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Sparse Observer-Based Sliding Mode Control For Networked Control Systems

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Abstract: This paper is devoted to the problem of designing a sparse distributed output feedback discrete-time sliding mode control (ODSMC) for the networked systems. A distributed structure is employed in the discrete-time sliding mode control framework by exploiting other sub-systems' information to improve the performance of each local controller/observer so that it can widen the applicability region of the given scheme. As the first step, a stability condition is derived for the overall closed-loop system obtained from applying ODSMC to the underlying interconnected system, by assuming a given structure for the control/observer network. In the second step, we explore a methodology to obtain a sparse control/observer network structure with the least possible number of communication links that satisfies the stability condition given in the first step. The boundedness of the obtained overall closed-loop system is analyzed and a bound is derived for the augmented system state which includes the closed-loop system state and the switching function.

Keywords: Networked control systems, discrete-time sliding mode control, disturbance observer, sparse systems, linear matrix inequality.

1. INTRODUCTION

Utilizing a centralized control scheme in networked control systems (NCSs), which requires the central controller to have access to the states of all subsystems' plants, is not practical as it needs a larger and more costly control network. Alternatively, decentralized or distributed control architectures have been proposed and used in the literature (Zhang et al., 2001). The general idea behind the decentralized control scheme is to use only the local state information in order to control the subsystems and thus there is no control network. This can be effective only when the interconnections between the subsystems are not strong. When the interconnections are strong, utilizing the distributed control frameworks has been considered to ensure stability of the overall system (Razeghi-Jahromi and Seyedi, 2015).

An outstanding research implemented on the sliding mode control (SMC) has been decentralized SMC for the large-scale interconnected systems; see Yan et al. (2004, 2006); Qureshi and Abido (2014); Mahmoud and Qureshi (2012) and the references therein. The distributed SMC has received less attention and hence it requires more investigations. Note that recently several work in the literature focus on the design of sparsely distributed controller/observer networks; see e.g. Razeghi-Jahromi and Seyedi (2015); Lin et al. (2011). The objective of this paper is to extend this idea to the field of Discrete-time SMC (DSMC) problems. This paper firstly explores the problem of designing a sparse DSMC network with an arbitrary but fixed topology for a given networked system. A methodology is provided to stabilize the underlying dynamics utilizing a (sparsely) distributed controller/observer network. We will show that the proposed observer-based DSMC has this ability to cover all the cases such as fully decentralized, fully distributed, and sparsely distributed topologies. Exploiting a sparse structure for the control network which is a subset of dynamics structure is crucial in the control system for large scale systems, for instance, the smart grids (Amin, 2011). The next arising problem is how to find a sparse control network structure that satisfies the control objective. One may resort to find the sparsest control/observer structure that can stabilize the interconnected system. This issue has been investigated in e.g. Lin et al. (2011) in order to find the suboptimal controllers that minimize a special objective function considering the sparsity of the feedback gain. However, this may result in a non-convex condition. Also, Razeghi-Jahromi and Seyedi (2015) have considered the problem of finding the sparsest control/observer network that satisfies a set of stability conditions, obtained through a Lyapunov direct scheme. In this paper, as the second step, we will search for a sparse control/observer network structure with the least possible number of links satisfying the given stability condition. To this end, a heuristic iterative algorithm will be proposed, distinguishing itself from a trial-and-error process which requires to check all the possible structures.

Disturbance observer-based control strategies have been exploited in different fields in the literature; see e.g. Li et al. (2014). The idea of using disturbance estimator in the DSMC has been developed in Su et al. (2000) in order to reduce the ultimate bound on the discrete-time system state. However, the disturbance estimator in Su et al. (2000) has been designed for the cases that the system states are entirely available and the system does not involve unmatched uncertainties. The method in Chang (2006) exploits only output information for discrete-time MIMO systems involving unmatched exogenous disturbances but without unmatched uncertainties, by using the so-called *proportional integral observer*. The distributed output feedback DSMC (ODSMC), presented in this paper, utilizes a disturbance observer in order to deal with the influences of the exogenous disturbances on the boundary layer thickness. This

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sparsely distributed ODSMC is designed by means of an LMI scheme.

Notation: $[\Sigma_{ij}]_{r\times r}$ is a block matrix with block entries Σ_{ij}, $i = 1,\cdots,r$, $j = 1,\cdots,r$. $\mathrm{diag}[\Sigma_{ii}]_{i=1}^{r}$ is a block-diagonal matrix with block entries Σ_{ii}, $i = 1,\cdots,r$. Moreover, $\mathrm{col}(\nu_i(k))_{i=1}^{r}$ denotes a block-vector with block entries $\nu_i(k)$, $i = 1,\cdots,r$. $\{\circ\}$ denotes an operator for $\Xi = [\xi_{ij}]_{h\times h}$ in which $\xi_{ij} \in \mathbb{R}$ and $W = [W_{ij}]_{h\times h}$ in which $W_{ij} \in \mathbb{R}^{r_i\times s_j}$ such that $\Xi \circ W = [\xi_{ij}W_{ij}]_{h\times h}$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a large scale networked system consisting of h sub-systems:

$$\begin{cases} x_i(k+1) = [A_{ii} + \Delta A_{ii}(k)]x_i(k) + B_i[u_i(k) + f_i(k)] \\ \qquad\qquad + \sum\limits_{j=1,\ j\neq i}^{h} [A_{ij} + \Delta A_{ij}(k)]x_j(k), \\ y_i(k) = C_i x_i(k), \qquad\qquad i = 1,\cdots,h, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $y_i \in \mathbb{R}^{p_i}$ and $u_i \in \mathbb{R}^{m_i}$ are the state vector, output vector and control input vector of the i-th sub-system, respectively. Without loss of generality, it is assumed that $m_i \le p_i \le n_i$, $\mathrm{rank}(B_i) = m_i$, $\mathrm{rank}(C_i) = p_i$. The term $\Delta A_{ii}(k)$ denotes the uncertainty of i-th sub-system and $\sum_{j=1, j\neq i}^{h} A_{ij}x_j(k)$, $\sum_{j=1, j\neq i}^{h} \Delta A_{ij}(k)x_j(k)$ are, respectively, a known interconnection and an uncertain interconnection of the i-th sub-system. We also assume

$$\Delta A_{ij}(k) = M_{ij}R_{ij}(k)N_{ij}, \tag{2}$$

where M_{ij} and N_{ij} are known matrices and $R_{ij}(k)$ is an unknown time varying matrix satisfying $R_{ij}^T(k)R_{ij}(k) \le I$. $f_i(k)$ is the matched external disturbances of the i-th sub-system with known bound.

