Optimal Design of a Linear Induction Motor Applied in Transportation

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Abstract- Several optimal design schemes of a single-sided linear induction motor (SLIM) adopted in linear metro are presented in this paper. Firstly, the equivalent circuit of SLIM fully considering the end effects, half-filled slots, back-iron saturation and skin effect is proposed, based on one-dimensional air-gap magnetic equations. In the circuit, several coefficients including longitudinal end effect coefficients \( K_x(s) \) and \( K_r(s) \), transversal edge effect coefficients \( C_x(s) \) and \( C_r(s) \), and skin effect coefficient \( K_u \) are achieved by using the dummy electric potential method and complex power equivalence between primary and secondary sides. Furthermore, several optimal design restraint equations of SLIM are provided in order to improve the operational efficiency and reduce the primary weight. These nonlinear equations are solved by using genetic algorithm and mixture penalty function method. The optimal schemes are compared with the original design of one company, where analysis on parameters is made in detail. These results show that the optimal schemes are reasonable for improving the performance of SLIM.

I. INTRODUCTION

In the recent years, the single-sided linear induction motor (SLIM) has been widely adopted in transportation systems, especially in intermediate speed range. Typical examples include the HSST in Japan and the linear metro in Canada, where power systems are both propelled by the SLIMs [1-3].

The SLIM has the following merits comparing with the rotary induction motor (RIM): greater ability to exert thrust on the secondary without mechanical contacts, higher acceleration or deceleration, less wear of the wheels, smaller turn circle radius, and more flexible road line. Because of its cut-open magnetic circuit, the linear induction motor (LIM) possesses the inherent characteristics such as longitudinal end-effect, transversal edge-effect and normal force. In addition, it also has half-filled slots in the primary ends. Therefore, an accurate equivalent circuit model of LIM is difficult to be obtained compared to that of RIM [4].

Many analysis techniques of SLIM have been studied and developed in the past years. However, effective methods on the design scheme of SLIM have not been obtained due to the following reasons. The selection of electric loading and magnetic loading by loading distribution is so difficult that one cannot calculate the apparent power (kVA) easily. The power factor and efficiency are affected by the end effect which is again affected by the design technique of SLIM.

The pole pitch can be varied structurally and the selection of frequency and rated slip is not easy with the decision on the rated velocity [5].

A new improved model is set out in this paper based on the air-gap flux density equation, which fully takes into account the end effects, half-filled slots, back-iron saturation and skin effect. Then, optimal design equations for achieving the maximum efficiency and minimum primary weight are built, which are nonlinearly constrained. It is necessary to choose a suitable solving method to obtain the independent variables, such as the rated slip, number of poles, pole pitch, stack height, ratio of slot width to slot pitch, secondary aluminum (copper?) thickness, tooth magnetic density, and secondary overhang [4-7].

II. PHYSICAL STRUCTURE AND MATHEMATICAL ANALYTIC MODEL

The structure of an SLIM and its equivalent circuit are shown separately in Fig. 1 and Fig. 2.

In the analytic circuit, \( K_f \) is the skin effect correction coefficient, \( K_r(s) \) and \( K_x(s) \) are the longitudinal end-effect coefficients, and \( C_x(s) \) and \( C_r(s) \) are the transversal end-effect coefficients [8]. These correction coefficients are expressed below.

The skin effect correction coefficient is

\[ K_f = \frac{1 + B_1^2 s h^2 (2 K \delta_x^e)}{A_1 [1 + B_1^2 s h^2 (2 K d)]} \]

where \( K \) is the function of slip and motor structure, \( d \) is the thickness of secondary metal sheet, \( \delta_x^e \) is the equivalent magnetic air gap length, and \( A_1 \) and \( B_1 \) are coefficients which are affected by slip and motor structure.

\( \delta_x^e \) can be expressed as

\[ \delta_x^e = K_c K_s g \]

where \( K_c \) is the Carter coefficient and \( K_s \) is the magnetic saturation coefficient.
Fig. 1. Simple construction diagram of LIM

(a) Longitudinal side view
(b) Transversal side view

Fig. 2. T-type equivalent circuit of LIM considering the back iron resistance

\[ A_1 \text{ and } B_1 \text{ are derived as} \]

\[
A_1 = ch^2(K_s) + \frac{K_p \rho_{\text{second}}}{S \mu_0 d} \]

\[
B_1 = \frac{S \mu_0 d}{2K_p \rho_{\text{second}}} [1 + \frac{K_p \rho_{\text{second}}}{S \mu_0 d}]^2
\]

where \( S \) is the slip, \( \omega \) is the primary electrical angular velocity, \( \rho_{\text{second}} \) is the resistivity of secondary electrical sheet, and \( \mu_0 \) is the permeability of air.

