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An Approximate Reasoning Based Linguistic Multi-Criteria Group Decision-Making Method

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Abstract. Team decision-making is a remarkable feature in complex dynamic decision context. Linguistic multi-criteria group decision-making (LMCGDM) is a widely-used form of team decision-making. The majority of existing LMCGDM methods are mainly conducted focusing on the selection of aggregation operators; but neglect the approximate reasoning in a decision-making procedure. The presented work analyses three drawbacks in aggregation-based LMCGDM methods and develops an LAR method to implement approximate reasoning in LMCGDM problems. In the LAR method, semantics of assessment and weights for criteria and evaluators are clarified; and decision processing is conducted under a framwork of approximate reasoning. The LAR method can effectively overcome those drawbacks in a reasonable way.
Keywords: group decision-making, approximate reasoning, aggregation operator, implication operator

1. Introduction

In a complex and dynamic decision environment (e.g., bushfire early warning and customer churn management), team decision-making has become a vital feature in the decision-making procedure. Team decision-making can effectively reduce drawbacks such as unbalances and incompleteness in individual decision-making procedure[1]. Linguistic multi-criteria group decision-making (LMCGDM) is one of typical team decision-making patterns, which has been extensively studied in business management [2, 3, 4], industrial producing [5], clinical diagnostics [6], emergency protection [7], as well as social activities [8, 9].

One of core issues of LMCGDM is dealing with uncertainty in natural or artificial languages. Resolutions of an LMCGDM problem depend on the effectiveness of representing and processing linguistic information which is generally expressed by linguistic terms. The majority of existing LMCGDM methods represent linguistic terms by fuzzy sets, in particular fuzzy numbers, on the real interval $[0, 1]$ [1, 2, 3, 4, 5] and process linguistic terms through aggregation operators, such as the OWA operator [10, 11] and 2-tuples operators [12, 13].

Aggregation is an important procedure and feature in human decision-making. Except for aggregation, reasoning is another crucial feature in human decision-making. However, the existing methods focus mainly on aggregation than reasoning. Much work is conducted on the designing of an elaborate computation model but neglects the real requirement of a decision problem. Aggregation cannot replace reasoning in decision-making because real problems is too complex to be modelled just by information aggregation and aggregated information is the antecedent of decision-making rather than the decision. Based on this idea, this paper first discusses three problems without sufficient concerns in existing LMCGDM methods; and then presents an approximate reasoning based LMCGDM method, called LAR. By the LAR method, weights of criteria and evaluators are divided into two basic forms, i.e., for evaluation and for judgment. Moreover, decision-making is described as an approximate reasoning based on assessments of evaluators being as facts and relationships between criteria decision target as inference rules. In terms of different interpretation of weights and assessments, corresponding processing is discussed in detailed.

The rest of the paper is organized as below. Section 2 the semantics of assessments and weights for criteria and evaluators; and discusses the possibility of using approximate reasoning in decision procedure. In Section 3, the LAR method is presented and its features are discussed. An illustration example is given in Section 4. Section 5 discusses some topics of our future work.

2. Three questions in existing LMCGDM

An MCGDM problem is conducted through three main steps [14]: 1) determine the relevant criteria and alternatives; 2) evaluate the relative impacts of alternatives on those criteria; and 3) determine a ranking of each alternative. Formally, a typical MCGDM model can be expressed by

$$\mathcal{M} = (\mathcal{C}, \mathcal{E}, \mathcal{A}, \mathcal{T}, \mathcal{OP}) \tag{1}$$

where $\mathcal{C} = \{(c_j, wc_j)|j = 1, 2, \ldots, n\}$ is a set of identified criteria with their weights; $\mathcal{E} = \{(e_k, we_k)|k = 1, 2, \ldots, m\}$ is a set of evaluators with their weights; $\mathcal{A} = \{a_i|i = 1, 2, \ldots, p\}$ is a set of alternatives (options) to be evaluated; $\mathcal{T} = \{T_i = (v_{jk}^{(i)})_{n\times m}|i = 1, 2, \ldots, p\}$ is a set of decision tables (i.e., evaluations) on those alternatives; and $\mathcal{OP}$ is a set of operators which implement assessments aggregation.