Notice that the most of the literature of SMC that considers the systems involving perturbations satisfying matching condition. In order to make the problem closer to practical cases and improve the generality of the controller synthesis problem, we consider mismatched uncertainties, i.e., ΔA_{ii} and ΔA_{ij} as shown in the system (1).

Define

$$\begin{aligned} &x(k) := \mathrm{col}(x_i(k))_{i=1}^{h},\ u(k) := \mathrm{col}(u_i(k))_{i=1}^{h}, \\ &y(k) := \mathrm{col}(y_i(k))_{i=1}^{h},\ f(k) := \mathrm{col}(f_i(k))_{i=1}^{h}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} &A := [A_{ij}]_{h\times h},\ \Delta A(k) := [\Delta A_{ij}(k)]_{h\times h}, \\ &B := \mathrm{diag}[B_i]_{i=1}^{h},\ C := \mathrm{diag}[C_i]_{i=1}^{h}. \end{aligned} \tag{4}$$

Also, the uncertainty matrix for overall system can be rearranged as:

$$\Delta A(k) = \sum_{i=1}^{h} M_i R_i(k) N_i, \tag{5}$$

in which

$$\begin{aligned} M_i =& [M_{rj}^i]_{h\times h},\ M_{rj}^i = \begin{cases} M_{ij} & \text{if } r = i, \\ 0 & \text{otherwise}, \end{cases} \\ R_i(k) =& \mathrm{diag}(R_{i1}(k),\cdots,R_{ih}(k)), \\ N_i =& \mathrm{diag}(N_{i1},\cdots,N_{ih}). \end{aligned}$$

Using (1), (3) and (4), the system in (1) at the network level can be written as:

$$\begin{cases} x(k+1) = [A + \Delta A(k)]x(k) + B[u(k) + f(k)] \\ y(k) = Cx(k). \end{cases} \tag{6}$$

Note that the proposed method is not restrictive to this ideal system, but can be readily extended to e.g. the system with time-delay and package losses; see Argha et al. (2015) for our relevant work in the same DSMC framework.

2.1 Preliminaries

Definition 1. A matrix is said to be *structure* matrix if its elements are either 0 or 1. The structure matrix of a block matrix $Y = [Y_{ij}]_{h\times h}$ with $Y_{ij} \in \mathbb{R}^{r_i\times s_j}$ is $\mathsf{S}(Y) \triangleq [s_{ij}]_{h\times h}$ with

$$s_{ij} = \begin{cases} 0 & \text{if } Y_{ij} = 0,\ i \neq j \\ 1 & \text{otherwise.} \end{cases}$$

Definition 2. Two matrices Y_1 and Y_2 are said to have the same structure if $\mathsf{S}(Y_1) = \mathsf{S}(Y_2)$.

Definition 3. The matrix Y_1 with $\mathsf{S}(Y_1) \triangleq [s_{ij}^1]_{h\times h}$ is said to be structurally subset of Y_2 with $\mathsf{S}(Y_2) \triangleq [s_{ij}^2]_{h\times h}$ while $s_{ij}^2 - s_{ij}^1 \ge 0$. We denote this as $\mathsf{S}(Y_1) \subseteq \mathsf{S}(Y_2)$.

Lemma 1. Consider

$$0 < W = \begin{bmatrix} \mathrm{diag}[\bar{W}_i]_{i=1}^{h} & \mathrm{diag}[\hat{W}_i]_{i=1}^{h} \\ \mathrm{diag}[\hat{W}_i]_{i=1}^{h\,T} & \mathrm{diag}[\tilde{W}_i]_{i=1}^{h} \end{bmatrix} \in \mathbb{R}^{(m+n)\times(m+n)},$$

with $\bar{W}_i \in \mathbb{R}^{n_i\times n_i}$, $\tilde{W}_i \in \mathbb{R}^{m_i\times m_i}$, $\hat{W}_i \in \mathbb{R}^{n_i\times m_i}$. We have

$$\mathsf{S}(W) = \mathsf{S}(W^{-1}).$$

Proof. This lemma can easily be proved by applying the block matrix inverse formula.

Lemma 2. Consider the matrix $0 < W \in \mathbb{R}^{(m+n)\times(m+n)}$ given in Lemma 1 and let Γ be a known structure matrix. For any $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$, where $Y_1 \in \mathbb{R}^{n\times p}$ and $Y_2 \in \mathbb{R}^{m\times p}$, while $\begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = W^{-1}Y$ and $\mathsf{S}(Y_1) \subseteq \Gamma$, $\mathsf{S}(Y_2) \subseteq \Gamma$, we have

$$\mathsf{S}(J_1) \subseteq \Gamma,$$
$$\mathsf{S}(J_2) \subseteq \Gamma.$$

Proof. Due to the simplicity, we omit the proof here.

Although the proofs of Lemmas 1, 2 are straightforward, these lemmas can be applied to relax the structure limitation from block diagonal to $\mathsf{S}(W)$, which tolerates the off-diagonal blocks given that they are also in block diagonal forms. To the best knowledge of the authors, these lemmas are new in literature.

