathbf{H}_{s,i,j}\mathbf{V}_{s,l}\mathbf{V}^H_{s,l}\mathbf{H}^H_{s,i,j} + \sigma^2\mathbf{I}_{N_r}.$$

Under the successive decoding and interference cancellation scheme described above, it follows that:

- UE (i, j) can decode its own common message $\mathbf{s}^c_{i,j}$ with the achievable rate (expressed in nats/s/channel-use):

$$r^c_{i,j}(\mathbf{V}) = \ln|\mathbf{I}_L + (\mathbf{V}^c_{i,j})^H\mathbf{H}^H_{i,i,j}\mathbf{M}^c_{i,j}(\mathbf{V})^{-1}\mathbf{H}_{i,i,j}\mathbf{V}^c_{i,j}|,$$

where $\mathbf{M}^c_{i,j}(\mathbf{V}) \triangleq \mathbf{M}_{i,j}(\mathbf{V}) - \mathbf{H}_{i,i,j}\mathbf{V}^c_{i,j}(\mathbf{V}^c_{i,j})^H\mathbf{H}^H_{i,i,j}$. Accordingly, $\mathbf{V}^c_{i,j} \equiv \mathbf{0}$, $r^c_{i,j}(\mathbf{V}) \equiv 0$ and $\mathbf{M}^c_{i,j}(\mathbf{V}) \equiv \mathbf{M}_{i,j}(\mathbf{V})$ if $a(i, j) = \emptyset$.
- UE (i, j) can decode the common message $\mathbf{s}^c_{\hat{i},\hat{j}}$ from the interfering user $(\hat{i}, \hat{j}) = a^{-1}(i, j)$ with the achievable rate:

$$r^a_{i,j}(\mathbf{V}) = \ln|\mathbf{I}_L + (\mathbf{V}^c_{\hat{i},\hat{j}})^H\mathbf{H}^H_{\hat{i},i,j}\mathbf{M}^a_{i,j}(\mathbf{V})^{-1}\mathbf{H}_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}|,$$

where $\mathbf{M}^a_{i,j}(\mathbf{V}) \triangleq \mathbf{M}^c_{i,j}(\mathbf{V}) - \mathbf{H}_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}(\mathbf{V}^c_{\hat{i},\hat{j}})^H\mathbf{H}^H_{\hat{i},i,j}$. Accordingly, $\mathbf{M}^a_{i,j}(\mathbf{V}) \equiv \mathbf{M}^c_{i,j}(\mathbf{V})$ if $a^{-1}(i, j) = \emptyset$.
- UE (i, j) can decode its own private message $\mathbf{s}^p_{i,j}$ with the achievable rate:

$$r^p_{i,j}(\mathbf{V}) = \ln|\mathbf{I}_L + (\mathbf{V}^p_{i,j})^H\mathbf{H}^H_{i,i,j}\mathbf{M}^p_{i,j}(\mathbf{V})^{-1}\mathbf{H}_{i,i,j}\mathbf{V}^p_{i,j}|,$$

where $\mathbf{M}^p_{i,j}(\mathbf{V}) \triangleq \mathbf{M}^a_{i,j}(\mathbf{V}) - \mathbf{H}_{i,i,j}\mathbf{V}^p_{i,j}(\mathbf{V}^p_{i,j})^H\mathbf{H}^H_{i,i,j}$.

Similar to [7, (10)–(12)], the achievable rate region under the successive decoding is given by:

$$\Big\{\boldsymbol{\nu} \triangleq [\nu_{i,j}]_{(i,j)\in\mathcal{I}\times\mathcal{J}} = [\nu^p_{i,j} + \nu^c_{i,j}]_{(i,j)\in\mathcal{I}\times\mathcal{J}} \; : \; \nu^p_{i,j} \leq r^p_{i,j}(\mathbf{V}), \nu^c_{i,j} \leq r^c_{i,j}(\mathbf{V}), \nu^c_{i,j} \leq r^a_{a(i,j)}(\mathbf{V})\Big\}.$$