An overall assessment v_i on alternative a_i based on model $\mathcal{M}$ is typically obtained by

$$v_i = (wc_1, wc_2, \ldots, wc_n) \circ \begin{pmatrix} v_{11}^{(i)} & v_{12}^{(i)} & \cdots & v_{1m}^{(i)} \\ v_{21}^{(i)} & v_{22}^{(i)} & \cdots & v_{2m}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1}^{(i)} & v_{n2}^{(i)} & \cdots & v_{nm}^{(i)} \end{pmatrix} \diamond \begin{pmatrix} we_1 \\ we_2 \\ \vdots \\ we_m \end{pmatrix}, \tag{2}$$

where $\circ$ and $\diamond$ are two selected operators which are always aggregation operators. It is obvious that the overall assessment may vary with the different selections of aggregation operators. Hence, the model is aggregation-based.

LMCGDM problems are extension of traditional MCGDM problems by taking linguistic information into account. Linguistic information is always represented by linguistic terms such as "high" and "important." Because linguistic information is of huge amount of uncertainties, fuzzy sets and fuzzy logic are accordingly used to express linguistic terms. Moreover, aggregation operators for numeric values in conventional MCGDM models are applied to or extended for linguistic terms. Literature indicates that most linguistic MCGDM models are still aggregation-based.

Study shows the following three questions are seldom concerned in existing LMCGDM methods:

1. Whether inputs of aggregation belong to same domain?

2. What is the semantic behind the weights of criteria and evaluators?

3. What is the semantic behind the assessments?

First, few aggregation-based linguistic MCGDM models clarify whether the inputs of an aggregation operator belong to the same domain. In our opinion, the question may be answered in three levels, i.e., operation level, structure level, and semantics level. At the operation level, existing methods cannot achieve closeness in aggregation operation with respect to given linguistic term set. Hence, the inputs do not belong to same domain at the operation level. A possible compromise is using the family $\mathcal{F}[0,1]$ of all fuzzy sets on [0,1] as linguistic term set. However, a potential difficulty is that so many terms are unnamed. At the structure level, existing methods seldom distinguish the difference in order, relationship, representation, and operation on linguistic terms between different criteria. In this sense, the inputs do not belong to the same domain at structure level. At the semantic level, the inputs still may not belong to the same domain because inputs may express relative or conflict semantics. For instance, the linguistic term "high" can be used to describe the "primary cost" and the "monthly fee" in a decision problem about customer churn management. However, it has remarkably different meanings. Aggregation operators essentially are a kind of averaging operation which accept a set of inputs in a given domain D and produce a synthesized value in the same domain D [15]. In mathematics, an aggregation operator requires that the inputs and output are closed with respect to the domain D. However, this requirement is not sufficient in a decision problem because the nature of inputs and output, which are affected by the nature of different criteria, must be concerned.

Second, few linguistic MCGDM models clearly identify the semantic of weights of criteria and evaluators. In some cases, weights are used as modifier of reliability of an assessment or reliability of an evaluator's assessment. For instance, "very high" is often used as a weight; but no clear explanation for its meaning. Sometimes, a weight may be used to illustrate the degree of influence on the decision, such as "important." In most existing methods, a weight is just assumed to be a set of numeric values or linguistic terms and processed simply by computations on real number or fuzzy sets without any interpretation. In our opinion, different explanations for the same weight may produce different processing strategies and bring different interpretation of obtained result. For example, if a weight is explained as modifier of reliability of an assessment. The aggregation result should still be explained as an assessment rather than a decision from that assessment. Obviously, this question has closed relationship with the first question and it will affect the construction of processing models.