Definition 4. The overall system (6) is said to be structurally controllable with respect to the structure matrix $\Gamma = [\gamma_{ij}]_{h\times h}$ if there exists $K = [K_{ij}]_{h\times h}$ with $\mathsf{S}(K) \subseteq \Gamma$ such that the modes of $A - BK$ are arbitrarily assignable.

Definition 5. The overall system (6) is said to be structurally observable with respect to the structure matrix $\Gamma = [\gamma_{ij}]_{h\times h}$ if there exists $L = [L_{ij}]_{h\times h}$ with $\mathsf{S}(L) \subseteq \Gamma$ such that the modes of $\Gamma \circ A - LC$ are arbitrarily assignable, where $\Gamma \circ A = [\gamma_{ij}A_{ij}]_{h\times h}$.

Assumption 1. The matrix triple (A, B, C) in (6) is structurally controllable and observable with respect to the given structure matrix $\Gamma = [\gamma_{ij}]_{h\times h}$.

Assumption 2. (Chang, 2006). The matrices A, B and C in the system (6) and the structure matrix Γ satisfy

$$\mathrm{rank}\left(\begin{bmatrix} \Gamma \circ A - I_n & B \\ C & 0 \end{bmatrix}\right) = n + m.$$

Notice that the above assumption is equivalent to not having transmission zero at 1.

Assumption 3. The exogenous disturbance $f_i(k)$ in (1) satisfies the Lipschitz continuity condition:

$$\left\|\tilde{f}_i(k)\right\| \le \tilde{L}_i T_s, \quad \forall k \ge 0, \tag{7}$$

where $\tilde{f}_i(k) = f_i(k) - f_i(k-1)$, $\tilde{L}_i > 0$ denotes the Lipschitz constant and T_s is the sampling time.

Here, it is supposed that $\tilde{L}_i$ is small. To this end, the sampling rate of the discrete signal processing system is assumed to be large enough compared to the maximum frequency component of the exogenous disturbance $f_i(k)$.

2.2 State and disturbance observer

In this note, we use the following estimation scheme to provide the i-th local controller by the system state information and disturbance estimate,

$$\begin{cases} \hat{x}_i(k+1) = A_{ii}\hat{x}_i(k) + B_i u_i(k) + \sum_{j=1,\ j\neq i}^{j=h} \gamma_{ij} A_{ij}\hat{x}_j(k) \\ \qquad + \sum_{j=1}^{j=h} \gamma_{ij} L_{ij}[y_j(k) - \hat{y}_j(k)] + B_i \hat{f}_i(k) \\ \hat{f}_i(k+1) = \hat{f}_i(k) + \sum_{j=1}^{j=h} \gamma_{ij} D_{ij}[y_j(k) - \hat{y}_j(k)] \\ \hat{y}_i(k) = C_i \hat{x}_i(k), \end{cases} \tag{8}$$

where $\hat{x}_i(k) \in \mathbb{R}^{n_i}$ is the state estimate of the i-th sub-system in (1), $\hat{y}_i(k) \in \mathbb{R}^{p_i}$ is the observer output, $L_{ij} \in \mathbb{R}^{n_i \times p_j}$ and $D_{ij} \in \mathbb{R}^{m_i \times p_j}$ are the local observer gains for the state and disturbance respectively. Here γ_{ij} denotes the availability of communication links among subsystems in the controller and observer design, that is, $\gamma_{ij} = 1$ if ij-th link exists in the control/observer network and $\gamma_{ij} = 0$ otherwise. Then the overall estimator is

$$\begin{cases} \hat{x}(k+1) = \Gamma \circ A\hat{x}(k) + Bu(k) + L_s[y(k) - \hat{y}(k)] + B\hat{f}(k) \\ \hat{f}(k+1) = \hat{f}(k) + D_s[y(k) - \hat{y}(k)] \\ \hat{y}(k) = C\hat{x}(k), \end{cases} \tag{9}$$

where $L_s := \Gamma \circ L$ with $L = [L_{ij}]_{h\times h}$, $D_s := \Gamma \circ D$ with $D = [D_{ij}]_{h\times h}$ and $\Gamma = [\gamma_{ij}]_{h\times h}$.

3. SPATIALLY DECENTRALIZED SLIDING MODE CONTROL

Consider the following linear sliding function

$$\sigma(k) = Sx(k), \tag{10}$$

where $\sigma(k) := \mathrm{col}(\sigma_i(k))_{i=1}^h$ and the block diagonal matrix $S := \mathrm{diag}[S_i]_{i=1}^h$ will be designed later such that SB is nonsingular. During the ideal sliding motion the sliding function satisfies:

$$\sigma(k) = 0,\ \forall k > k_s, \tag{11}$$

where $k_s > 0$ denotes the time that sliding motion starts. One may obtain from (6) and (10) that

$$\sigma(k+1) = S[A + \Delta A]x(k) + SB[u(k) + f(k)]. \tag{12}$$

Remark 1. Notice that since this sliding function will not be used in the variable structure discontinuous component of the ODSMC, the sliding surface is not required to be designed by utilizing known information of the system. Instead, it is only required to be ensured that the system state trajectories can be steered into a boundary layer around the sliding surface and be kept there thereafter. This is the key feature of the ODSMC presented in this note for NCSs. This can also lead to a considerable extension to the applicable region of the framework given in this paper compared to the existing literature for the continuous-time counterpart. The same manner can be seen in Lai et al. (2006) for the static ODSMC.