The longitudinal end-effect coefficients \( K_r(s) \) and \( K_(s) \) are denoted by

\[
K_r(s) = \frac{SG}{2P_s r \sqrt{1 + (SG)^2}} \frac{C_1^2 + C_2^2}{C_1}
\]

\[
K_s(s) = \frac{1}{2P_s r \sqrt{1 + (SG)^2}} \frac{C_1^2 + C_2^2}{C_2}
\]

where \( C_1 \) and \( C_2 \) are the functions of slip \( S \) and goodness factor \( G \), \( P_s \) is the number of equivalent pole pairs, and \( r \) is the pole pitch.

The transversal end-effect coefficients \( C_r(s) \) and \( C_s(s) \) are given by

\[
C_r(s) = \frac{SG[R_r^2[T] + I_m^2[T]]}{R_r[T]} \]

\[
C_s(s) = \frac{R_s^2[T] + I_m^2[T]}{I_m[T]}
\]

where \( T \) is the function of slip, goodness factor and motor structure, and \( R_r \) and \( I_m \) are the real part and image part of complex \( T \), respectively.

\[ T = f \left[ y^2 + (1 - y^2) \frac{\lambda}{aa} \right] th(\alpha \alpha) \]

where \( a \) is half the primary lamination width, \( \alpha \) is the ratio of \( c \) to \( \tau \), and \( c \) is half the width of the secondary sheet overhang. \( \gamma \) and \( \lambda \) can be obtained by

\[
\lambda = \frac{1}{1 + \frac{1}{y} th(\alpha \alpha) th(K(c - a))}
\]

\[
\gamma^2 = \frac{1}{1 + jSG}
\]

The five parameters in the \( T \)-circuit: primary resistance \( r_1 \), primary leakage reactance \( x_1 \), secondary resistance \( r_2 \), secondary leakage reactance \( x_2 \) and exciting reactance \( x_m \), are derived below.

The primary resistance \( r_1 \) is

\[
r_1 = \rho_{\text{cu}} \times 2l_{av} W_1 / S_{\text{cu}}
\]

where \( \rho_{\text{cu}} \) is the resistivity of copper, \( l_{av} \) is half the average length of the primary winding loop, \( W_1 \) is the number of turns of the primary winding in series, and \( S_{\text{cu}} \) is the cross sectional area of the primary winding conductor.

The primary leakage reactance \( x_1 \) is

\[
x_1 = 0.158 f W_1^2 a_1 \left[ \frac{\lambda_s + \lambda_e + \lambda_c}{q} + \frac{\lambda_s}{P_e} \right]
\]

where \( f \) is the primary frequency, \( a_1 \) is the width of primary lamination, \( P \) is the number of actual pole pairs, \( \lambda_s \) is the primary slot leakage magnetic conductance, \( \lambda_e \) is primary teeth leakage magnetic conductance, \( \lambda_c \) is primary winding end leakage magnetic conductance, \( \lambda_d \) is primary harmonic leakage magnetic conductance.

The secondary resistance is composed of those of conducting sheet and back iron because the flux can penetrate through the aluminum or copper sheet \([8]\), and then enter the back iron.

The depth of flux density into back iron, \( d_{Fe} \), is

\[
\Delta d_{Fe} = \frac{2 \rho_{Fe}}{\sqrt{\omega \mu_{Fe}}}
\]

where \( \rho_{Fe} \) is back iron resistivity, \( \mu_{Fe} \) is the permeability of back iron.

The resistance of secondary conducting sheet \( r_{2\text{sheet}} \) is
\[ r_{2\text{sheet}} = 4m_i \rho_{\text{second}} \left( W_l K_{W_l} \right)^2 \frac{a_i}{2P_e} - \Delta d_{l_s} \tau \]  

where \( m_i \) is the number of primary winding phases, \( \rho_{\text{second}} \) is the resistivity of the secondary electric sheet, and \( K_{W_l} \) is primary winding coefficient.

