A Novel Small Signal Model of Multi-Bus Microgrids for Modeling Interaction of Droop Controllers through the Power Network

Mohsen Eskandari, Li Li,
Faculty of Engineering and Information Technology, University of Technology Sydney, Australia
Email: Mohsen.Eskandari@student.uts.edu.au

***Abstract*-- Microgrid (MG) consists of distributed generation (DG) units, which supply MG loads via the power network. Droop-based control system has been proposed for power sharing implementation among DG units in MGs. The droop control performance is based on the power network variables, which makes it easy to implement. As the control system at the primary level, the droop controller plays a major role in secure operation of MGs in terms of stability concern. However, the lack of high band-width communication links, low X/R ratio of grid impedance and fast response power converters make the droop-based MGs exposed to instability. In this work, a suitable small signal model is developed to assess the droop control system performance and stability. In the proposed model, droop controllers' interaction through the transmission network is modeled appropriately. The proposed model reveals unstable regions which have not been discovered by the conventional parallel-based small signal models.**

***Index Terms*-- Dynamic Stability, Distributed Generation, Microgrid, Power Sharing, Voltage Regulation,.**

I. INTRODUCTION

The two major features of microgrid (MG), which are introduced as promising means of proper integration of renewable energy resources (RES) into power systems, are islanded operation capability and efficiency improvement [1]. The former enhances the reliability for sensitive loads, and the latter is achieved by implementing energy management system which dispatches power among installed distributed generation (DG) units in MGs. Providing these services simultaneously, i.e. implementing energy management in an islanded MG, requires employing an efficient control system at the primary level of hierarchical control structure of MGs [2]. Droop-based control method, inspired from synchronous generators behavior in the conventional power systems, is proposed as affordable method for power sharing implementation in MGs [3].

According to autonomous operation capability of MGs, as isolated power system, all necessities in conventional power networks, such as energy management and economic dispatch, bidding and participating in the market, demand response, and voltage and frequency control, are needed to establish in MGs [4]. So, the hierarchical control structure has been proposed for MGs. The hierarchical control system of MGs consists of three layers. Tertiary layer is specifically designed for energy management process, and making decision of connected or islanded operation of MG [5]. This level of control generally influences MG's steady state operation and does not affect MG dynamic performance. Secondary level is in charge of power quality improvement by restoring voltage and frequency deviations. Primary level, on the other hand, is responsible for dynamic stability of MG as well as implementing power sharing among DG units to maintain the production and consumption balance [6]-[8]. So, the control strategy adopted in this level performs a vital role in the safe operation of islanded MGs [9]. Droop control is the most straightforward way for this purpose, by which the MG designer extricates from expensive and less reliable high band-width communication structure.

However, removing communication links as well as low X/R ratio of grid impedance reduce stability margin of droop-based multi-bus MGs (MBMGs). So, an appropriate small signal model for droop-based control systems in MBMGs is essential for the sake of stability analysis. The existing models in the literatures, mostly deal with parallel-based (P-based) MGs in which DG units are connected to the MG main bus, the point of common coupling, via power converters, LC filters and interconnecting power lines [10]-[13]. The parallel connection of installed DG units in a radial or meshed distribution grid is not very common [14]-[16]. Besides, the small signal models of individual voltage source inverters (VSI) are transformed to a common reference frame by transforming the output current state variable. Therefore, f/P and V/Q droop controllers are not modeled properly as the control variables are the phase angle and magnitude of the output voltage, respectively. In some studies, effective elements are not inserted into the model [17]-[19].

In this work, a power network-based (PN-based) small signal model for MBMGs is proposed. In the developed model, current and power flow through the power network are obtained as function of voltage magnitude and phase angle of the voltage at MBMG buses which, in turn, are determined by droop controllers. This method has two main advantages: 1) This strategy reduces the order of the model in comparison to conventional parallel-based models. 2) The interaction of droop controllers are modeled properly. The eigenvalue location of the obtained state matrix of typical MBMGs shows some unstable poles in relation to grid impedance and phase angle variations.

II. MBMG Control System

A. MBMG Structure

The MBMG architecture is depicted in Fig. 1. Each DG unit is connected to its related bus via power converter interface and LC filter. For the sake of simplicity, the LC filters related to other converters are not shown in the figure. Every bus could be either generation bus or load bus or both. Adjacent buses are connected to each other by low voltage interconnecting power lines, in arbitrary radial or meshed topology, as well as low band-width communication links. In this way, the entry of adjacency matrix *(σ_{ij})* is defined by 1 if the i^{th} bus is connected to the j^{th} bus and 0 otherwise.