As $r^a_{a(i,j)}(\mathbf{V})$ is the achievable rate of decoding the common message $\mathbf{s}^c_{i,j}$ of UE (i, j) by UE $(\tilde{i}, \tilde{j}) = a(i, j)$, the constraint $\nu^c_{i,j} \leq r^a_{a(i,j)}(\mathbf{V})$ arises only when $a(i, j) \neq \emptyset$. Therefore, by defining the nonsmooth functions

$$r_{i,j}(\mathbf{V}) = r^p_{i,j}(\mathbf{V}) + \min\left\{r^c_{i,j}(\mathbf{V}), r^a_{a(i,j)}(\mathbf{V})\right\}, \tag{1}$$

we formulate the problem of sum-rate maximization over the achievable rate region as:

$$\max_{\mathbf{V}} \; \mathcal{P}(\mathbf{V}) \triangleq \sum_{(i,j)\in\mathcal{S}} r_{i,j}(\mathbf{V}) \quad \text{s.t.} \tag{2a}$$

$$\sum_{j\in\mathcal{J}} \langle \mathbf{V}_{i,j}\mathbf{V}^H_{i,j}\rangle \leq P^{\max}_i, \forall i \in \mathcal{I}, \tag{2b}$$

where $P^{\max}_i$ is the maximum transmit power of BS i.

Problem (2) is a challenging nonconvex optimization problem because its objective function is nonconvex and even nonsmooth. It should be emphasized that the available sum-rate maximization solutions (see, e.g., [10]) are only suitable for the conventional coordinated precoding approach, which corresponds to $r_{i,j}(\mathbf{V}) \equiv r^p_{i,j}(\mathbf{V})$ and $\mathbf{V} = [\mathbf{V}^p]_{(i,j)\in\mathcal{S}}$ (i.e., there is no split of UE's data). On the other hand, the covariance optimization approach in [8], [9] is applicable for the case of $L = N_t$ only, i.e., one independent data stream is sent per one transmit antenna. In this particular case, (2) can be equivalently transformed to the optimization of a d.c. function in the rank-free outer products $\mathbf{Q}^x_{i,j} = \mathbf{V}^x_{i,j}(\mathbf{V}^x_{i,j})^H \succeq \mathbf{0}$, $x \in \{p, c\}$. The computational complexity of each d.c. iteration in [8] is high, because it involves the maximization of a logarithmic-determinant function under semi-definite constraints—a difficult convex optimization problem with unknown polynomial computational complexity. Whenever $L < N_t$, such a variable change leads to the additional difficult rank constraints $\text{rank}(\mathbf{Q}^x_{i,j}) \leq L$ for which there is no available solution method. Indeed, there is no effective d.c. representation of each rate function in (1) even for the simplest case of $L = 1$.

In what follows, we will develop an efficient successive optimization algorithm of low computational complexity to solve problem (2). Our solution works for both cases of $L = N_t$ and $L < N_t$.

Remark. We adopt the pairing rule proposed in [7]. At the receiver of each UE, the optimal solution of (2) for $\mathbf{V}^c_{i,j} \equiv \mathbf{0}$ is used to identify the interference power from the UEs of other cells. Then, each UE is paired with the UE from a different cell that introduces the strongest interference.

3. Proposed Precoder Design

Suppose that $\mathbf{V}^{(\kappa)} \triangleq [\mathbf{V}^{p,(\kappa)}_{i,j} \quad \mathbf{V}^{c,(\kappa)}_{i,j}]_{(i,j)\in\mathcal{S}}$ is a feasible point found at $(\kappa-1)$-th iteration. Define the following quadratic functions:

$$\begin{aligned}
r^{c,(\kappa)}_{i,j}(\mathbf{V}) &\triangleq r^c_{i,j}(\mathbf{V}^{(\kappa)}) + 2\Re\{\langle \mathcal{A}^{c,(\kappa)}_{i,j}, \mathbf{V}^c_{i,j} - \mathbf{V}^{c,(\kappa)}_{i,j}\rangle\} \\
&\quad - \langle \mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}_{i,j}(\mathbf{V}^{(\kappa)})^{-1}, \\
&\quad\quad \mathbf{M}_{i,j}(\mathbf{V}) - \mathbf{M}_{i,j}(\mathbf{V}^{(\kappa)})\rangle, \\
r^{a,(\kappa)}_{i,j}(\mathbf{V}) &\triangleq r^a_{i,j}(\mathbf{V}^{(\kappa)}) + 2\Re\{\langle \mathcal{A}^{a,(\kappa)}_{\hat{i},\hat{j}}, \mathbf{V}^c_{\hat{i},\hat{j}} - \mathbf{V}^{c,(\kappa)}_{\hat{i},\hat{j}}\rangle\} \\
&\quad - \langle \mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})^{-1}, \\
&\quad\quad \mathbf{M}^c_{i,j}(\mathbf{V}) - \mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})\rangle, \\
r^{p,(\kappa)}_{i,j}(\mathbf{V}) &\triangleq r^p_{i,j}(\mathbf{V}^{(\kappa)}) + 2\Re\{\langle \mathcal{A}^{p,(\kappa)}_{i,j}, \mathbf{V}^p_{i,j} - \mathbf{V}^{p,(\kappa)}_{i,j}\rangle\} \\
&\quad - \langle \mathbf{M}^p_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})^{-1}, \\
&\quad\quad \mathbf{M}^a_{i,j}(\mathbf{V}) - \mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})\rangle,
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A}^{c,(\kappa)}_{i,j} &\triangleq \mathbf{H}^H_{i,i,j}\mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})^{-1}\mathbf{H}_{i,i,j}\mathbf{V}^{c,(\kappa)}_{i,j}, \\
\mathcal{A}^{a,(\kappa)}_{\hat{i},\hat{j}} &\triangleq \mathbf{H}^H_{\hat{i},i,j}\mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})^{-1}\mathbf{H}_{\hat{i},i,j}\mathbf{V}^{c,(\kappa)}_{\hat{i},\hat{j}}, \\
\mathcal{A}^{p,(\kappa)}_{i,j} &\triangleq \mathbf{H}^H_{i,i,j}\mathbf{M}^p_{i,j}(\mathbf{V}^{(\kappa)})^{-1}\mathbf{H}_{i,i,j}\mathbf{V}^{p,(\kappa)}_{i,j}.
\end{aligned}$$

Note that all the above functions are concave in $\mathbf{V}$ because $\mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}_{i,j}(\mathbf{V}^{(\kappa)})^{-1} \succeq \mathbf{0}$, $\mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}^c_{i,j}(\mathbf{V}^{(\kappa)})^{-1} \succeq \mathbf{0}$, $\mathbf{M}^p_{i,j}(\mathbf{V}^{(\kappa)})^{-1} - \mathbf{M}^a_{i,j}(\mathbf{V}^{(\kappa)})^{-1} \succeq \mathbf{0}$.

The following result shows that the complicated function defined by (1) is lower bounded by a concave quadratic function.

Algorithm 1 Proposed SCQP Algorithm

Initialization: Initialize a feasible point $\mathbf{V}^{(0)}$ that satisfies power constraint (2b).

κ-th iteration: Solve convex quadratic program (4) to find an optimal solution $\mathbf{V}^*$. If $|(\mathcal{P}(\mathbf{V}^*) - \mathcal{P}(\mathbf{V}^{(\kappa)}))/\mathcal{P}(\mathbf{V}^{(\kappa)})| \leq \epsilon$, terminate. Otherwise, set $\kappa := \kappa + 1$, $\mathbf{V}^{(\kappa)} := \mathbf{V}^*$ and continue.

Theorem 1: For

$$r^{(\kappa)}_{i,j}(\mathbf{V}) \triangleq r^{p,(\kappa)}_{i,j}(\mathbf{V}) + \min\{r^{c,(\kappa)}_{i,j}(\mathbf{V}), r^{a,(\kappa)}_{a(i,j)}(\mathbf{V})\},$$

it is true that

$$r_{i,j}(\mathbf{V}^{(\kappa)}) = r^{(\kappa)}_{i,j}(\mathbf{V}^{(\kappa)}) \quad \text{and} \quad r_{i,j}(\mathbf{V}) \geq r^{(\kappa)}_{i,j}(\mathbf{V}), \ \forall \ \mathbf{V}. \quad (3)$$

Proof. The proof is given in Appendix. □

In Algorithm 1, we propose a successive convex quadratic programming (SCQP) algorithm to solve problem (2). Given a feasible point $\mathbf{V}^{(\kappa)}$, this algorithm iteratively generates a feasible point $\mathbf{V}^{(\kappa+1)}$ as the optimal solution to the following optimization problem at the κ-th iteration:

$$\max_{\mathbf{V}} \ \mathcal{P}^{(\kappa)}(\mathbf{V}) \triangleq \sum_{(i,j)\in\mathcal{S}} r^{(\kappa)}_{i,j}(\mathbf{V}) \quad \text{s.t.} \quad (2b). \qquad (4)$$