The resistance of secondary back iron \( r_{2Fe} \) is

\[ r_{2Fe} = 4m_i \rho_{Fe} \left( W_l K_{W_l} \right)^2 \frac{a_i}{2P_e} - \Delta d_{l_s} \tau \]  

Therefore, the secondary equivalent resistance \( r_2 \) is

\[ r_2 = \frac{r_{2\text{sheet}} r_{2Fe}}{r_{2\text{sheet}} + r_{2Fe}} \]  

The secondary leakage reactance is

\[ X_2 = K_s \frac{r_2}{S} B_s h \sin(2Kd) \]  

The exciting reaction is

\[ X_m = 4m_i \mu_0 \left( W_l K_{W_l} \right)^2 \frac{V_s}{2\pi\delta_e P_e} \]  

where \( V_s \) is the synchronous velocity of primary side.

The characteristics of SLIM are fully considered in the above circuit. When \( K_{s(s)}=K_{e(s)}=C_{s(s)}=C_{e(s)}=1 \), the circuit can be simplified as the same as that of RIM. Therefore, it is very convenient to analyze the performance of SLIM in the same way as that of RIM based on the circuit [9].

### III. Optimal Design Equations of SLIM

As an example, this paper does optimal design on an SLIM, which was produced by a company and adopted in a linear metro but does not meet the anticipated requirement. According to their demand, a further optimization on original dimensions has been made. The efficiency and primary weight are regarded as objective functions under the given thrust as follows.

\[
\begin{align*}
\min f_1(x) &= -\eta = -\frac{P_2}{P_1} \\
\min f_2(x) &= G_{\text{teeth}} + G_{\text{york}} + G_{\text{winding}}
\end{align*}
\]

where \( P_1 \) is the input active power, \( P_2 \) is the output active power, \( G_{\text{teeth}} \) is the primary teeth weight, \( G_{\text{york}} \) is the primary yoke weight, and \( G_{\text{winding}} \) is the primary winding weight.

The following parameters are selected as the constraint variables, including efficiency (\( \eta \)), power factor (\( \cos \varphi \)), product of power factor and efficiency (\( \eta \cos \varphi \)), flux density in primary tooth (\( B_s \)), primary length (\( L \)), primary weight (\( W_p \)), thrust (\( F_s \)), vertical force (\( F_h \)), primary phase voltage (\( U \)), primary phase current (\( I \)), and primary conductor current density (\( J \)) [10].

The other parameters are taken as design variables, such as slip (\( \delta \)), stack height (\( h_s \)), slot depth (\( b_s \)), slot width (\( b_h \)), primary height (\( h_p \)), pole pitch (\( \tau \)), short pitch factor (\( \beta \)), and number of turns per phase in series (\( N_p \)) [11]. The parameters involving subscript “0” are original ones, and the remaining are the optimal results.

The constraint conditions are given by

\[
\begin{align*}
g_1(x) &= (\eta_0 - \eta) / \eta_0 \\
g_2(x) &= (\cos \varphi_0 - \cos \varphi) / \cos \varphi_0 \\
g_3(x) &= (\varphi_0 - \varphi) / \varphi_0 \\
g_4(x) &= (B_s - B_{10}) / B_{10} \\
g_5(x) &= (L - L_0) / L_0 \\
g_6(x) &= (W_p - W_{p0}) / W_{p0} \\
g_7(x) &= (F_s - F_{s0}) / F_{s0} \\
g_8(x) &= (F_h - F_{h0}) / F_{h0} \\
g_9(x) &= (U - U_0) / U_0 \\
g_{10}(x) &= (I - I_0) / I_0 \\
g_{11}(x) &= (J - J_0) / J_0
\end{align*}
\]

The above equations (20)-(31) are nonlinear functions influenced by many variables, such as structural parameters and electrical variables. They are solved by both genetic algorithm (GA) and mixture penalty function method. The GA can convert the nonlinear constraint functions into linear sub-problems so as to find a reasonable range of final results very quickly. The mixture penalty function can keep the solutions back to rationale scope by rectification when they are out of the constraint extent [11].

During the optimization process, the rated velocity is set as 36 km/h.