A third question is, what is the semantic of assessments? Similar to the semantic of weights, an assessment may provide support information for a final decision or just be an evaluation in terms of a given criterion. From the viewpoint of logic, an assessment on final decision is the truth value of the statement $p(x)$ in which x represents an alternative; while an assessment on criteria is the truth value of the statement $c(x)$ where c is a given criterion. Distinguishing of these two different semantics will also lead to different process models. This point will be discussed in the followed sections.

One of the reasons resulting in above questions, in our opinion, is the ignorance of approximate reasoning in the decision procedure. Decision-making in complex and dynamic situation requires not only aggregating multi-sources information but also reasoning based on domain knowledge and experience. Without reasoning, a decision is obtained too simple to be trusted. In fact, aggregation itself reflects a certain of reasoning because an aggregation operator for specific domain can be estimated through a series of hypothesis and testing based on domain observations and knowledge [16]. However, it should be pointed out that aggregation cannot completely take the place of reasoning because: 1) a randomly given aggregation operator may not be appropriate for a special decision problem due to its domain exclusion; and 2) a specific decision problem may use aggregation and reasoning alternatively [17, 18, 19]. Explicitly processing approximate reasoning in decision-making procedure has some benefits. It can provide rational solutions to above listed questions. The following section provides a possible implementation. It can also employ aggregation operators which are not usable in aggregation-based models to real decision problems. Finally, it can take advantage of existing studies in non-classical logic, in particular implication operators which are effective utilities for expressing human reasoning in different domains.

3. An approximate reasoning based LMCGDM method

3.1. Overview

Based on aforementioned questions and discussions, this paper presents an approximate reasoning based LMCGDM method – LAR. The LAR method depicts a decision-making procedure as repetition of reasoning and aggregation. The LAR method is implemented through clarifying the semantics behind the assessments; treating weights of criteria and evaluators as truth-value of specific logical rules; and describing decision procedure as reasoning.

3.2. Semantics of assessments

In the LAR method, an assessment has two types of semantics. The first one is evaluation on criteria, i.e, an assessment is treated as the truth-value of a statement (logic formula) of form $c(x)$ where x is an alternative and c is an evaluated criterion. For instance, "monthly fee" is an evaluated criterion for "customer churn prediction problem" in telecoms. Thus an assessment expressed by "very high" for an alternative in terms of "monthly fee" means "the monthly fee of the alternative is very high" under this kind of semantic, i.e., the truth-value of statement "monthly fee(alternative)" is "very high". The second type of semantics is the truth-value of a statement of form $c(x) \to p(x)$, where p represents the decision target. Continue the "monthly fee" example. An assessment "very high" under this semantic means "based on observation on monthly fee of it, the alternative is of very high possiblity to churn", i.e., the truth-value of statement "alternative will churn based on monthly fee" is "very high". In the following, $c(x) \to p(x)$ is also denoted by $(c \to p)(x)$.

3.3. Weights of criteria and evaluators

In the LAR method, weights for criteria have two kinds of semantics corresponding to the semantics of assessments respectively. The first kind of semantics is the truth-value of the statement $(\forall x)c(x)$; and the second kind of semantics is the truth-value of the statement $(\forall x)(c(x) \to p(x))$ (also denoted by $(\forall x)(c \to p)(x)$.

There are also two kinds of interpretations to weights of evaluators. The first one interprets weights for evaluators as the acceptability of their evaluations. In this sense, weights of evaluators can be seen as modifiers to their evaluations, i.e., a weight is the truth-value of the statement $(\forall x)(c(x)|_{e_k} \to c(x))$ where $c(x)|_{e_k}$ is the assessment from e_k and $c(x)$ is the assessment to be used in decision procedure. A second interpretation treats weights as influences of evaluators on the final decision.

Under this interpretation, a weight describes the relationship between evaluators and final decision, i.e., a weight is the truth-value of rule $(\forall x)(c(x)|_{e_k} \to p(x))$.

Different interpretations of weights for criteria and evaluators will lead to different choices of processing operations and procedures. In the following, the semantic having relationship with the decision problem (i.e., p occurs in the statements) is called with respect to the decision problem (denoted by $v \to p$); otherwise is called with respect to alternative (denoted by v).