Problem (4) is a convex quadratic with $m = N + 2KN$ quadratic constraints and $n = 2NKN_tL + KN$ real decision variables. The complexity for computing its optimal solution $\mathbf{V}^{(\kappa+1)}$ is thus $O(n^2m^{2.5} + m^{3.5})$. Note that $\mathbf{V}^{(\kappa)}$ is also feasible to (4) with $\mathcal{P}(\mathbf{V}^{(\kappa)}) = \mathcal{P}^{(\kappa)}(\mathbf{V}^{(\kappa)})$ by the equality in (3). It is then true that $\mathcal{P}^{(\kappa)}(\mathbf{V}^{(\kappa+1)}) > \mathcal{P}^{(\kappa)}(\mathbf{V}^{(\kappa)}) = \mathcal{P}(\mathbf{V}^{(\kappa)})$ whenever $\mathbf{V}^{(\kappa+1)} \neq \mathbf{V}^{(\kappa)}$. Together with $\mathcal{P}(\mathbf{V}^{(\kappa+1)}) \geq \mathcal{P}^{(\kappa)}(\mathbf{V}^{(\kappa)})$ according to the inequality in (3), we have that $\mathcal{P}(\mathbf{V}^{(\kappa+1)}) > \mathcal{P}(\mathbf{V}^{(\kappa)})$, i.e., the optimal solution $\mathbf{V}^{(\kappa+1)}$ of the convex quadratic problem (4) is a better point of the nonconvex nonsmooth optimization problem (2) than $\mathbf{V}^{(\kappa)}$. Therefore, once initialized from an achievable sum-rate $\mathcal{P}(\mathbf{V}^{(0)})$, the sequence $\{\mathcal{P}(\mathbf{V}^{(\kappa)})\}$ obtained by solving (4) is guaranteed to improve at each iteration and it eventually converges to at least a local optimum of (2) [11].

4. Numerical Results

In our simulations, we assume $\mathbf{H}_{i,s,l} \triangleq \sqrt{\eta_{i,s,l}}\tilde{\mathbf{H}}_{i,s,l}$ where $\tilde{\mathbf{H}}_{i,s,l} \in \mathbb{C}^{N_r \times N_r}$ with $N_t = 4$ and $N_r = 2$ represents the normalized MIMO channel, the entries of which are independent and identically distributed complex Gaussian variables with zero-mean and unit variance. Following [4], [12] the direct channel powers $\eta_{i,i,l}$ are fixed, while the interfering channel powers $\eta_{i,j,\ell}$ $(i \neq j)$ are varied to cover all the environment-dependent channel effects, including path loss and shadowing. The simulation scenarios thus vary from weak MIMO INs to mixed MIMO INs, for which the Han-Kobayashi strategy is advantageous. The notation 'H-K' refers to the Han-Kobayashi strategy whereas 'coordinated' refers to the conventional coordinated precoding approach which only involves private message precoding. It can be

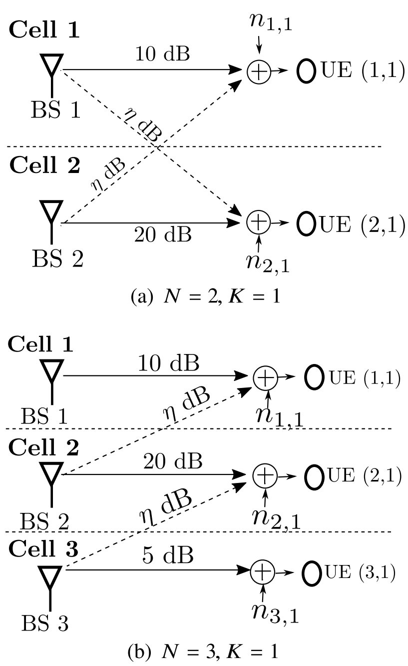

Fig. 1 Network configurations used in the simulations.

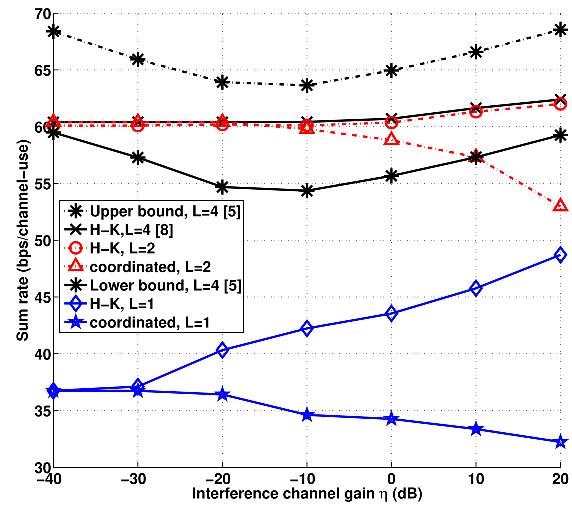

Fig. 2 Performance results for the network in Fig. 1(a).