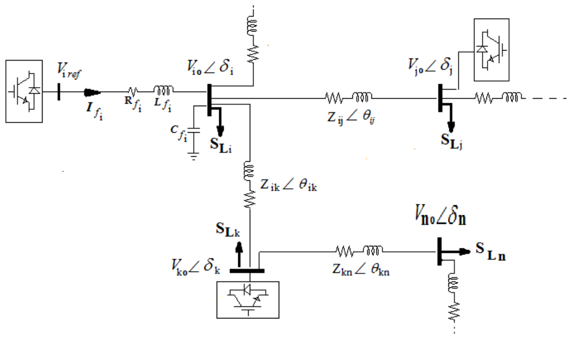


Fig. 1. MBMG topology

B. Power Flow Analysis

In order to obtain the MBMG small signal model, first we need to obtain the power flow equations. For the power flowing from bus i to bus j, we have:

$$\tilde{I}_{ij} = \frac{V_i\angle\delta_i - V_j\angle\delta_j}{Z_{ij}\angle\theta_{ij}} \tag{1}$$

$$\tilde{S}_{ij} = \tilde{V}_i.\tilde{I}_{ij}^{\,*} = \frac{V_i^2\angle\theta_{ij} - V_i.V_j\angle(\delta_i - \delta_j + \theta_{ij})}{Z_{ij}} \tag{2}$$

$$P_{ij} = \frac{V_i^2.\cos(\theta_{ij})}{Z_{ij}} - \frac{V_i.V_j.\cos(\delta_i - \delta_j + \theta_{ij})}{Z_{ij}} \tag{3}$$

$$Q_{ij} = \frac{V_i^2.\sin(\theta_{ij})}{Z_{ij}} - \frac{V_i.V_j.\sin(\delta_i - \delta_j + \theta_{ij})}{Z_{ij}} \tag{4}$$

where i and j indexes denote the i^{th} and j^{th} buses, V_i is the voltage magnitudes, δ_i is the voltage phase angle, $\tilde{I}_{ij}$, $\tilde{S}_{ij}$, P_{ij} & Q_{ij} are the complex current, complex power, active power and reactive power, respectively, Z_{ij} and θ_{ij} are the magnitude and the phase angle of the interconnecting line impedance, respectively. By extending the second terms of right hand sides of (3)-(4) and rearranging (3)-(4), we obtain:

$$P_{ij} = \frac{V_i^2.\cos(\theta_{ij})}{Z_{ij}} - \frac{V_i.V_j.\cos(\delta_{ij}).\cos(\theta_{ij})}{Z_{ij}} + \frac{V_i.V_j.\sin(\delta_{ij}).\sin(\theta_{ij})}{Z_{ij}} \tag{5}$$

$$Q_{ij} = \frac{V_i^2.\sin(\theta_{ij})}{Z_{ij}} - \frac{V_i.V_j.\cos(\delta_{ij}).\sin(\theta_{ij})}{Z_{ij}} - \frac{V_i.V_j.\sin(\delta_{ij}).\cos(\theta_{ij})}{Z_{ij}} \tag{6}$$

where $\delta_{ij} = \delta_i - \delta_j$. For each bus, the power flow equations would be:

$$0 = -P_i + \sum_{j=1}^{n} \sigma_{ij}.P_{ij} + P_{Li} \tag{7}$$

$$0 = -Q_i + \sum_{j=1}^{n} \sigma_{ij}.Q_{ij} + Q_{Li} \tag{8}$$

where P_i and Q_i are the injected active and reactive powers, respectively, P_{Li} and Q_{Li} are the active and reactive loads, and n is the number of MBMG buses. Equations (5)-(6) demonstrate how the power flows in the interconnecting line as function of the terminal buses variables, i. e. the voltage magnitude and phase angle.

III. Small Signal Model

In order to assess the stability performance of droop-based MG control system, a novel small signal model of MBMG is developed in which MG topology, grid impedance, the placement of MG elements and droop controller have been taken into account. For conventional *P/f* and *Q/V* droop control to control the output active and reactive power of DG units in MGs, for each VSI we have;

$$w_i - w_0 = -k_{p_i}(P_i - P_{0i}) \tag{9}$$

$$V_{i,ref} - V_0 = -k_{qi}(Q_i - Q_{0i}) \tag{10}$$

where w_i & $V_{i,ref}$ are the operating radian frequency and voltage magnitude, respectively, w_0 & V_0 are the nominal radian frequency and voltage magnitude, respectively, k_{pi} & k_{qi} are the *P/f* and *Q/V* droop coefficients respectively, P_i & Q_i are the output active and reactive powers of DG units, respectively, and P_{0i} & Q_{0i} are the reference active and reactive powers of DG units, respectively, which are equal to zero in the islanded mode operation. In order to define state variables of the MBMG, first we can define:

$$\dot{\delta}_i = w_i - w_0 \tag{11}$$

Substituting (11) into (9) and considering the islanded mode operation of MBMG we have:

$$\dot{\delta}_i = -k_{pi}.\overline{P}_i \tag{12}$$

$$V_{i,ref} = V_{i0} - k_{qi}.\overline{Q}_i \tag{13}$$

where $\overline{P}_i$ and $\overline{Q}_i$ are average active and reactive power obtained from passing instantaneous active and reactive power through first order filter. For the LC filter we can write:

$$\dot{I}_{fid} = -\frac{R_{fi}}{L_{fi}} I_{fid} + w_0.I_{fiq} + \frac{1}{L_{fi}} V_{i,ref} - \frac{1}{L_f} V_{iod} \tag{14}$$

$$\dot{I}_{fiq} = -\frac{R_{fi}}{L_{fi}} I_{fiq} - w_0.I_{fid} - \frac{1}{L_{fi}} V_{ioq} \tag{15}$$

$$\dot{V}_{iod} = w_0.V_{ioq} + \frac{1}{C_{fi}} I_{fid} - \frac{1}{C_{fi}} I_{iod} \tag{16}$$

$$\dot{V}_{ioq} = -w_0.V_{iod} + \frac{1}{C_{fi}} I_{fiq} - \frac{1}{C_{fi}} I_{ioq} \tag{17}$$

where R_{fi}, L_{fi}, and C_{fi} are the resistance, inductance and capacitance of LC filter respectively, I_{fid} and I_{fiq} are the direct and quadrature (d-q) components of LC filter inductance currents, respectively, V_{iod} and V_{ioq} are d-q components of DG unit, respectively, I_{oid} and I_{oiq} are d-q components of DG unit output current. In order to derive the DG output current according to power line parameters of the grid, from Park transformation theory for a simple rL circuit, we have:

$$\begin{bmatrix} V_{iod} - V_{jod} \\ V_{ioq} - V_{joq} \end{bmatrix} = \begin{bmatrix} r_{ij} & -wL_{ij} \\ wL_{ij} & r_{ij} \end{bmatrix} \begin{bmatrix} I_{ijd} \\ I_{ijq} \end{bmatrix} \quad (18)$$

From (18) we have:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \frac{1}{r^2 + w^2 L^2} \begin{bmatrix} r & wL \\ -wL & r \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \ or \ I_{dq} = Y.V_{dq} \quad (19)$$

where Y is the power line admittance matrix. For the output current of DG unit and from (19), we have:

$$I_{io,dq} = \sum_{j=1}^{n} \sigma_{ij} . I_{ij,dq} = \sum_{j=1}^{n} \left[\sigma_{ij} . Y_{ij} \left(V_{io,dq} - V^i_{jo,dq} \right) \right] \quad (20)$$

where $I_{io,dq}$ is the i^{th} DG unit outgoing current in the dq reference frame, $I_{ij,dq}$ is the current flowing from bus i to bus j, and $V_{io,dq}$ is the output voltage at i^{th} DG unit, and $V^i_{jo,dq}$ is the d-q components of the voltage at the j^{th} bus which is transformed to the i^{th} reference frame, and a_{ij} is the ij^{th} entry of the adjacency matrix , and n is the number of MBMG buses. To make (20) consistence with power flow equation, we know that:

$$r = Z.\cos(\theta) \ \& \ wL = Z.\sin(\theta) \quad (21)$$

And accordingly:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \quad (22)$$

So from (19)-(22) we can obtain:

$$\begin{aligned} I_{io,d} = & \sum_{j=1}^{n} \frac{\sigma_{ij}}{Z_{ij}} \left[\cos(\theta_{ij}) V_{iod} + \sin(\theta_{ij}) V_{ioq} \right] \\ & - \sum_{j=1}^{n} \frac{\sigma_{ij}}{Z_{ij}} \left[\cos(\theta_{ij}) V^i_{jod} + \sin(\theta_{ij}) V^i_{joq} \right] \end{aligned} \quad (23)$$

$$\begin{aligned} I_{io,q} = & \sum_{j=1}^{n} \frac{\sigma_{ij}}{Z_{ij}} \left[-\sin(\theta_{ij}) V_{iod} + \cos(\theta_{ij}) V_{ioq} \right] \\ & - \sum_{j=1}^{n} \frac{\sigma_{ij}}{Z_{ij}} \left[-\sin(\theta_{ij}) V^i_{jod} + \cos(\theta_{ij}) V^i_{joq} \right] \end{aligned} \quad (24)$$