3.4. Decision-making procedure as approximate reasoning

Based on the semantics of assessments and weights for criteria and evaluators, the LAR is implemented by approximate reasoning models. There are eight kinds of models as shown in Table 1. Because the processing for these eight models are similar, this paper just takes model 1 for example.

Table 1. Four cases of processing model

	interpretations		
case id	assessment	criteria weights	evaluator weights
1	$v \to p$	$v \to p$	v
2	$vc \to p$	$v \to p$	$v \to p$
3	v	v	v
4	v	v	$v \to p$
5	v	$v \to p$	v
6	v	$v \to p$	$v \to p$
7	$v \to p$	v	v
8	$v \to p$	v	$v \to p$

In model 1, the decision problem is read: based on the facts

$$c_j(x)|_{e_k}, \quad j = 1, 2, \ldots, n; k = 1, 2, \ldots, m; x \in \mathcal{A}$$
$$((\forall x)(\forall k)(c_j \to p)(x), wc_j)$$
$$((\forall x)(\forall j)(c_j(x)|_{e_k} \to c_j(x)), we_k),$$

and inference rule (Modus Ponens)

$$\frac{P, P \to Q}{Q}, \tag{3}$$

to get the order of $p(x)$.

For resolving this decision problem, the LAR method uses the following steps:

1. Get $c_j(x)$ from $c_j(x)|_{e_k}$ and $((\forall x)(\forall j)(c_j(x)|_{e_k} \to c_j(x)), we_k)$. The following deduction sequence is a typical reasoning in classical and non-classical logic to achieve this goal:

$$\begin{aligned}&(\forall x)(\forall j)(c_j(x)|_{e_k} \to c_j(x)) \to (c_j(x)|_{e_k} \to c_j(x))\\&(\forall x)(\forall j)(c_j(x)|_{e_k} \to c_j(x))\\&c_j(x)|_{e_k} \to c_j(x)\\&c_j(x)|_{e_k}\\&c_j(x).\end{aligned}$$

   Accordingly, a truth-value $\alpha_j(x)$ of $c_j(x)$ is obtained. In general, the value of $\alpha_j(x)$ is calculated and determined by the truth-value operations defined in the underlying logic system. In most cases, those operations can be determined by the truth-value operation for implication operator. Hence, $\alpha_j(x)$ is determined by implication operator. Moreover, the inference rule, i.e. the Module Ponens, also exerts influence on $\alpha_j(x)$. Therefore, the computation of $\alpha_j(x)$ can be given once the underlying logic is fixed. Simply speaking, this step implements approximate reasoning

$$\{((\forall x)(\forall j)(c_j(x)|_{e_k} \to c_j(x)), we_k), (c_j(x)|_{e_k}, v(x)_{jk})\} \vdash (c_j(x), \alpha_j(x)), \quad x \in \mathcal{A} \tag{4}$$

   where $\alpha_j(x)$ is the truth-valued obtained with the reasoning consequences.

2. Get $p_j(x)$ from $c_j(x)$ and $((\forall x)(\forall k)(c_j \to p)(x), wc_j)$, $j = 1, 2, \ldots, n$. By similar deduction in step 1, the goal can be achieved. Hence, this step implements approximate reasoning for each j

$$\{(c_j(x), \alpha_j(x)), ((\forall x)(\forall k)(c_j \to p)(x), wc_j)\} \vdash (p(x), \beta_j(x)), \quad x \in \mathcal{A}, \tag{5}$$

   where $\beta_j(x)$ is the truth-valued obtained with the reasoning consequences. As the obtained $p(x)$ is related to j, it will denoted by $p_j(x)$.

3. Get $p(x)$ from $p_j(x)$ $(j = 1, \ldots, n)$ for each $x \in \mathcal{A}$ by aggregations. This step is implemented through selecting appropriate aggregation operators.