The controller is assumed to have the following structure:

$$u_i(k) = -(S_iB_i)^{-1}[(S_iA_{ii} - \Phi_i S_i)\hat{x}_i(k) + S_i \sum_{j=1,\ j\neq i}^{j=h} \gamma_{ij} A_{ij}\hat{x}_j(k)] - \hat{f}_i(k), \tag{13}$$

where $\Phi_i \in \mathbb{R}^{m_i\times m_i}$ is a stable matrix and aims to govern the state convergence rate. Here, similar to Edwards (2004), it is assumed that $\Phi_i = \lambda_i I_{m_i}$, where $0 \le \lambda_i < 1$ is a given constant value. The control law $u_i(k)$ in (13) can be written as

$$u_i(k) = -(S_iB_i)^{-1}S_i[A_{ii}^{\lambda_i}\hat{x}_i(k) + \sum_{j=1,\ j\neq i}^{j=h} \gamma_{ij}A_{ij}\hat{x}_j(k)] - \hat{f}_i(k), \tag{14}$$

where $A_{ii}^{\lambda_i} = A_{ii} - \lambda_i I_{n_i}$. Then the compact control law is

$$u(k) = -(SB)^{-1}S(\Gamma \circ A_\lambda)\hat{x}(k) - \hat{f}(k), \tag{15}$$

where $A_\lambda = A - \mathrm{diag}[\lambda_i I_{n_i}]_{i=1}^h$. It is worth mentioning that, referring to e.g. Lai et al. (2006); Su et al. (2000); Chang (2006), the DSMC does not necessarily require switching component and the linear part in (15) leads to a boundary layer with thickness $O(T_s)$. By proper consideration of the sampling phenomenon in the discrete-time sliding mode control design, the boundary layer thickness can be reduced to $O(T_s^2)$ (Chang, 2006). Moreover, the controller in (15) is indeed based on the equivalent control, by removing unknown uncertainty terms and taking into account the structure constraint. The removed terms can be taken care of by robust control techniques.

Remark 2. With different structure matrix Γ, the above controller and the observer in (9) can explain various topologies. The decentralized control strategy can be obtained by $\gamma_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{otherwise} \end{cases}$. In this case, there is no control network in the system. When $\Gamma = \mathsf{S}(A)$, which means that we may have a fully distributed control system, where each subsystem uses its own state as well as the states of all other physically coupled subsystems. As the third alternative, the structure matrix Γ can generate a middle-of-the-road solution, between fully distributed control approaches and decentralized ones, regarded as sparsely distributed control systems.

Define the overall state estimation error as

$$e(k) := x(k) - \hat{x}(k), \tag{16}$$

and disturbance estimation error as

$$e_f(k) := f(k) - \hat{f}(k). \tag{17}$$

The overall closed-loop system is obtained by applying the controller (15) to (6) and using (16), (17) and (9), as

$$\begin{cases} x(k+1) = (A + \Delta A - \hat{A})x(k) + B(SB)^{-1}S\begin{bmatrix}\Gamma\circ A_\lambda & B\end{bmatrix}e_t(k) \\ e_t(k+1) = \begin{bmatrix}A+\Delta A - \Gamma\circ A \\ 0\end{bmatrix}x(k) + (A_t - L_tC_t)e_t(k) + \bar{f}(k+1), \end{cases} \tag{18}$$

where $\hat{A} = B(SB)^{-1}S(\Gamma \circ A_\lambda)$, $\bar{f}(k+1) = \begin{bmatrix}0 \\ \tilde{f}(k+1)\end{bmatrix}$ with $\tilde{f}(k) = \mathrm{col}(\tilde{f}_i(k))_{i=1}^h$, $e_t(k) = \begin{bmatrix}e(k) \\ e_f(k)\end{bmatrix}$, $A_t = \begin{bmatrix}\Gamma\circ A & B \\ 0 & I_m\end{bmatrix}$ with $m = \sum_{i=1}^h m_i$, $L_t = \begin{bmatrix}L_s \\ D_s\end{bmatrix}$ and $C_t = \begin{bmatrix}C & 0\end{bmatrix}$.

Lemma 3. (Chang, 2006). If the matrix pair $(\Gamma \circ A,\ C)$ is observable and $(\Gamma \circ A,\ B,\ C)$ satisfies the rank condition in Assumption 2, then the matrix pair (A_t, C_t) is observable.

Also, it can simply be found that

$$\sigma(k+1) = S(\Delta A + A - \Gamma \circ A_\lambda)x(k) + S\begin{bmatrix}\Gamma\circ A_\lambda & B\end{bmatrix}e_t(k). \tag{19}$$

4. STABILITY ANALYSIS

In the case of applying DSMC to the system involving exogenous disturbances, it can only ensure the state trajectories to be driven into a boundary layer around the ideal sliding surface $\sigma(k) = 0$. This issue is indeed regarded as the quasi sliding mode (QSM) in the literature. The following theorem considers a method to analyze simultaneously the reachabiltiy of QSM and the stability of the system states utilizing a discrete-time Lyapunov stability method, in the absence of exogenous disturbances. The characterization of the bounds on the closed-loop system states and sliding function's boundary layer are presented separately later in Theorem 2. Furthermore, as Theorem 2 needs to derive the cross terms between the system state (sliding function) and the components $\bar{f}(k+1)$, in order to avoid unnecessary repetition of the technical manipulations, we will start the proof of Theorem 1 more generally (with the external disturbance and the component $\hat{f}(k)$ in the controller) for the sake of Theorem 2. We then let $f(k) = 0$, $\tilde{f}(k) = 0$, and thus, $\bar{f}(k+1) = 0$ to derive the LMI condition for the stability analysis, and control/observer synthesis.