### IV. Optimal Results

#### TABLE I

<table>
<thead>
<tr>
<th>Schemes</th>
<th>Items</th>
<th>Original</th>
<th>Optimal 1</th>
<th>Optimal 2</th>
<th>Optimal 3</th>
<th>Optimal 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stack height (mm)</td>
<td>266</td>
<td>266</td>
<td>266</td>
<td>266</td>
<td>266</td>
<td></td>
</tr>
<tr>
<td>Primary length (mm)</td>
<td>1950</td>
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<td>2333</td>
<td>2333</td>
<td>2459</td>
<td></td>
</tr>
<tr>
<td>Primary depth (mm)</td>
<td>98.6</td>
<td>119.6</td>
<td>114.8</td>
<td>114.6</td>
<td>106.8</td>
<td></td>
</tr>
<tr>
<td>Slot width (mm)</td>
<td>15.87</td>
<td>19.3</td>
<td>19</td>
<td>18.8</td>
<td>19.6</td>
<td></td>
</tr>
<tr>
<td>Tooth width (mm)</td>
<td>8.4</td>
<td>10.2</td>
<td>10.4</td>
<td>10.6</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Slot depth (mm)</td>
<td>58</td>
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<td>79.1</td>
<td>74.2</td>
<td>63.6</td>
<td></td>
</tr>
<tr>
<td>Pole pitch (mm)</td>
<td>291.2</td>
<td>265.5</td>
<td>264.6</td>
<td>264.6</td>
<td>275.4</td>
<td></td>
</tr>
<tr>
<td>No. of slots</td>
<td>80</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>
The main design parameters of SLIM are listed in Table I. Four rational optimal schemes are gained, which are compared with the original one. The thrust curve is shown in Fig. 3. The efficiency, power factor, and product of power factor and efficiency are illustrated in Figs. 4-6, respectively.

In Fig. 3, the thrust is decided by working requirement, which can overcome the wind and friction resistances, at the same time also produce definite acceleration. Below the base velocity, the thrust is kept almost constant though the phase current increases. However, beyond the base speed, the phase voltage reaches its rate value but the total reactance continues to increase, so the phase current gradually decreases as well as the thrust.
From Fig. 4 to Fig. 6, we can find clearly the trend of efficiency, power factor and product of power factor and efficiency. After optimal design, the efficiency of new schemes has been improved in the whole operational range compared with the original scheme. However, the power factor in Fig. 5 encounters certain reduction in the optimal schemes. In order to attain high efficiency, the slot width, slot depth and pole pitch are changed greatly as indicated in Table I. Finally, the ratio of total resistance to reactance becomes smaller, which reduces the power factor. In Fig. 6, restrictions are made to the product of power factor and efficiency in the new schemes so as to ensure that the optimal results are not worse than the original one. 

In the linear drive transportation system, the inverter is placed on the vehicle. Under the same output power, the product of power factor and efficiency decides the converter rating power. That is to say, it will affect the weight of converter. Hence, it is significant to improve the efficiency while not decreasing the product of power factor efficiency. In this way, it can ensure the maximum neat load [8, 10].

Among the four new designs, the third is the best according to user demand. Afterwards, we make a new motor design based on the structural parameters in Table I. The performance curves are shown in Fig. 7.

In the motor design scheme, there are two regions: “constant current” and “constant power”. Below the base speed of 36 km/h, the primary phase current is kept constant (570 A), and the slip frequency is also constant (6 Hz). Due to power conditioner voltage limit, the constant voltage operation (with phase voltage of 306 V) is applied above base speed [11].

The phase current decreases very quickly when the resistance increases. In order to meet the operating requirement, it is necessary to increase the slip frequency linearly for avoiding quick reduction of thrust. The maximum slip frequency is 12.5 Hz at 115 km/h.

V. CONCLUSIONS

In this paper, maximum efficiency optimal design of SLIM for a subway is performed. By the optimal schemes, several positive conclusions can be drawn below. Firstly, when the number of poles is larger and the pole and slip are smaller, the efficiency will be improved. Secondly, the efficiency is contradictory to power factor. It is necessary to limit the product of power factor and efficiency so as not to increase the inverter weight under the same output power. The results indicate that the equivalent circuit and optimal methods are rational.

VI. ACKNOWLEDGEMENT

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