4. Order $p(x)$ for all $x \in \mathcal{A}$. This step ranks alternatives by $p(x)$ with consideration of real problem.

Remark 3.1 *Because a real decision problem may be involved multi-level criteria or evaluators, the step (1), (2), and (3) may be repeated many times before entering step (4).*

Remark 3.2 *Above processing method applies aggregation to the final decisions $p_j(x)$. This has some merits. First, all $p_j(x)$ belong to the same domain which satisfies the demand of an aggregation operator. Second, it can use various aggregation operators. In general, aggregation operators can explicitly or implicitly use weights. For instance,* max *and* min *are two typical aggregations without explicitly usage of weights and the weighted-sum is a typical aggregation with explicitly usage of weight.*[1] *A decision procedure may be involved in a varieties of aggregation operators for reasons such as different evaluators may have different preferences and different criteria may have different demand.*

Remark 3.3 *The reasoning (i.e., the deduction sequences) in step 1 and step 2 holds in a great number of classical and non-classical logical systems. The first advantage we can take from those logical systems is applying different logical systems simultaneously for the reasons that different person has different thinking pattern, in other words, different evaluators may make decision based on different logic. Another advantage is that we can use various implication operators to obtain the truth-values for judgments on $c_j(x)$ and $p(x)$. Implication operator is a kind of logical connectives which can be used to depict the cause-and-effect between facts and consequences. Applying implication operators to decision making procedure is rational because decision is established on the analysis of predictive consequences of obtained observations. Moreover, the final decision can obtained support from logical system and can be explained in a rational way.*

4. A case study

In the section, we use a typical LMCGDM problem as an example to illustrate the LAR method.

4.1. Problem description

A company needs to update its operating system. There are four possible options:

a_1	a_2	a_3	a_4
Linux	Windows	Solaris	VMS

The company has consulted four consultancies experts from "cost analysis"(c_1; e_1), "performance analysis"(c_2; e_2), "security analysis"(c_3; e_3), and "technique support analysis"(c_4; e_4) fields, respectively.

[1]Although the OWA operator can cover some aggregation operators without explicitly using weights such as max and min through change the value and order of weights for inputs, we still refer it as operator with explicitly using weights because the values of weights for inputs seldom change in a real application.

Suppose the assessments are expressed by linguistic terms

$S = \{\text{some}(s_0), \text{very low}(s_1), \text{low}(s_2), \text{medium}(s_3), \text{high}(s_4), \text{very high}(s_5), \text{perfect}(s_6)\}$

and they are given in Table 2.

Table 2. Assessments for operating systems

	a_1	a_2	a_3	a_4
e_1	s_1	s_3	s_3	s_2
e_2	s_3	s_2	s_1	s_4
e_3	s_2	s_3	s_3	s_3
e_4	s_4	s_4	s_2	s_2

Without loss of generality, suppose (1) all experts have same weights which have semantic with respecto to alternatives; and (2) the weights for criteria have semantic with respect to decision problem.

Under above setting, the decision problem satisfies the model 6 and the decision target is determing which one is the best option.

4.2. Decision facts analysis

According to the settings of criteria, the semantics for criteria c_1 and c_3 are different from those of c_2 and c_4 because the expected order of linguistic terms are different. For c_1 and c_3, linguistic term with smaller index is expected; while c_2 and c_4 prefer linguistic term with bigger index. The following facts analysis should consider this point.

Take the option "Linux" (a_1) for example. The assessments for a_1 are formalized by the following logical formulae:

$$(c_1|_{e_1}(a_1), s_1), \qquad (c_2|_{e_2}(a_1), s_3), \qquad (c_3|_{e_3}(a_1), s_2), \qquad (c_4|_{e_4}(a_1), s_4).$$

Because weights for criteria are interpreted with respect to decision problem, the following facts hold:

$$(\forall x)(c_j(x) \to p(x), wc_j), \quad j = 1,2,3,4; x = a_1, a_2, a_3, a_4.$$
$$(c_1(a_1) \to p(a_1), wc_1), \quad (c_2(a_1) \to p(a_1), wc_2),$$
$$(c_3(a_1) \to p(a_1), wc_3), \quad (c_4(a_1) \to p(a_4), wc_1).$$