Theorem 1. In the absence of disturbance $f(k)$, the linear part of the control law (15) can drive the system state onto the ideal sliding surface (10), and the system state is stabilized, if there exist matrices $P = \mathrm{diag}[P_i]_{i=1}^h$, with $0 < P_i := U_i^T \left[\begin{smallmatrix} P_{i_{11}} & 0 \\ 0 & P_{i_{22}} \end{smallmatrix}\right] U_i$, $P_i \in \mathbb{R}^{n_i \times n_i}$, $Q = \begin{bmatrix} \mathrm{diag}[\bar{Q}_i]_{i=1}^h & \mathrm{diag}[\hat{Q}_i]_{i=1}^h \\ \mathrm{diag}[\hat{Q}_i]_{i=1}^{h^T} & \mathrm{diag}[\tilde{Q}_i]_{i=1}^h \end{bmatrix} > 0$, with $\bar{Q}_i \in \mathbb{R}^{n_i \times n_i}$, $\tilde{Q}_i \in \mathbb{R}^{m_i \times m_i}$, $\hat{Q}_i \in \mathbb{R}^{n_i \times m_i}$, X_1, X_2 and $X_3 = \begin{bmatrix} \Gamma \circ X_L \\ \Gamma \circ X_D \end{bmatrix}$, with $X_L \in \mathbb{R}^{n \times p}$, $X_D \in \mathbb{R}^{m \times p}$, and scalars $\varepsilon_{ij} > 0$, $i = 1, \cdots, h$, $j = 1, \cdots, h$, and $\rho > 0$ satisfying the LMI in (20), where $0 < P_{i_{11}} \in \mathbb{R}^{m_i \times m_i}$, $0 < P_{i_{22}} \in \mathbb{R}^{(n_i - m_i) \times (n_i - m_i)}$ and $U_i \in \mathbb{R}^{n_i \times n_i}$ is defined in Lemma 5 in Argha et al. (2016a), $\check{\chi}_{11} = -P + X_2^T B^T + B X_2 + \rho I + \sum_{i=1}^h \Upsilon_i N_i^T N_i$, with $\Upsilon_i = \mathrm{diag}[\varepsilon_{ij} I_{n_j}]_{j=1}^h$, $\bar{\chi}_{22} = -Q + \rho I$, $\Gamma = [\gamma_{ij}]_{h \times h}$ is a given structure matrix and $\{\star\}$ denotes the symmetric elements in a symmetric matrix. Here $S = B^T P$ and the observer gain is

$$L_t = Q^{-1} X_3. \tag{21}$$

Proof. Define

$$V(\zeta(k)) = x^T(k) P x(k) + e_t^T(k) Q e_t(k) + \sigma^T(k)(SB)^{-1}\sigma(k), \tag{22}$$

where $\zeta(k) = \left[x^T(k) \; e_t^T(k) \; \sigma^T(k) \right]^T$, $P > 0$ and $Q > 0$ are symmetric matrices and $S = B^T P$. Note that the inclusion of both state $x(k)$ and sliding function $\sigma(k)$ in the Lyapunov candidate function makes it possible to analyze simultaneously the reachabiltiy of QSM as well as the boundedness of the system state and sliding function, as will be seen later in the proof of Theorem 2. Defining $\varpi = \left[x^T(k) \; e_t^T(k) \; \bar{f}(k+1) \right]^T$, it can be written

$$\begin{aligned} \Delta V(\zeta(k)) &\triangleq V(\zeta(k+1)) - V(\zeta(k)) \\ &= \varpi^T(k) \left[\chi_{ij} \right]_{3 \times 3} \varpi(k). \end{aligned} \tag{23}$$

where

$$\begin{aligned}
\chi_{11} =& (A+\Delta A)^T P (A+\Delta A) - (A+\Delta A)^T PB(B^T PB)^{-1} B^T P (A+\Delta A) \\
& - PB(B^T PB)^{-1} B^T P - P + 2(\Delta A + A - \Gamma \circ A_\lambda)^T PB(B^T PB)^{-1} \\
& \times B^T P (\Delta A + A - \Gamma \circ A_\lambda) \\
& + \begin{bmatrix} \Delta A + A - \Gamma \circ A_\lambda \\ 0 \end{bmatrix}^T Q \begin{bmatrix} \Delta A + A - \Gamma \circ A_\lambda \\ 0 \end{bmatrix}, \\
\chi_{12} =& 2(\Delta A + A - \Gamma \circ A_\lambda)^T PB(B^T PB)^{-1} B^T P \left[\Gamma \circ A_\lambda \;\; B \right] \\
& + \begin{bmatrix} \Delta A + A - \Gamma \circ A_\lambda \\ 0 \end{bmatrix}^T Q (A_t - L_t C_t), \\
\chi_{13} =& \begin{bmatrix} \Delta A + A - \Gamma \circ A_\lambda \\ 0 \end{bmatrix}^T Q, \\
\chi_{22} =& 2 \left[\Gamma \circ A_\lambda \;\; B \right]^T S^T (SB)^{-1} S \left[\Gamma \circ A_\lambda \;\; B \right] \\
& + (A_t - L_t C_t)^T Q (A_t - L_t C_t) - Q, \\
\chi_{23} =& (A_t - L_t C_t)^T Q, \\
\chi_{33} =& Q.
\end{aligned}$$

Now, in order to analyze the system stability let $\bar{f}(k+1) = 0$. The system will be stable if

$$\Xi := [\chi_{ij}]_{2 \times 2} < -\rho I, \tag{24}$$

where $\rho > 0$ is a scalar variable. To consider the feasibility of (24), by the Schur complement, (24) is equivalent to

$$\begin{bmatrix} \bar{\chi}_{11} & \star & \star & \star \\ 0 & \bar{\chi}_{22} & \star & \star \\ \sqrt{2} B^T P (A + \Delta A - \Gamma \circ A) & \sqrt{2} B^T P \left[\Gamma \circ A_\lambda \;\; B \right] & -B^T PB & \star \\ Q \left[\begin{smallmatrix} A + \Delta A - \Gamma \circ A \\ 0 \end{smallmatrix}\right] & Q(A_t - L_t C_t) & 0 & -Q \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned} \bar{\chi}_{11} =& (A+\Delta A)^T P (A+\Delta A) - (A+\Delta A)^T S^T (SB)^{-1} S (A+\Delta A) \\ & - S^T (SB)^{-1} S - P + \rho I, \\ \bar{\chi}_{22} =& -Q + \rho I. \end{aligned}$$