Or in a matrix form we have:

$$I_{io,dq} = \left(\sum_{j=1}^{n} \sigma_{ij} \begin{bmatrix} \alpha_{ij} & \beta_{ij} \\ -\beta_{ij} & \alpha_{ij} \end{bmatrix} \right) V_{io,dq} - \left(\sum_{j=1}^{n} \sigma_{ij} \begin{bmatrix} \alpha_{ij} & \beta_{ij} \\ -\beta_{ij} & \alpha_{ij} \end{bmatrix} \right) V^i_{jo,dq} \quad (25)$$

$$I_{io,dq} = \left(\sum_{j=1}^{n} \sigma_{ij} . Y_{ii,dq} \right) . V_{io,dq} - \sum_{j=1}^{n} \sigma_{ij} . Y_{ij,dq} . V^i_{jo,dq} \quad (26)$$

where $\alpha_{ij}=cos(\theta_{ij})/Z_{ij}$ & $\beta_{ij}=sin(\theta_{ij})/Z_{ij}$, index i and j denote the i^{th} and j^{th} DG units. So the small signal model of I_{ijd} and I_{ijq} are:

$$I_{ij,d} = \alpha_{ij} . V_{io,d} + \beta_{ij} . V_{io,q} - \alpha_{ij} . V^i_{jo,d} - \beta_{ij} . V^i_{jo,q} \quad (27)$$

$$I_{ij,q} = -\beta_{ij} . V_{io,d} + \alpha_{ij} . V_{io,q} + \beta_{ij} . V^i_{jo,d} - \alpha_{ij} . V^i_{jo,q} \quad (28)$$

We obtained the flowing current between two buses in terms of the voltage magnitude and interconnecting power line parameters. The other factor which should also be modeled in the current equation is the phase angle. Two i and j buses have different phase angle δ_i and δ_j which relate the two local reference frames. So in order to transform the j^{th} reference frame to the i^{th} one we use the relevant transformation matrix as in (29)

$$\begin{bmatrix} V^i_{jo,d} \\ V^i_{jo,q} \end{bmatrix} = \begin{bmatrix} \cos(\delta_{ij}) & -\sin(\delta_{ij}) \\ \sin(\delta_{ij}) & \cos(\delta_{ij}) \end{bmatrix} \begin{bmatrix} V_{jo,d} \\ V_{jo,q} \end{bmatrix} \quad (29)$$

Accordingly, (27) and (28) would be:

$$\begin{aligned} I_{ij,d} = & \alpha_{ij} . V_{io,d} + \beta_{ij} . V_{io,q} \\ & - \alpha_{ij} . \cos(\delta_{ij}) . V_{jo,d} - \beta_{ij} . \sin(\delta_{ij}) . V_{jo,d} \\ & + \alpha_{ij} . \sin(\delta_{ij}) . V_{jo,q} - \beta_{ij} . \cos(\delta_{ij}) . V_{jo,q} \end{aligned} \quad (30)$$

$$\begin{aligned} I_{ij,q} = & -\beta_{ij} . V_{io,d} + \alpha_{ij} . V_{io,q} \\ & + \beta_{ij} . \cos(\delta_{ij}) . V_{jo,d} - \alpha_{ij} . \sin(\delta_{ij}) . V_{jo,d} \\ & - \beta_{ij} . \sin(\delta_{ij}) . V_{jo,q} - \alpha_{ij} . \cos(\delta_{ij}) . V_{jo,q} \end{aligned} \quad (31)$$

The inner voltage and current control loops in VSI have faster dynamics in comparison to droop control, so for the sake of simplification are ignored here. The other factor which influences the dynamic of the MG control system is the low-pass filter which is used to obtain the average values of active and reactive powers for using in (12)-(13). In order to embed the first order filter effect, with the transfer function of $w_c/(s+w_c)$, in the control system model, for average active and reactive powers we have:

$$\overline{P} = \frac{w_c}{s + w_c} \left[\sum_{j=1}^{n} \left(\sigma_{ij} . P_{ij} \right) + P_{Lii} \right] \quad (32)$$

$$\overline{Q} = \frac{w_c}{s + w_c} \left[\sum_{j=1}^{n} \left(\sigma_{ij} . Q_{ij} \right) + Q_{Lii} \right] \quad (33)$$

From (32)-(33) we have:

$$\dot{\overline{P}}_i = -w_c . \overline{P}_i + w_c . \left[\sum_{j=1}^{n} \left(\sigma_{ij} . P_{ij} \right) + P_{Lii} \right] \quad (34)$$

$$\dot{\overline{Q}}_i = -w_c . \overline{Q}_i + w_c . \left[\sum_{j=1}^{n} \left(\sigma_{ij} . Q_{ij} \right) + Q_{Lii} \right] \quad (35)$$