Similar, because weights for experts are interpreted with respect to alternatives and each criterion is evaluated by a unique expert, the weight we_k is interpretated as "no modification" (denoted by I). Thus, the following facts hold:

$$\begin{aligned}
&(\forall x)(c_j|_{e_j}(x) \to c_j(x), I), \quad j = 1,2,3,4; x = a_1, a_2, a_3, a_4\\
&(c_1|_{e_1}(a_1) \to c_1(a_1), I),\\
&(c_2|_{e_2}(a_1) \to c_2(a_1), I),\\
&(c_3|_{e_3}(a_1) \to c_3(a_1), I),\\
&(c_4|_{e_4}(a_1) \to c_4(a_1), I).
\end{aligned}$$

4.3. Decision process steps

Based on facts analysis of assessments, weights of criteria and evaluators, the decision process for "Linux" is as follows.

Step 1: the $c_1(a_1)$ is obtained by

$$\frac{(c_1|_{e_1}(a_1), s_1), (c_1|_{e_1}(a_1) \to c_1(a_1), I)}{(c_1(a_1), s_1 \otimes I)}, \tag{6}$$

where $s_1 \otimes I$ is a truth-value operation[2)]. Because I means "no modification", $s_1 \otimes I = s_1$. Similar, we have

$$(c_2(a_1), s_3), \qquad (c_3(a_1), s_2), \qquad (c_4(a_1), s_4).$$

Step 2: the $p_1(a_1)$ is obtained by

$$\frac{(c_1(a_1), s_1), (c_1(a_1) \to p(a_1), wc_1)}{(p_1(a_1), s_1 \otimes_1 wc_1)}, \tag{7}$$

where $\otimes_1$ is a truth-value operation for criterion c_1. Notice that the less the cost is the better, we can take design a operation such that $s_i \otimes_1 wc_1 \leq s_j \otimes_1 wc_1$ if $i \geq j$. Here, for the illustration purpose, let $s_i \otimes_1 wc_1 = s_{6-i}$ for criterion c_1. Similarly, for other three criteria, let $s_i \otimes_2 wc_2 = s_i$, $s_i \otimes_3 wc_3 = s_{6-i}$, and $s_i \otimes_4 wc_4 = s_i$. Then we have

$$(p_1(a_1), s_5), \qquad (p_2(a_1), s_2), \qquad (p_3(a_1), s_4), \qquad (p_4(a_1), s_4).$$

Step 3: the $p(a_1)$ is obtained by a selected aggregation operator. Suppose the max is used for this purpose. Then $p(a_1) = s_5$. For other three options, we can obtained

$$p(a_2) = s_4, \qquad p(a_3) = s_3, \qquad p(a_4) = s_4.$$

Step 4: order $p(a_1)$, $p(a_2)$, $p(a_3)$, and $p(a_4)$. Suppose the ranking standard is that "the bigger index the better." Then a_1, i.e., "Linux" is the best option.

[2)] In fact, $\otimes$ is a induced operation in the underlying logic which can be defined by implication operator and is affected by inference rules. For instance, $s_1 \otimes I$ can be defined by ordinary product $s_1 \cdot I$.

5. Conclusions and future works

Linguistic MCGDM problems are ubiquitous in complex and dynamic decision environment. Traditional LMCGDM methods mainly focus on aggregation, but neglect the influence of approximate reasoning. This paper presented an approximate reasoning based LMCGDM method (LAR) to explicitly process reasoning in decision procedure. The LAR method is established on the analysis of semantics of assessments and weights of criteria and evaluators; and treats a decision procedure as a repetition of aggregation and reasoning. The LAR method can take advantage of existing studies on logic, approximate reasoning, and aggregation. It also provides rational explanation to the obtained result.