Consequently, Lemma 2.1 in Argha et al. (2016b) can be used to show that the feasibility of (25) is equivalent to that of

$$\begin{bmatrix} \hat{\chi}_{11} & \cdots \\ \vdots & \ddots \end{bmatrix} < 0, \tag{26}$$

with

$$\begin{aligned} \hat{\chi}_{11} =& (A + \Delta A + BF_1)^T P (A + \Delta A + BF_1) - P \\ & + F_2^T (B^T PB) F_2 + F_2^T B^T P + PBF_2 + \rho I, \end{aligned} \tag{27}$$

where F_1 and F_2 are auxiliary variables (Li et al., 2007) and note that except $\hat{\chi}_{11}$, other elements of (26) are the same as those in (25). Therefore, using Lemma 5 in Argha et al. (2016a), $\hat{\chi}_{11}$ in (27) can be rearranged as

$$\begin{aligned} \hat{\chi}_{11} =& [P(A + \Delta A) + BZF_1]^T P^{-1} [P(A + \Delta A) + BZF_1] - P \\ & + F_2^T Z^T B^T P^{-1} BZF_2 + F_2^T Z^T B^T + BZF_2 + \rho I, \end{aligned} \tag{28}$$

where Z satisfies $PB = BZ$. Defining $X_1 = ZF_1$, $X_2 = ZF_2$ and $X_3 = QL_t$, with the help of the Schur complement and Lemma 1 in Argha et al. (2016a), it can be seen that (26) is sufficed by the LMI in (20).

The following theorem aims to characterize the boundedness of the obtained overall closed-loop system state and corresponding sliding function in the presence of disturbance $f(k)$.

Theorem 2. In the presence of disturbance $f(k)$, if the LMI in (20) is feasible, for the obtained P, Q, $L_t = Q^{-1} X_3$ and ρ, the controller (15) satisfying (7) will lead to a bound on the augmented system state $\zeta(k) = [x^T(k), e_t(k), \sigma^T(k)]^T$ as follows:

$$\forall \varsigma > 0, \; \exists k^\star > 0, \text{ s.t. } \forall k > k^\star, \;\; \|\zeta(k)\|^2 \leq \frac{\lambda_{\max}(\mathbf{M})}{\hat{\rho} \lambda_1} \delta + \varsigma, \tag{29}$$

where $\lambda_1 = \lambda_{\min}(\mathrm{diag}(P, Q, (B^T PB)^{-1}))$, $\mathbf{M} = \mathrm{diag}(M_P, Q)$, $M_P = PB(B^T PB)^{-1} B^T P + P$, and $\delta = \|\Pi + Q\| \sum_{i=1}^h \tilde{L}_i^2 T_s^2$; here the scalar variable $\hat{\rho} > 0$ and matrix variable $\Pi > 0$ are obtained from solving the following LMI:

$$\begin{bmatrix} \check{\chi}_{11} & \star & \star & \star & \star & \star & \star & \cdots & \star \\ 0 & \bar{\chi}_{22} & \star & \star & \star & \star & \star & \cdots & \star \\ \sqrt{2}B^T P(A-\Gamma\circ A_\lambda) & \sqrt{2}B^T P\left[\begin{smallmatrix}\Gamma\circ A_\lambda & B\end{smallmatrix}\right] & -B^T PB & \star & \star & \star & \star & \cdots & \star \\ Q\left[\begin{smallmatrix}A-\Gamma\circ A \\ 0\end{smallmatrix}\right] & QA_t - X_3C_t & 0 & -Q & \star & \star & \star & \cdots & \star \\ PA+BX_1 & 0 & 0 & 0 & -P & \star & \star & \cdots & \star \\ BX_2 & 0 & 0 & 0 & 0 & -P & \star & \cdots & \star \\ 0 & 0 & \sqrt{2}M_1^T PB & \left[\begin{smallmatrix}M_1^T & 0\end{smallmatrix}\right]Q & M_1^T P & 0 & -\Upsilon_1 & \cdots & \star \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \sqrt{2}M_h^T PB & \left[\begin{smallmatrix}M_h^T & 0\end{smallmatrix}\right]Q & M_h^T P & 0 & 0 & \cdots & -\Upsilon_h \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \Omega_1 & \star & \star & \star & \cdots & \star \\ 0 & (\hat{\rho}-\rho)I & \star & \star & \cdots & \star \\ \bar{\chi}_{13}^T & \chi_{23}^T & -\Pi & \star & \cdots & \star \\ 0 & 0 & \left[\begin{smallmatrix}M_1^T & 0\end{smallmatrix}\right]Q & -\bar{\Upsilon}_1 & \cdots & \star \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \left[\begin{smallmatrix}M_h^T & 0\end{smallmatrix}\right]Q & 0 & \cdots & -\bar{\Upsilon}_h \end{bmatrix} < 0, \tag{30}$$

where $\bar{\chi}_{13} = \left[\begin{smallmatrix}A-\Gamma\circ A_\lambda \\ 0\end{smallmatrix}\right]^T Q$, $\chi_{23} = (A_t - L_tC_t)^T Q$, $\Omega_1 = (\hat{\rho} - \rho)I + \sum_{i=1}^{h} \bar{\Upsilon}_i N_i^T N_i$ and $\bar{\Upsilon}_i = \text{diag}[\bar{\varepsilon}_{ij} I_{n_j}]_{j=1}^{h}$ in which $\bar{\varepsilon}_{ij} > 0$, $i = 1, \cdots, h,\ j = 1, \cdots, h$ are scalar variables.