In order to achieve the small signal model of P_{ij} and Q_{ij}, we know:

$$P = V_d I_d + V_q I_q \quad \& \quad Q = V_q I_d - V_d I_q \quad (36)$$

From (36) for power flowing from bus i to bus j in the MG we have:

$$P_{ij} = V_{io,d} I_{ij,d} + V_{io,q} I_{ij,q} \ \& \ Q_{ij} = V_{io,q} I_{ij,d} - V_{io,d} I_{ij,q} \quad (37)$$

By substituting (30) and (31) into (37) and a little bit of simplification, we have:

$$\begin{aligned} P_{ij} = & \alpha_{ij} V^2_{io,d} \\ & - \alpha_{ij} . \cos(\delta_{ij}) . V_{io,d} V_{jo,d} - \beta_{ij} . \sin(\delta_{ij}) . V_{io,d} V_{jo,d} \\ & + \alpha_{ij} . \sin(\delta_{ij}) . V_{io,d} V_{jo,q} - \beta_{ij} . \cos(\delta_{ij}) . V_{io,d} V_{jo,q} \\ & + \beta_{ij} . \cos(\delta_{ij}) . V_{io,q} V_{jo,d} - \alpha_{ij} . \sin(\delta_{ij}) . V_{io,q} V_{jo,d} \\ & - \beta_{ij} . \sin(\delta_{ij}) . V_{io,q} V_{jo,q} - \alpha_{ij} . \cos(\delta_{ij}) . V_{io,q} V_{jo,q} \\ & + \alpha_{ij} V^2_{io,q} \end{aligned} \quad (38)$$

$$
\begin{aligned}
Q_{ij} = \beta_{ij} V_{io,d}^{2} \\
& - \beta_{ij}.\cos(\delta_{ij}).V_{io,d} V_{jo,d} + \alpha_{ij}.\sin(\delta_{ij}).V_{io,d} V_{jo,d} \\
& + \beta_{ij}.\sin(\delta_{ij}).V_{io,d} V_{jo,q} + \alpha_{ij}.\cos(\delta_{ij}).V_{io,d} V_{jo,q} \\
& - \alpha_{ij}.\cos(\delta_{ij}).V_{io,q} V_{jo,d} - \beta_{ij}.\sin(\delta_{ij}).V_{io,q} V_{jo,d} \\
& + \alpha_{ij}.\sin(\delta_{ij}).V_{io,q} V_{jo,q} - \beta_{ij}.\cos(\delta_{ij}).V_{io,q} V_{jo,q} \\
& + \beta_{ij} V_{io,q}^{2}
\end{aligned} \tag{39}
$$

P_{ij} & Q_{ij} are obtained based on the voltage magnitude and phase angle at the i^th and j^th buses later. P_{Li} & Q_{Li} are related to the local bus load and are obtained by:

$$
\begin{bmatrix} P_{Li} \\ Q_{Li} \end{bmatrix} = \begin{bmatrix} V_{iod} & V_{ioq} \\ V_{ioq} & -V_{iod} \end{bmatrix} \begin{bmatrix} I_{iLd} \\ I_{iLq} \end{bmatrix} \tag{40}
$$

where V_{iod} and V_{ioq} are d-q components of the voltage at the i^th DG unit, respectively, I_{iLd} and I_{iLq} are d-q components of the local load current of the i^th DG unit which are considered as disturbances in this paper.

In (38) and (39), we obtained the formula for power flow based on the d-q reference frame which is consistent with the power flow equations (5)-(6) (as function of the voltage magnitude and phase angle of both buses). Now we define the state variables matrix *x* as:

$$
for \;\; i = 1:n
$$

$$
x_i = \left[\Delta\delta \; \Delta i_{fd} \; \Delta i_{fq} \; \Delta V_{od} \; \Delta V_{oq} \; \Delta\overline{P} \; \Delta\overline{Q} \right]_i^T \tag{41}
$$

From (8), (11)-(17), (30)-(31), (34)-(35) and (38)-(39) the small signal mathematical model of MBMG is obtained as:

$$
for \;\; i = 1:n
$$

$$
\dot{x}_i = \left[J_{ii}.x_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(J_{ij}.x_j \right) \right] + F_i.\Delta I_{iL,dq} \tag{42}
$$

where J_{ii} and J_{ij} is the Jacobian matrices for i^th DG unit obtained from (12)-(17), (30)-(31), (34)-(35) and (38)-(40) as (43)-(44). $\Delta I_{iL,dq}$ are the d-q components variation of the local load current as inputs, which are considered as a disturbance to the system, and F_i is the input matrix at bus *i* defined by (45).