Table 1 Average number of iterations by Algorithm 1 for the network in Fig. 1(a).

		η (dB)			
		−10	0	10	20
$\epsilon = 10^{-2}$	Coordinated, $L = 1$	5.6	3.1	2	2
	H-K, $L = 1$	5.6	3.2	2.2	2.3
	Coordinated, $L = 2$	5.7	3.2	2.1	2.6
	H-K, $L = 2$	6.1	4.2	2.7	2.7
$\epsilon = 10^{-3}$	Coordinated, $L = 1$	5.5	2.5	2	2
	H-K, $L = 1$	15.2	20.7	15.6	23.6
	Coordinated, $L = 2$	23.88	15.5	10	7.1
	H-K, $L = 2$	32.4	16.1	11.9	8.7
$\epsilon = 10^{-4}$	Coordinated, $L = 1$	8.1	6.7	8.8	15.5
	H-K, $L = 1$	28.1	35.2	35.2	41.3
	Coordinated, $L = 2$	43.8	31.8	34.1	27.1
	H-K, $L = 2$	41.5	55.3	34.6	38.1

seen from (2) that the IN sum rate monotonically increases in the number of involved data streams L. Since the covariance optimization approach in [8] is suitable for $L = N_t = 4$, it is used for performance evaluation in this case. The comparison between the 'H-K' and 'coordinated' schemes is to show the capability of the H-K strategy in mitigating the intercell interferences. Each result in the Monte-Carlo simulation is obtained upon averaging over 100 random network realizations. We set the error tolerance as $\epsilon = 10^{-4}$ and $\sigma^2 = 1$, $P_i^{\max} = 10^3$, $\forall i$ in (2). We divide the achieved sum-rate results by ln(2) to arrive at the unit of bps/channel-use for binary communications.

First, we consider the two-cell two-UE MIMO network depicted in Fig. 1(a). The values of direct channel gains $\eta_{1,1,1}$ and $\eta_{2,2,1}$ are indicated in the figure, while interference channel gains $\eta_{1,2,1} = \eta_{2,1,1} = \eta$ are varied from −40 dB to 20 dB. In this scenario, UEs do not experience any intracell interference and the only UE pairing possibilities are $a(1,1) = (2,1)$ and $a(2,1) = (1,1)$. Figure 2 also includes the curve of the theoretical lower bound and upper bound by solving the linear inequality [5, (52a)–(52i)] and [5, (11)–(17)], respectively. While the performance of the H-K strategy with $L = 4$ or $L = 2$ is above the lower bound for the whole range of η considered, the performance of the 'coordinated' scheme is worse than the lower bound for $\eta \geq 10$ dB. More importantly, the H-K strategy in all considered cases of L is able to achieve better sum rate performance even when the interference channel gain increases. As seen, the 'H-K' scheme offers a substantial performance gain over the 'coordinated' counterpart, especially for large interference channel gain η. In particular, an improvement of up to 30% is observed for $L = 1$ and $\eta = 20$ dB. In addition, the performance of the H-K strategy with $L = 2$ and the proposed precoding matrices is very close to the performance of the scheme in [8] under the covariance matrix design and $L = 4$. This means that our solution approaches the global optimum of problem (2).

Table 1 shows the average number of iterations required for Algorithm 1 to converge, which is similar to the convergence result in [8, Table IV] for the SDP-based covariance optimization algorithm. Since our proposed algorithm is an iterative procedure, the value of ϵ determines the number of iterations and the performance of the algorithm. In general, choosing which value of ϵ is governed the tradeoff between performance and complexity one wants to make. Our numerical results show that the performance of the proposed algorithm is indeed improved as ϵ decreases. For clarity of presentation, we only include the performance of the proposed algorithm with $\epsilon = 10^{-4}$ and focus on performance comparison between the H-K strategy and the 'coordinated' scheme. However, for all the values of ϵ considered, the H-K solutions outperform the solutions corresponding to the coordinated scheme.