Proof. Defining $\nu(k) = \left[\begin{smallmatrix}x^T(k) & e_t^T(k)\end{smallmatrix}\right]^T$ and $\chi_\nu = \left[\begin{smallmatrix}\chi_{13}^T & \chi_{23}^T\end{smallmatrix}\right]^T$ and according to Lemma 4 in Niu et al. (2004) it can be written that

$$\begin{aligned} 2\nu^T(k)\chi_\nu \bar{f}(k+1) \leq & \nu^T(k)\chi_\nu \Pi^{-1}\chi_\nu^T \nu(k) \\ & + \bar{f}^T(k+1)\Pi\bar{f}(k+1), \end{aligned} \tag{31}$$

where $\Pi > 0$ is of appropriate dimension. It follows from (23), (24) and (31) that

$$\begin{aligned} \Delta V(\zeta(k)) \leq & -\nu^T(k)[\rho I - \chi_\nu \Pi^{-1}\chi_\nu^T]\nu(k) \\ & + \bar{f}^T(k+1)[\chi_{33} + \Pi]\bar{f}(k+1). \end{aligned} \tag{32}$$

If we choose $\Pi > 0$ such that

$$\hat{\rho} I < \rho I - \chi_\nu \Pi^{-1}\chi_\nu^T, \tag{33}$$

where $\rho > \hat{\rho} > 0$, which is always possible if $\rho > 0$ exists, then, it follows from (32) that

$$\Delta V(\zeta(k)) \leq -\hat{\rho}\nu^T(k)\nu(k) + \bar{f}^T(k+1)[\chi_{33} + \Pi]\bar{f}(k+1). \tag{34}$$

Moreover, note that

$$V(\zeta(k)) = \nu^T(k)\mathbf{M}\nu(k), \tag{35}$$

where $\mathbf{M} = \text{diag}(M_P, Q)$, and $M_P = PB(B^T PB)^{-1}B^T P + P$, hence,

$$\lambda_{\min}(\mathbf{M})\|\nu(k)\|^2 \leq V(\zeta(k)) \leq \lambda_{\max}(\mathbf{M})\|\nu(k)\|^2. \tag{36}$$

Additionally, it is known that

$$\lambda_1\|\zeta(k)\|^2 \leq V(\zeta(k)) \leq \lambda_2\|\zeta(k)\|^2. \tag{37}$$

where $\lambda_1 = \lambda_{\min}(\text{diag}(P, Q, (B^T PB)^{-1}))$ and $\lambda_2 = \lambda_{\max}(\text{diag}(P, Q, (B^T PB)^{-1}))$. Hence, from (34) and (36) one can derive that

$$\Delta V(\zeta(k)) \leq -\frac{\hat{\rho}}{\lambda_{\max}(\mathbf{M})}V(\zeta(k)) + \delta, \tag{38}$$

where $\delta = \|\Pi + Q\| \sum_{i=1}^{h} \tilde{L}_i^2 T_s^2$. Moreover, from (24) it can simply be written that $\forall\ \nu(k) \neq 0$

$$\begin{aligned} \nu^T(k)\Xi\nu(k) = & V(\zeta(k+1))\big|_{\bar{f}(k+1)=0} - V(\zeta(k)) \\ < & -\rho\nu^T(k)\nu(k). \end{aligned} \tag{39}$$

It is known that $V(\zeta(k+1))\big|_{\bar{f}(k+1)=0} \geq 0$, and thus, from (39) and (36), it can be claimed that $\rho < \lambda_{\max}(\mathbf{M})$. Therefore, $\frac{\hat{\rho}}{\lambda_{\max}(\mathbf{M})} < 1$. Finally, from Khalil (2002) (Theorem 5.1, Corollaries 5.1, 5.2) and (38), one can find the bound given in (29). Moreover, to find $\Pi > 0$ in (33), for given $P > 0$, $Q > 0$, $L_t = Q^{-1}X_3$ and $\rho > 0$, by using Lemma 1 in Argha et al. (2016a), we can show that (33) is sufficed by the LMI in (30).

As seen in the proposed sparse distributed ODSMC, local controllers/observers are able to utilize some interconnections in the nominal A matrix, and the remaining interconnections in A matrix together with ΔA are considered as the uncertainties of the overall system. The second step of this paper will consider the issue of minimizing the costs of a control/observer network utilized for the stabilizing distributed ODSMC. This will be the subject of the next section.

Remark 3. It is easy to realize from Lemma 1 and 2 that $\mathsf{S}(Q^{-1}) = \mathsf{S}(Q)$, and thus $\mathsf{S}(L_s) \subseteq \Gamma$, $\mathsf{S}(D_s) \subseteq \Gamma$.

5. SPARSIFYING THE CONTROL NETWORK STRUCTURE

Previous sections have studied the problem of designing ODSMC for NCSs with imposing *a priori* constraints on communication requirements between sub-systems. This section aims to design a control network with a minimum number of links that satisfies the stability condition (20). We formulate this problem as

$$\begin{aligned} \min_{P,Q,X_1,X_2,X_3,\rho,\Upsilon_1,\cdots,\Upsilon_h} \quad & \mathbf{card}(\Gamma) \\ \text{subject to (20) and } & \Gamma \subseteq \mathsf{S}(A), \end{aligned} \tag{40}$$

where $\Gamma = [\gamma_{ij}]_{h\times h}$ and $\mathbf{card}(\cdot)$ denotes the cardinality function (the number of nonzero elements of a matrix). The above optimization problem is a convex mixed-binary problem which is broadly speaking NP-hard. A number of exact schemes for addressing the convex mixed-binary programs are considered (Grossmann, 2002). However, exploiting these schemes are computationally expensive for large networks and in the worst case it may require solving 2^{N} convex problems, where N denotes the number of physical interconnections in the plant network. Instead, in this note, we will consider a heuristic sub-optimal scheme to deal with this problem.