$$
J_{ij} = \sigma_{ij} \begin{bmatrix}
\multicolumn{7}{c}{0_{3\times 7}} \\
\frac{-1}{C_{fi}} \frac{\partial I_{ij,d}}{\partial \delta_j} & 0 & 0 & \frac{-1}{C_{fi}} \frac{\partial I_{ij,d}}{\partial V_{jod}} & \frac{-1}{C_{fi}} \frac{\partial I_{ij,d}}{\partial V_{joq}} & 0 & 0 \\
\frac{-1}{C_{fi}} \frac{\partial I_{ij,q}}{\partial \delta_j} & 0 & 0 & \frac{-1}{C_{fi}} \frac{\partial I_{ij,q}}{\partial V_{jod}} & \frac{-1}{C_{fi}} \frac{\partial I_{ij,q}}{\partial V_{joq}} & 0 & 0 \\
w_c \frac{\partial P_{ij}}{\partial \delta_j} & 0 & 0 & w_c \frac{\partial P_{ij}}{\partial V_{jod}} & w_c \frac{\partial P_{ij}}{\partial V_{joq}} & 0 & 0 \\
w_c \frac{\partial Q_{ij}}{\partial \delta_j} & 0 & 0 & w_c \frac{\partial Q_{ij}}{\partial V_{jod}} & w_c \frac{\partial Q_{ij}}{\partial V_{joq}} & 0 & 0
\end{bmatrix} \tag{44}
$$

$$
F_i = \begin{bmatrix}
0 & 0 & 0 & \frac{-1}{C_{fi}} & 0 & w_c V_{iod} & w_c V_{ioq} \\
0 & 0 & 0 & 0 & \frac{-1}{C_{fi}} & w_c V_{ioq} & -w_c V_{iod}
\end{bmatrix}^T \tag{45}
$$

Given the state variables defined in (41), and according to the small signal mathematical model of MBMG (42)-(45), we are dealing with a control system with absolutely nonlinear behavior. The block diagram of MBMG control system is depicted in Fig. 2.

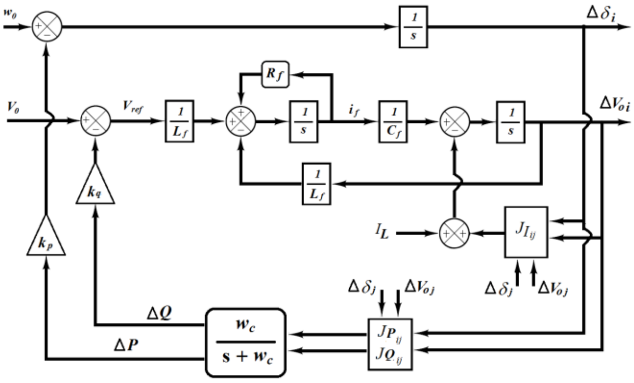


Fig. 2. Block diagram of droop-based control system of MBMG.

Fig. 2 shows that the control system of conventional droop-based MBMGs consists of the *P/f* and *Q/V* loops. The overlapping point between two loops is the Jacobian matrix. According to the equations governing the Jacobian matrices (30)-(31), (38)-(39), if the dominant grid impedance is either inductive or resistive, the control loop will be decoupled.

$$
J_{ii} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & -k_{pi} & 0 \\
0 & \frac{-R_{fi}}{L_{fi}} & w_0 & \frac{-1}{L_{fi}} & 0 & 0 & \frac{-k_{qi}}{L_{fi}} \\
0 & -w_0 & \frac{-R_{fi}}{L_{fi}} & 0 & \frac{-1}{L_{fi}} & 0 & 0 \\
\frac{-1}{C_{fi}} \sum_{j \neq i} \sigma_{ij} \frac{\partial I_{ij,d}}{\partial \delta_i} & \frac{1}{C_{fi}} & 0 & \frac{-1}{C_{fi}} \sum_{j=1}^{n} \sigma_{ij} \frac{\partial I_{ij,d}}{\partial V_{iod}} & \left(w_0 - \frac{1}{C_{fi}} \sum_{j=1}^{n} \sigma_{ij} \frac{\partial I_{ij,d}}{\partial V_{ioq}} \right) & 0 & 0 \\
\frac{-1}{C_{fi}} \sum_{j \neq i} \sigma_{ij} \frac{\partial I_{ij,q}}{\partial \delta_i} & 0 & \frac{1}{C_{fi}} & \left(-w_0 - \frac{1}{C_{fi}} \sum_{j=1}^{n} \sigma_{ij} \frac{\partial I_{ij,q}}{\partial V_{iod}} \right) & \frac{-1}{C_{fi}} \sum_{j=1}^{n} \sigma_{ij} \frac{\partial I_{ij,q}}{\partial V_{ioq}} & 0 & 0 \\
w_c \sum_{j \neq i} \sigma_{ij} \frac{\partial P_{ij}}{\partial \delta_i} & 0 & 0 & w_c \left(\sum_{j=1}^{n} \sigma_{ij} \frac{\partial P_{ij}}{\partial V_{iod}} \right) & w_c \left(\sum_{j=1}^{n} \sigma_{ij} \frac{\partial P_{ij}}{\partial V_{ioq}} \right) & -w_c & 0 \\
w_c \sum_{j \neq i} \sigma_{ij} \frac{\partial Q_{ij}}{\partial \delta_i} & 0 & 0 & w_c \left(\sum_{j=1}^{n} \sigma_{ij} \frac{\partial Q_{ij}}{\partial V_{iod}} \right) & w_c \left(\sum_{j=1}^{n} \sigma_{ij} \frac{\partial Q_{ij}}{\partial V_{ioq}} \right) & 0 & -w_c
\end{bmatrix} \tag{43}
$$