Next, we consider the three-cell three-UE MIMO network depicted in Fig. 1(b). The values of direct channel gains are indicated in the figure. Following [8, Fig. 5(b)],

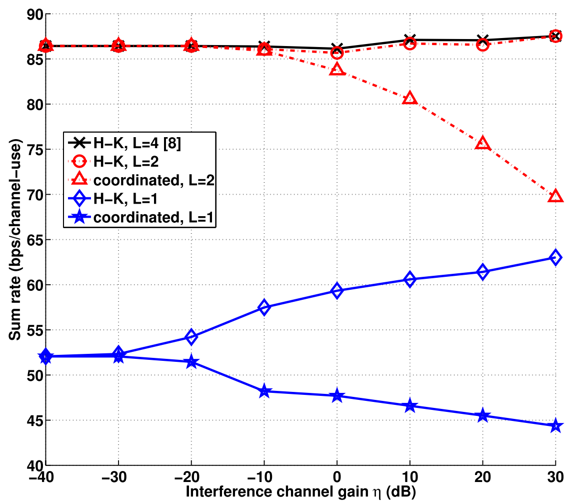

Fig. 3 Performance results for the network in Fig. 1(b).

we set $\eta_{1,2,1} = \eta_{2,3,1} = \eta_{3,1,1} = 0$, while varying other interfering channel gains $\eta_{2,1,1} = \eta_{3,2,1} = \eta$ from -40 dB to 30 dB. In this case, the obvious choice for UE pairing is $a(1,1) = (2,1)$, $a(2,1) = (3,1)$. Figure 3 demonstrates the H-K strategy is again able to improve the sum-rate performance under stronger channel interferences for all cases of L. The performance gap between 'H-K' and 'coordinated' curves is widened especially in the high interference region of $\eta \geq 20$ dB.

It is worth noting that in both considered examples, the performances of 'H-K' scheme are not much distinguishable for $L = 2$ and $L = 4$, although the optimal covariance matrices $\mathbf{Q}^x_{i,j} \in \mathbb{C}^{4\times4}$, $x \in \{p, c\}$ are not necessarily of rank $N_r = 2$ [8]. This result implies that using $L = 2$ data streams gives a performance that is close to the best sum-rate performance. For $L = 1$, an improvement is still observed in the region $\eta \in [-20, -10]$ dB, where the interference per antenna at each UE is at least 14.47 dB and 8.27 dB for the networks in Fig. 1(a) and Fig. 1(b), respectively. These levels of interference are well above the background noise power of 0 dB. It is sufficient for the low interference regime condition to be satisfied only for the optimal input covariance matrices (those that maximize the achievable sum rate assuming Gaussian inputs and treating interference as noise) if they are full rank [13].

Remark 1: Since the focus of this work is to find the optimal precoding matrices under the H-K strategy for a given user pairing choice, the considered simulation scenarios are limited to having one interferer. In the presence of multiple inter-cell interferers, the H-K strategy only mitigates one paired source of inter-cell interference. The H-K strategy may not be better than the conventional scheme if there is no dominant interferer or if the pairing choices are not optimized. In general, the H-K strategy is most efficient when a user has a dominant source of inter-cell interference. A similar result was obtained in [8] for the case of covariance matrices design.

5. Conclusions

This paper has addressed the problem of precoder design for both common and private messages in MU-MIMO multicell networks under the Han-Kobayashi strategy. Our aim is to find the optimal precoding matrices for network sum-rate maximization. We have proposed a successive convex quadratic programming algorithm to solve the nonconvex optimization problem in the precoder matrices. Numerical results have confirmed the potential advantages of our proposed approach and also the ability of the Han-Kobayashi to mitigate the intercell interference, which leads to even better sum-rate despite an increase in channel interference.

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Appendix: Proof of Theorem 1

Due to space limitation, we only provide an outline of this quite complicated proof. The equality in (3) is obvious because $r^x_{i,j}(\mathbf{V}^{(\kappa)}) = r^{x,(\kappa)}_{i,j}(\mathbf{V}^{(\kappa)})$ for $x \in \{c, a, p\}$. To prove the inequality in (3), it suffices to show that

$$r^x_{i,j}(\mathbf{V}) \geq r^{x,(\kappa)}_{i,j}(\mathbf{V}) \ \forall\ \mathbf{V}, x \in \{c, a, p\}. \qquad (\mathrm{A}\cdot 1)$$

We will prove (A· 1) for $x = c$ only, as the proof for $x = a$ and $x = p$ is similar.