Aggregation and reasoning are two crucial features in human decision-making. Just emphasizing one of them but neglecting the other is insufficient. Therefore, how to combine these two aspects effectively is a natural question. Although the presented work provided and illustrated a possible way, much work is required. This will be a research task in our future studies. Moreover, aggregation plays important role in LMCGDM problem. But few rational methods are presented to determine how to select an aggregation for a specific problem. This will be another research topic in the future. Finally, how to establish and select an appropriate domain specific logic, particularly in dynamic decision environment, is also a study issue of our future work.

References

[1] Lu, J., Zhang, G., Ruan, D., Wu, F.: Multi-Objective Group Decision Making-Methods, Software and Applications with Fuzzy Set Technology. Imperial College Press, London (2007).

[2] Delgado, M., Herrera, F., Herrera-Viedma, E., Martínez, L.: Combining numerical and linguistic information in group decision making. Information Sciences 107 (1998) 177-94.

[3] Herrera, F., Herrera-Viedma, E.: Linguistic decision analysis: steps for solving decision problems under linguistic information. Fuzzy Sets and Systems 115 (2000) 45-65.

[4] Herrera-Viedma, E.,Chiclana, F., Herrera, F., Alonso, S.: Group decision-making model with incomplete fuzzy preference relations based on additive consistency. IEEE Transactions on Systems, Man, and Cybernetics Part B: Cybernetics 37(1) (2007) 176-189.

[5] Maschio, I.: A decision-support system for safeguards information analysis. International Journal of Nuclear Knowledge Management 2(4) (2007) 410-421.

[6] Kerre, E.E.: The use of fuzzy set theory in electrocardiological diagnostics. In Gupta, M.M., Sanchez, E., eds.: Approximate Reasoning in Decision Analysis. North-Holland, Amsterdam (1982) 277-282.

[7] Lindell, M.K.: An overview of protective action decision-making for a nuclear power plant emergency. Journal of Hazardous Materials 75(2-3) (2000) 113-129.

[8] Munda, G.: A conflict analysis approach for illuminating distributional issues in sustainability policy. European Journal of Operational Research 194(1) (2009) 307-322.

[9] Huynh, V.N., Nakamori, Y.: A satisfactory-oriented approach to multiexpert decision-making with linguistic assessments. IEEE Transactions on Systems, Man, and Cybernetics -Part B: Cybernetics 35(2) (2005) 184-196.

[10] Yager, R.R.: On ordered weighted averaging aggregation operators in multi-criteria decision making. IEEE Transactions on Systems, Man, and Cybernetics 18(1) (1988) 183-190.

[11] Yager, R.R.: OWA aggregation over a continuous interval argument with applications to decision making. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics 34(5) (2004) 1952-1963.

[12] Herrera, F., Martínez, L.: A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems 8(6) (2000) 746-752.

[13] Herrera, F., Martínez, L.: A model based on linguistic 2-tuples for dealing with multi-granular hierarchical linguistic contexts in multi-expert decision-making. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics 31(2) (April 2001) 227-234.

[14] Triantaphyllou, E.: Multi-Criteria Decision Making Methods: A Comparative Study. Kluwer Academic Publishers, Dordrecht/Boston/London (2000).

[15] Mesiar, R., Kolesárová, A., Calvo, T., Komorníková, M.: A review of aggregation functions. In Bustince, H., Herrera, F., Montero, J., eds.: Fuzzy Sets and Their Extensions: Representation, Aggregation and Models. Volume 220 of Studies in Fuzziness and Soft Computing. Springer Berlin/Heidelberg (2008) 121-144.

[16] Beliakov, G.,Warren, J.: Appropriate choice of aggregation operators in fuzzy decision support systems. IEEE Transactions on Fuzzy Systems 9(6) (2001) 773-784.

[17] Wang, G.: Order of aggregate and infer. Journal of Shaanxi Normal University (Natural Science Edition) 27(1) (1999) 1-5.

[18] Buckley, J.J., Hayashi, Y.: Can approximate reasoning be consistent? Fuzzy Sets and Systems 65 (1994) 13-18.

[19] Buckley, J.J.: Erratum to "can approximate reasoning be consistent?" Fuzzy Sets and Systems 103 (1999) 189-190.