IV. SIMULATION AND EIGENVALUE ANALYSIS

The MBMG, depicted in Fig. 3, is simulated in MATLAB\Simulink to evaluate the effectiveness of the proposed method. The relevant data is given in Table I.

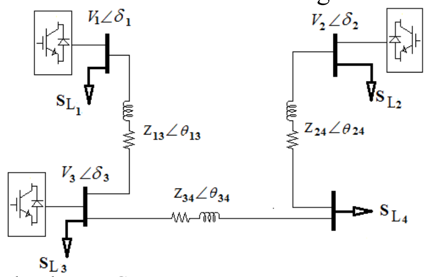


Fig. 3. Simulated MBMG

TABLE I: MBMG DATA

DG unit	DG 1	DG 2	DG 3
k_p	0.5e-5	1e-5	1e-5
k_q	0.25e-3	0.5e-3	0.5e-3
Grid line	Line 13	Line 34	Line 24
Resistance(Ω)	0.060	0.054	0.072
Inductance(H)	3.50e-4	3.15e-4	4.2e-4

Eigenvalue analysis: the eigenvalue location of the state matrix of the small signal model related to the MBMG in Fig. 4 is obtained using MATLAB code. The dominant poles of the MBMG are those related to f/P and V/Q droop control loops, which are obtained in relation to variations of the corresponding droop gains. So, the droop control loops determine the performance and stability of the droop-based control system for MBMG. On the other hand, the droop control operation is based on the power network and power flow variables, i. e. voltage, frequency, active and reactive power. So the presented model is developed based on the relationship of these variables.

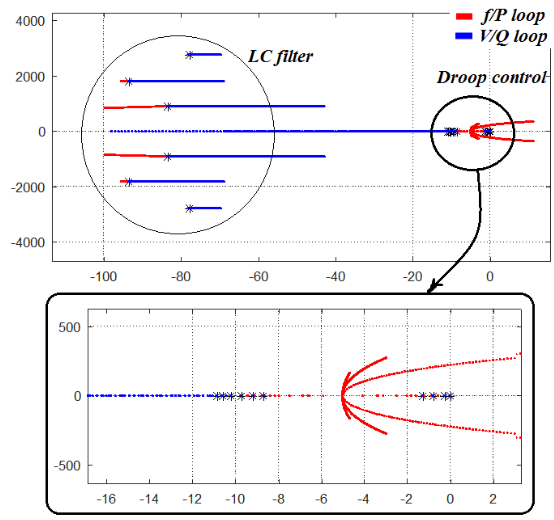


Fig. 4. Eigenvalue location of the MBMG

Effectiveness of the developed model (power network-based model) is compared with the conventional parallel-based model, presented in [10], in terms of the eigenvalue location which is illustrated in Fig. 5. The developed model shows the stability boundaries of the system in relation to grid impedance parameters, by decreasing the Z amplitude as well as X/R ratio of the transmission line, is better than the parallel-based model. Furthermore, the root loci of phase angle difference at MBMG buses, as a consequence of increasing load, are depicted in Fig. 5 (b) as well. While, the parallel-based model does not reveal the unstable regions accurately.