Define

$$g(\mathbf{V}^c_{i,j}, \mathbf{M}) \triangleq \ln |\mathbf{I}_L - h(\mathbf{V}^c_{i,j}, \mathbf{M})|$$

and

$$h(\mathbf{V}^c_{i,j}, \mathbf{M}) \triangleq (\mathbf{V}^c_{i,j})^H \mathbf{H}^H_{i,i,j} \mathbf{M}^{-1} \mathbf{H}_{i,i,j} \mathbf{V}^c_{i,j}$$

in the domain

$$\mathcal{U} \triangleq \{(\mathbf{V}^c_{i,j}, \mathbf{M})\ : \mathbf{M} \succ \mathbf{H}_{i,i,j} \mathbf{V}^c_{i,j} (\mathbf{V}^c_{i,j})^H \mathbf{H}^H_{i,i,j}\}.$$

By [14, Appendix C], $h(.,.)$ is convex-valued in $\mathcal{U}$, i.e.,

$$\begin{aligned} &\alpha h(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta h(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}) \quad \succeq \\ &h(\alpha(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)})), \end{aligned}$$

for all $\alpha \geq 0, \beta \geq 0, \alpha + \beta = 1$. It follows that

$$\begin{aligned} &g(\alpha(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)})) && = \\ &\ln |\mathbf{I}_{N_r} - h(\alpha(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}))| && \geq \\ &\ln |\mathbf{I}_{N_r} - (\alpha h(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta h(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}))| && \geq \\ &\alpha \ln |\mathbf{I}_{N_r} - h(\mathbf{V}^c_{i,j}, \mathbf{M})| + \beta \ln |\mathbf{I}_{N_r} - h(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)})| && = \\ &\alpha g(\mathbf{V}^c_{i,j}, \mathbf{M}) + \beta g(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}), \end{aligned}$$

i.e. $g(.,.)$ is concave in $\mathcal{U}$.

Based on the fact that the first order approximation of a concave function at a given point is its global upper bound [15], we have

$$g(\mathbf{V}^c_{i,j}, \mathbf{M}) \leq g^{(\kappa)}(\mathbf{V}^c_{i,j}, \mathbf{M}), \qquad (\mathrm{A}\cdot 2)$$

where using a standard differential calculus of log-det functions yields

$$\begin{aligned} &g^{(\kappa)}(\mathbf{V}^c_{i,j}, \mathbf{M}) && \triangleq \\ &g(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}) \\ &+\langle \nabla g(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}), (\mathbf{V}^c_{i,j}, \mathbf{M}) - (\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)})\rangle && = \\ &g(\mathbf{V}^{c,(\kappa)}_{i,j}, \mathbf{M}^{(\kappa)}) \\ &-2\Re\{\langle \mathbf{H}^H_{i,i,j}(\mathbf{M}^{(\kappa)})^{-1}\mathbf{V}^{c,(\kappa)}_{i,j}\mathbf{H}_{i,i,j}, \mathbf{V}_{i,j} - \mathbf{V}^{c,(\kappa)}_{i,j}\rangle\} \\ &+\langle(\mathbf{M}^{(\kappa)} - \mathbf{H}_{i,i,j}\mathbf{V}^{c,(\kappa)}_{i,j}(\mathbf{V}^{c,(\kappa)}_{i,j})^H\mathbf{H}^H_{i,i,j})^{-1} \\ &-(\mathbf{M}^{(\kappa)})^{-1}, \mathbf{M} - \mathbf{M}^{(\kappa)}\rangle, \end{aligned}$$

Then (A· 1) for $x = c$ follows by substituting $\mathbf{M} = \mathbf{M}_{i,j}(\mathbf{V})$ and $\mathbf{M}^{(\kappa)} = \mathbf{M}_{i,j}(\mathbf{V}^{(\kappa)})$ in (A· 2) and verifying that

$$r^c_{i,j}(\mathbf{V}) = -g(\mathbf{V}^c_{i,j}, \mathbf{M}_{i,j}(\mathbf{V}))$$

and

$$r^{c,(\kappa)}_{i,j}(\mathbf{V}) = -g^{(\kappa)}(\mathbf{V}^c_{i,j}, \mathbf{M}_{i,j}(\mathbf{V})).$$

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