The method to address the minimization problem (40) is encapsulated in the algorithm below. Note that in this algorithm, we relax the constraint on $\gamma_{ij}, i \neq j$ from binary variables to the constraint $0 \leq \gamma_{ij} \leq 1$.

Algorithm 1.

1) Set $\Gamma = I$. If the LMI (20) is feasible, $\{\gamma_{ij}^\star\} \leftarrow \{\gamma_{ij}\}$, no control network is required and the sparsest structure is the decentralized structure. Terminate the search and go to Step 6.
2) Initialize $\Gamma = \mathsf{S}(A)$ and $l = 1$, in which l denotes the iteration number.

3) Solve the LMI (20). If it is feasible, $\{\gamma_{ij}^{\star}\} \leftarrow \{\gamma_{ij}\}$. Otherwise, if $l = 1$ terminate the search and the problem has no solution, or else go to Step 6.
4) With known $P, Q, X_1, X_2, X_3, \rho, \Upsilon_1, \cdots, \Upsilon_h$ and replacing those entries $\gamma_{ij} = 1,\ i \neq j$ with the relaxed constraint $0 \leq \gamma_{ij} \leq 1$, minimize $\sum_{i,j=1,\ i\neq j}^{h} \gamma_{ij}$ subject to the LMI (20) and $0 \leq \gamma_{ij} \leq 1$ to find the relaxed γ_{ij}^{r}. Sort the set $\{\gamma_{ij}^{r}\}$ in ascending order.
5) Set γ_{ij} corresponding to the first entry of $\{\gamma_{ij}^{r}\}$ to zero and $l = l+1$. If $l < \mathbf{Card}(\mathsf{S}(A)) - n$ return to Step 3, otherwise go to Step 6.
6) Return $\gamma_{ij}^{\star}$.

In the above algorithm, Step 4 characterizes the contribution of each link in the stability of the overall system. Moreover, as seen, the algorithm searches for the sparsest structure using the sorted set $\{\gamma_{ij}^{r}\}$. In the worst case, in order to find the solution, $2\left(\mathbf{Card}(\mathsf{S}(A)) - n\right)$ convex problems may be addressed. Finally, it should be stressed that this alternate scheme is only a sub-optimal method to deal with the sparsification problem considered in this section. Broadly speaking, to obtain the optimal solution, one should solve the original mixed-binary convex problem in (44), which is NP-hard.

6. NUMERICAL EXAMPLES

Consider an interconnected system consisting of three inverted pendulums that are mounted on coupled carts (Razeghi-Jahromi and Seyedi, 2015; Ogata, 1997). The linearized equations of motions are also given in Razeghi-Jahromi and Seyedi (2015). Define $x_i = [x_{i,1}, x_{i,2}, x_{i,3}, x_{i,4}]^T = [\theta_i, \dot{\theta}_i, \mathbf{x}_i, \dot{\mathbf{x}}_i]^T$,

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M_i + m}{M_i \ell} g & 0 & \frac{k_i}{M_i \ell} & \frac{c_i + b_i}{M_i \ell} \\ 0 & 0 & 0 & 1 \\ \frac{-m}{M_i} g & 0 & \frac{-k_i}{M_i} & \frac{-c_i - b_i}{M_i \ell} \end{bmatrix},\ A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-k_{ij}}{M_i \ell} & \frac{-b_{ij}}{M_i \ell} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_{ij}}{M_i} & \frac{b_{ij}}{M_i} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & \frac{-1}{M_i \ell} & 0 & \frac{1}{M_i} \end{bmatrix}^T,\ C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

for $(i,j) \in \mathbf{E} \triangleq \{(1,2),(2,1),(2,3),(3,2)\}$. Here $k_i = \sum_{j \in \mathsf{J}_i} k_{ij}$ and $b_i = \sum_{j \in \mathsf{J}_i} b_{ij}$, where $\mathsf{J}_i := \{j \mid (i,j) \in \mathbf{E}\}$. Besides, c_i, $b_{ij} = b_{ji}$, $k_{ij} = k_{ji}$ and ℓ are the friction, damper, spring coefficients and pendulum length respectively. It is also assumed that the moment of inertia of the pendulums are zero. The numerical system parameters are assumed as $M_1 = 2,\ M_2 = 1,\ M_3 = 3,\ m = 0.5,\ g = 10,\ \ell = 0.5,\ k_{12} = k_{21} = 5,\ k_{23} = k_{32} = 15,\ b_{12} = b_{21} = 1,\ b_{23} = b_{32} = 5,\ c_1 = 4,\ c_2 = 2$ and $c_3 = 1$. A discretized representation based on a sample interval of 0.005 s is obtained. We set $\lambda_i = 0.7$. In order to check the robustness properties of the controller, the following uncertainty parameters are considered: $M_{ij} = 0.01 \times \mathbf{1}_{4\times 1},\ N_{ij} = -0.01 \times \mathbf{1}_{1\times 4},\ i,j = 1,2,3$. Algorithm 1 is solved and then it is found that the most sparse structure that can satisfy the rank condition in Assumption 2 and, more importantly, the stability condition in the LMI (20) is the decentralized structure. For comparison, we then exploit an exhaustive search on the binary variables, followed by convex optimization of other variables, and the obtained structure is also the decentralized one. We can see that the proposed sub-optimal algorithm leads to the same result as the optimal solution.

7. CONCLUSIONS

This paper firstly, with assuming a priori known structure for the control/observer network, proposes a DSMC by utilizing only control system sensors' signals for the networked systems. A unified framework is derived for the observer-based controller design, with the aid of an LMI scheme. Furthermore, our sparse ODSMC reduces the conservatism of the existing methods in the literature for the LMI based DSMC. Then, this paper explores the solution to the problem of finding the sparsest control/observer network structure that satisfies the LMI stability condition obtained in the first part.

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