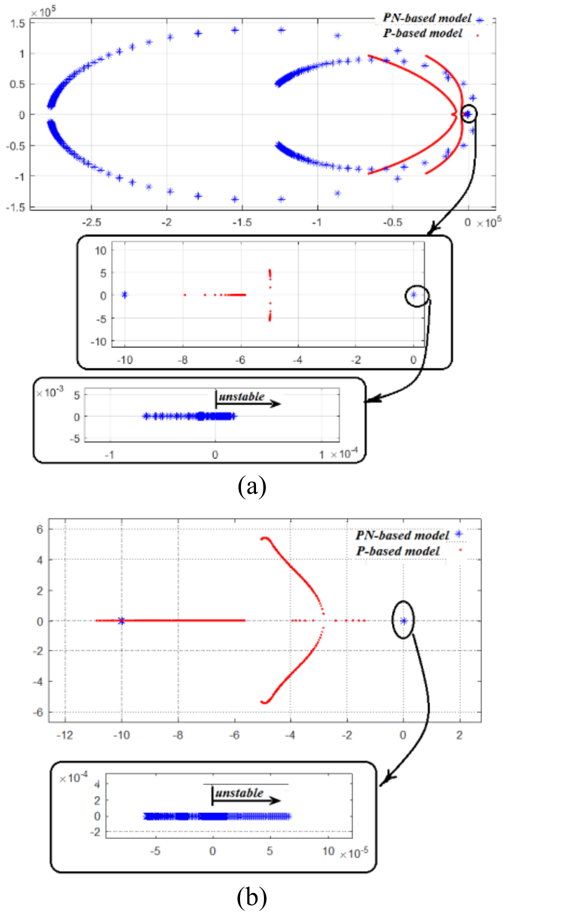
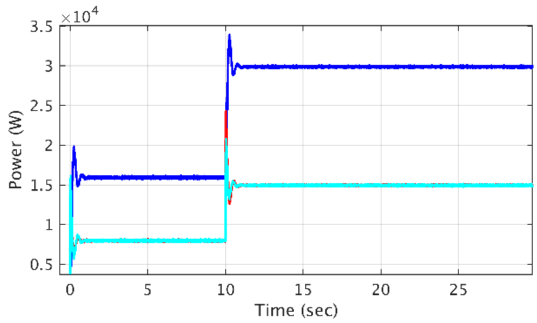
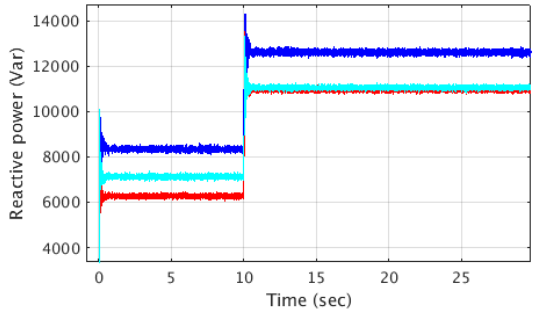


Fig. 5. Dominant eigenvalue comparison. (a) Decreasing inductance of the grid impedance (and X/R ratio as well), (b) Increasing $\Delta\delta_{ij}$ (phase angle). The red points are eigenvalues of power network-based (PN-based) model, and blue points are related to parallel-based (P-based) model.

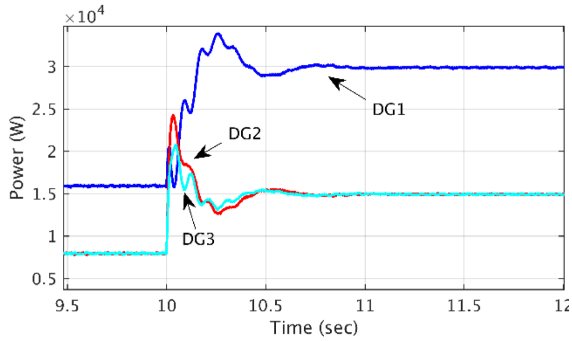
Simulation results: the MBMG in Fig. 3 is simulated in the Simulink platform. The controller is designed based on the proposed model. Although the reactive power sharing is not implemented well, which is the drawback of droop control, the control system is stable. The performance of the f/P and /Q control loop is shown in Figs. 6 (c), (d) which is consistence with the eigenvalue location.



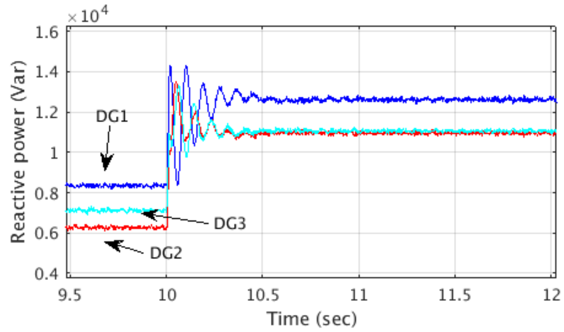
(a)



(b)



(c)



(d)

Fig. 6. Simulation results, (a), (b) active and reactive power sharing by droop control, (c), (d), dynamic response of f-P and V-Q control loops.

V. CONCLUSION

In this work, a new small signal model of MBMG is developed where the interaction of conventional droop control through the transmission network is modelled, to design and assess the control system. First the d-q components of current flowing between DG units located at the end buses of a given power line, is obtained as functions of phase angle and voltage magnitude. Then the power flow equations are developed using the current formula and as functions of d-q components of voltage magnitudes of terminal buses as well as the phase angles. The dominant eigenvalues of the developed model show that the PN-based model can appropriately discover the unstable regions, in relation to grid impedance variation and phase angle difference of bus voltages, while the P-based model fails to do so.

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